1. $\vec{r}(t) = (2\cos t, 2\sin t, t)$

$\vec{r}\,'(t) = (-2\sin t, 2\cos t, 1)$ $\quad \|\vec{r}\,'(t)\| = \sqrt{4\sin^2 t + 4\cos^2 t + 1}$

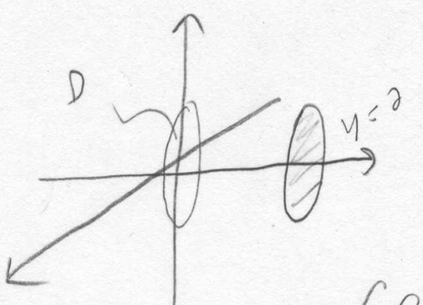
$$\vec{T}(t) = \left(\frac{-2\sin t}{\sqrt{5}}, \frac{2\cos t}{\sqrt{5}}, 1/\sqrt{5} \right) \qquad = \sqrt{5}$$

2. $$L = \int_0^{\pi/2} \sqrt{(3)^2 + (-2\sin t)^2 + (2\cos t)^2}\, dA$$

$$= \int_0^{\pi/2} \sqrt{9 + 4\sin^2 t + 4\cos^2 t}\, dA = \int_0^{\pi/2} \sqrt{13}\, dt$$

$$= \frac{\sqrt{13}\,\pi}{2}$$

3.



S: $h(x,z) = y = 2$

D: $x^2 + z^2 \le 2$

$0 \le r \le 2$

$0 \le \theta \le 2\pi$

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_D (2xy, 2y, 2xz) \cdot (-h_x, 1, -h_z)\, dA$$

$$= \iint_D (4x, 4, 2xz) \cdot (0, 1, 0)\, dA = \iint_D 4\, dA$$

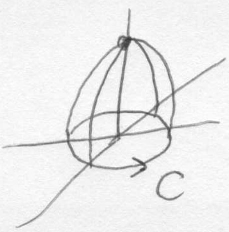
$$= 4\int_0^{2\pi}\int_0^{2} r\, dr\, d\theta = 4\int_0^{2\pi} \left(\frac{r^2}{2} \Big|_0^2 \right) d\theta = 4\int_0^{2\pi} 2\, d\theta = 8\int_0^{2\pi} d\theta$$

$$= 16\pi$$

orientée vers la gauche donc $\iint_S \vec{F} \cdot d\vec{S} = \boxed{-16\pi}$

4. On doit calculer $\int_C \vec F \cdot d\vec r$.

C: $\vec r(t) = (\cos t, \sin t, 0)$, $0 \le t \le 2\pi$

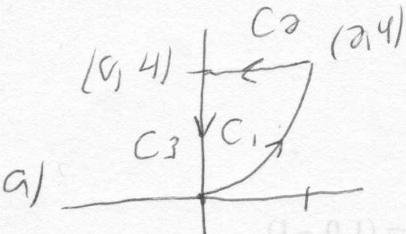


$$\int_C \vec F \cdot d\vec r = \int_0^{2\pi} (\cos^2 t \sin 0, \sin^2 t, \cos t \sin t) \cdot (-\sin t, \cos t, 0)\, dt$$

$$= \int_0^{2\pi} (0, \sin^2 t, \cos t \sin t) \cdot (-\sin t, \cos t, 0)\, dt$$

$$= \int_0^{2\pi} \cos t \sin^2 t\, dt = \frac{\sin^3 t}{3}\Big|_0^{2\pi} = 0$$

5.



a)

C_1: $\vec r(t) = (t, t^2)$, $0 \le t \le 2$

C_2: $\vec r(t) = (2,4) + t(-2,0)$, $0 \le t \le 1$
$= (2-2t, 4)$, $0 \le t \le 1$

C_3: $\vec r(t) = (0,4) + t(0,-4)$, $0 \le t \le 1$
$= (0, 4-4t)$

$$\int_{C_1} xy\,dx + y\,dy = \int_0^2 t^3\,dt + t^2\, 2t\,dt = \int_0^2 3t^3\,dt = \frac{3t^4}{4}\Big|_0^2 = 12$$

$$\int_{C_2} xy\,dx + y\,dy = \int_0^1 4(2-2t)(-2\,dt) + 4(0) = \int_0^1 -8(2-2t)\,dt$$
$$= \int_0^1 (-16+16t)\,dt = -16t + 8t^2\Big|_0^1 = -8$$

$$\int_{C_3} xy\,dx + y\,dy = \int_0^1 0 + (4-4t)(-4)\,dt = \int_0^1 (-16+16t)\,dt$$
$$= -16t + 8t^2\Big|_0^1 = -8$$

$$\boxed{\int_C xy\,dx + y\,dy = 12 - 8 - 8 = -4.}$$

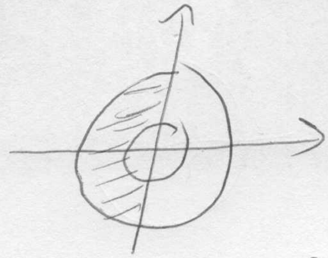
b) $P = xy$, $Q = y$

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \iint_D -x\,dA$$

$$= \cancel{\int_0^2 \int_{x^2}^4 -x\,dy\,dx} = \int_0^2 (-4x) - (-x^3)\,dx = \int_0^2 (x^3 - 4x)\,dx = \frac{x^4}{4} - 2x^2 \Big|_0^2 = 4 - 8 = -4$$

$$= \int_0^2 (-4x + x^3)\,dx = \left(-2x^2 + \frac{x^4}{4}\right)\Big|_0^2 = -4$$

6.



