Physics 206
Solutions
Assignment 1

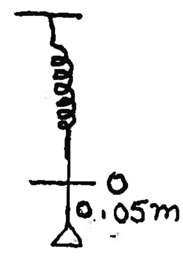
Q1 A body of mass 5 kg is suspended by a spring, which stretches 0.1 m when the body is attached. It is then displaced downward as additional 0.05m and released. Find the amplitude, period, and frequency of the resulting Simple Harmonic Motion.

(ans: A=0.05 m, T=0.635 s, f=1.57 Hz)

Solution:

Amplitude → maximum displacement →

$A = 0.05 m$



spring Constant → $k' = \frac{mg}{\Delta l} = \frac{(5 \times 9.8)}{(0.1)}$

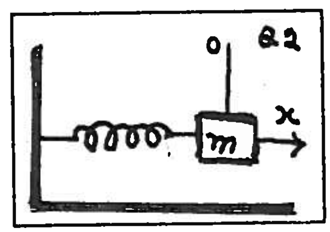
$= 490 N/m$

Time Period of Oscillation $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{5/490}$

$= 0.633 S.$

Frequency → $f = \frac{1}{T} = \frac{1}{0.633} = 1.57 Hz.$

Q 2a A spring is mounted as shown in the figure. By attaching a spring balance to the free end and pulling sideways, we determine that the force is proportional to the displacement, a force of 4.0 N causing a displacement of 0.02 m. We attach a 2 kg body to the end, pull it aside a distance of 0.04 m and released. Find the force constant of the spring. (ans: 200 N/m)



Solution:

Force Constant of the spring → $k = \frac{\text{Tension}}{\text{Extension}}$

$\therefore k' = \frac{F}{x} = \frac{4.0N}{0.02m} = 200 N/m$

2b Find the frequency of vibration. Time Period → $T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{2}{200}}$

$= 0.628 S$

$\therefore$ frequency → f → $f = \frac{1}{T} = \frac{1}{0.628} = 1.59 Hz.$

Angular frequency → $\omega = 2\pi f = \sqrt{\frac{k'}{m}} = 10 rad/s.$

2c Compute the maximum velocity attained by the vibrating body. ∴ Max velocity occurs at equilibrium position, where $x = 0$. velocity is given by $v = \pm\omega\sqrt{A^2 - x^2}$,

For $x = 0$, $v = v_{max} = \pm\omega A = (10) \times (0.04) = \pm 0.4 m/s.$

2d Compute the maximum acceleration. (ans: $\pm 4.0\ m/s^2$)

Acceleration $\rightarrow a$ is given by $a = -\frac{kx}{m} = -\omega^2 x$

Max. acceleration occurs at the end of path, where $x = \pm A$

$\therefore a_{max} = \pm \omega^2 A = (10)^2(0.04) = \pm 4\ m/s^2$

2e Compute the velocity and acceleration when the body has moved halfway in towards the centre from its initial position. (ans: v= -0.346 m/s, a= -2.0 m/s^2)

when the body has moved half way, $x = \frac{A}{2} = \frac{0.04}{2} = 0.02m$

$\therefore$ velocity at $(x = 0.02) \rightarrow v(0.02) = \pm 10\sqrt{(0.04)^2 - (0.02)^2} = \pm 0.35\ m/s$

Since the body is moving towards the Center, $v = -0.35\ m/s.$

and acceleration at $(x = 0.02m)$ is

At $x = 0.02$, $a = -\omega^2 x = -(10)^2(0.02) = -2.0\ m/s^2$

2f How much time is required for the body to move halfway into the centre from its initial position? (ans: $\pi/30$ s).

position at any time, t is given by,

$x = A\cos(\omega t + \phi)$, At $t = 0$, $x = A$, $\therefore A = A\cos\phi$ $\therefore \phi = 0$

$\therefore x = A\cos\omega t$

$\therefore$ At $x = A/2$, $A/2 = A\cos 10t$

or $10t = \cos^{-1}\frac{1}{2} = \pi/3$

$\therefore t = \pi/30\ s = 0.1\ s$

Q-3

The system in Q.2 is given an initial displacement of 0.05 m and initial velocity of 2m/s. Find the amplitude, the phase angle, and the total energy of the motion, and with an equation for the position as a function of time.

(ans: A=0.206 m, θ_0=-76° = -1.33 rad, E=4.25 J, x=(0.206) cos{(10)t-1.33 rad})

<u>Solution:</u>

S. H. M. is given by the equation:

$$x = A\cos(\omega t + \phi) \quad (1)$$

At $t = 0$, initial displacement $\rightarrow x_0 = 0.05\ m.$

$\therefore$ From (1) $x_0 = 0.05 = A\cos\phi$ (2) (Put $t = 0$ in (1))

Also, at $t = 0$, initial velocity $\rightarrow v_0 = 2\ m/s$

From (1) $v = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$ (3)

$\therefore \quad v_0 = 2 = -A\omega \sin\phi \quad (4) \quad (\text{Put } t=0 \text{ in } (3))$

From (2) and (4) we get

$$x_0^2 + \frac{v_0^2}{\omega^2} = A^2$$

$$\therefore \text{ Amplitude} \rightarrow A = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}$$

$$= \sqrt{(0.05)^2 + \left(\frac{2}{10}\right)^2} = 0.21\ m$$

Also from (2) and (4) we get

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = -\frac{v_0/\omega}{x_0} = \frac{-2/10}{0.05} = \frac{-2}{(10)(0.05)}$$

$$\therefore \phi = \tan^{-1}\left(\frac{-2}{(10)(0.05)}\right) = -76.0^\circ = -1.33\ rad$$

$\therefore$ Initial Phase Angle $\rightarrow \phi = -1.33$ rad.

and the position as a function of time is given as:

$$x = (0.21).\cos\{\underbrace{10t - 1.33}_{\text{radians}}\}$$

Total Energy $\rightarrow E =$ Kinetic. E + Potential energy

$$= \frac{1}{2} m v_0^2 + \frac{1}{2} k' x_0^2$$

$$= \frac{1}{2}(2)(2)^2 + \frac{1}{2}(200)(0.05)^2$$

$$= 4.25\ J$$

Note: If we take $x = A \sin(\omega t + \phi')$ as Eq(1) of S.H.M

$$x_0 = 0.05 = A \sin\phi'$$

$$\text{and } v_0 = 2 = A\omega \cos\phi'$$

$$\therefore \tan\phi' = \frac{2}{(10)(0.05)} \quad \text{and} \quad \phi' = 1.33\ rad.$$

$$\therefore \boxed{x = (0.21 \quad \sin(\omega t + 1.33\ rad)}$$

4a A certain string has a linear mass density of 0.25 g/m and is stretched with a tension of 25 N. One end is given a sinusoidal motion with frequency 5 Hz and amplitude 0.01m. At time t=0, the end has zero displacement and is moving in the positive y-direction. Find the wave speed, amplitude, angular frequency, period, wavelength, and wave number. (ans: v=10 m/s, A=0.01m, ω=31.4 rad/s, T=0.2 s, λ=2.0 m, k=3.1 4/m).

Solution: The wave speed in a stretched string is given by $\rightarrow$ $v = \sqrt{\frac{T}{\mu}}$ where $T \rightarrow$ tension in the string, $\mu \rightarrow$ mass per unit length

$$\therefore \quad v = \sqrt{\frac{25}{0.25}} = 10 \frac{m}{s}$$

The amplitude, A is just the amplitude of motion of the end point of the string i.e $A = 0.01m$.

$\therefore$ The angular velocity is $\rightarrow$ $\omega = 2\pi f = 2\pi(5) = 31.4 \frac{rad}{s}$.

Time Period $\rightarrow$ $T = \frac{1}{f} = 1/5 = 0.2S$

wavelength $\rightarrow$ $\lambda = \frac{v}{f} = \frac{10}{5} = 2m$

wave number $\rightarrow$ $k = \frac{2\pi}{\lambda} = \frac{2\pi}{2} = 3.14/m$

Also $k = \omega/v = \frac{31.4}{10} = 3.14/m$.

4b Write the wave function describing the wave. (ans: (0.01m) Sin{(3.14 /m) x- (31.4 rad/s) t}.)

wave function is given by $\boxed{y = A \sin(kx - \omega t)}$

$\because$ At $t=0$, $x=0$, y is positive
$\therefore$ $v_y = \frac{\partial y}{\partial t}$ must be positive

$\rightarrow y = A \sin(\omega t - kx)$ gives v_y +ve

$\therefore$ Wave function describing the wave with the Condition that at $t=0, x=0, y=+ve$ is given by, $\boxed{y = 0.01 \sin(31.4t - 3.14x)}$

4c Find the position of the point at x= 0.25 m, and t= 0.1 s. (ans 0.00707 m).