$1 \le r \le 3$

$\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$

$P = xy$

$Q = x + y$

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \iint_D (1 - x)\, dA = \int_{\pi/2}^{3\pi/2}\int_1^3 (1 - r\cos\theta)\, r\, dr\, d\theta$$

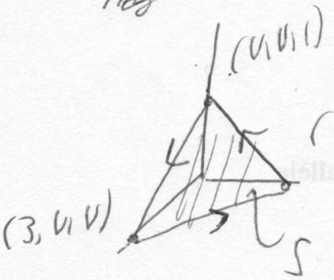
$$= \int_{\pi/2}^{3\pi/2}\int_1^3 (r - r^2\cos\theta)\, dr\, d\theta = \int_{\pi/2}^{3\pi/2} \left(\frac{r^2}{2} - \frac{r^3}{3}\cos\theta\right)\Big|_1^3 d\theta$$

$$= \int_{\pi/2}^{3\pi/2} \left(\frac{9}{2} - 9\cos\theta\right) - \left(\frac{1}{2} - \frac{1}{3}\cos\theta\right) d\theta = \int_{\pi/2}^{3\pi/2} \left(4 - \frac{26}{3}\cos\theta\right) d\theta$$

$$= \left(4\theta - \frac{26}{3}\sin\theta\right)\Big|_{\pi/2}^{3\pi/2} = \left(6\pi + \frac{26}{3}\right) - \left(2\pi - \frac{26}{3}\right)$$

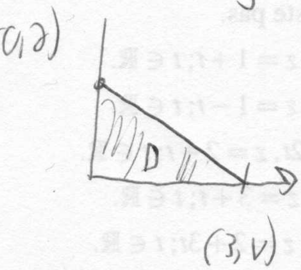
$$= 4\pi + \frac{52}{3}$$

7.



S: $6z = 6 - 2x - 3y$

$z = 1 - \frac{x}{3} - \frac{y}{2} = f(x,y)$



$0 \le x \le 3$

$0 \le y \le -\frac{2}{3}x + 2$

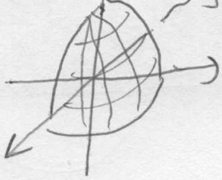
$$\text{Rot}\,\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \sin(x^2) & e^{y^2} + x^2 & z^4 + 2x^2 \end{vmatrix}$$

$$= (0, -4x, 2x)$$

$$\iint_S \text{Rot}\,\vec{F} \cdot d\vec{S} = \iint_D (0, -4x, 2x) \cdot (1/3, 1/2, 1)\, dA = \iint_D (-2x + 2x)\, dA$$

$$= \iint_D 0\, dA = 0$$

8)



$\underline{S}$: $0 \leq \rho \leq 2$

$0 \leq \theta \leq \pi/2$

$0 \leq \phi \leq \pi/2$

$\operatorname{div} \vec{F} = 2x + 0 + 3 = 2x+3$

$$\iint_S \vec{F} \cdot d\vec{S} = \iiint_E \operatorname{div} \vec{F} \, dV = \iiint_E (2x+3) \, dV$$

$$= \int_0^{\pi/2} \int_0^{\pi/2} \int_0^2 (2\rho \cos\theta \sin\phi + 3) \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

$$= \int_0^{\pi/2} \int_0^{\pi/2} \int_0^2 (2\rho^3 \cos\theta \sin^2\phi + 3\rho^2 \sin\phi) \, d\rho \, d\theta \, d\phi$$

$$= \int_0^{\pi/2} \int_0^{\pi/2} \left(\frac{\rho^4}{2} \cos\theta \sin^2\phi + \rho^3 \sin\phi \Big|_0^2 \right) d\theta \, d\phi$$

$$= \int_0^{\pi/2} \int_0^{\pi/2} (8 \cos\theta \sin^2\phi + 8 \sin\phi) \, d\theta \, d\phi$$

$$= \int_0^{\pi/2} \left(8 \sin\theta \sin^2\phi + 8\theta \sin\phi \Big|_0^{\pi/2} \right) d\phi = \int_0^{\pi/2} (8 \sin^2\phi + 4\pi \sin\phi) \, d\phi$$

$$= \int_0^{\pi/2} (4 - 4\cos 2\phi + 4\pi \sin\phi) \, d\phi$$

$$= (4\phi - 2\sin 2\phi - 4\pi \cos\phi) \Big|_0^{\pi/2} = (2\pi - 0 - 0) - (0 - 0 - 4\pi)$$

$$= 6\pi$$