To find the position at point $x = 0.25\ m$ at $t = 0.1$, put the values of x and t in the wave equation.

$$\therefore y = 0.01 \sin(31.4 \times (0.1) - 3.14 \times 0.25)$$

$$= 0.01 \sin(2.355\ rad)$$

$$= +0.007\ m.$$

4d

Find the transverse velocity of the point, x=0.25 m at time t=0.1s. $\left(-0.22 \frac{m}{s}\right)$

$$y = 0.01 \sin(31.4t - 3.14x)$$

$$\frac{\partial y}{\partial t} = (0.01)(31.4)\cos(31.4t - 3.14x)$$

At $x = 0.25$ and $t = 0.1s$

$$\frac{\partial y}{\partial t} = (0.01)(31.4)\cos(31.4 \times 0.1 - 3.14 \times 0.25)$$

$$= 0.314 \cos(2.355)$$

$$= -0.22 \text{ m/s}.$$

4e

Find the slope of the string at the point x=0.25m at time t=0.1s. (ans: 0.022).

Slope at any point at any time is given by

$$\frac{\partial y}{\partial x} = \frac{\partial}{\partial x}\left[(0.01)\sin(-3.14x + 31.4t\right]$$

$$= -(0.01)(3.14)\cos(-3.14 \times .25 + 31.4 \times 0.1)$$

$$= (0.01)(3.14)\cos(+2.355) = 0.022$$

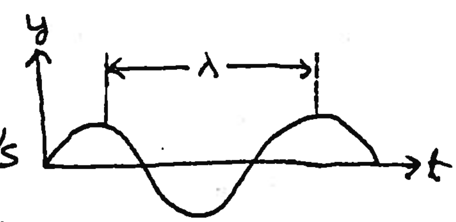
Q5

It is 16m from crest to crest in a system of waves. If 20 waves pass a given point each minute, find the speed of waves. (Ans: 5.33 m/s)

Solution $\lambda = 16 m$

Frequency $\rightarrow f = \frac{20}{60} = \frac{1}{3}$ Hz

Speed of wave $\rightarrow v = \lambda f = \frac{16}{3} = 5.33$ m/s



Q6

If the displacement along a wave is given by y=0.05 sin (20πt-0.1πx), where distances are in meters, and times in seconds, what are the amplitude, frequency, wavelength, and speed of the wave motion? (ans: A=0.05 m, t=10 s, λ=20 m, v=200m/s)

Solution: The wave equation is, $y = 0.05 \sin(20\pi t - 0.1\pi x)$

Compare it with $y = A\sin(\omega t - kx)$

$\therefore$ Amplitude $\rightarrow A = 0.05$ m

$\omega = 2\pi f = 20\pi$, $\therefore f = 10$ Hz

$k = \frac{2\cancel{\pi}}{\lambda} = 0.1\cancel{\pi}$, $\therefore \lambda = \frac{2}{0.1} = 20$ m

Speed $\rightarrow v = \lambda f = (20)(10) = 200$ m/s

Q7 The displacement of a particle in centimetres is given by y=8 Sin (2 πft). If f=20 Hz, find the displacement and the velocity of the particle at a time t=0.01 s and 0.07 s.
(ans: y=7.61 cm, v=3.11 m/s, y=4.70 cm, v=-8.13 m/s)

Solution: The S.H.M of the particle is given by

$$y = 8 \sin 2\pi f t \quad \text{—(1)}$$

The displacement for $t = 0.01\,s$ is for $f = 20\,Hz$,

$$y = 8 \sin (2)(\pi)(20)(0.01)$$
$$= 8 \sin 1.257 \text{ rad} = 7.61 \text{ Cm}$$

velocity of the particle → $v = \frac{dy}{dt} = \frac{d}{dt}[8 \sin 2\pi f t]$

$$v = (16\pi f) \cos 2\pi f t \quad (2)$$

For $t = 0.01\,S$, $v = 16\pi(20) \cos 2\pi(20)(0.01)$

$$= 320\pi \cos(1.257 \text{ rad}) = 311 \text{ Cm/S}$$
$$= 3.11 \text{ m/s}$$

For $t = 0.07\,S$, from (1), $y = 8 \sin 2\pi(20)(0.07)$

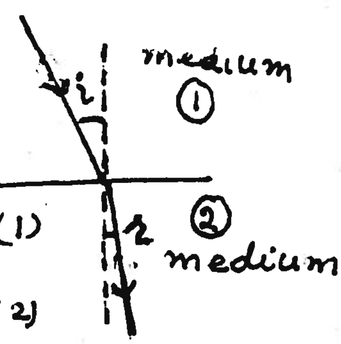
$$= 8 \sin 8.8 \text{ rad} = 4.70 \text{ Cm}$$

For $t = 0.07\,S$, from (2) $v = 320\pi \cos 2\pi(20)(0.07)$

$$= 320\pi \cos(8.8 \text{ rad})$$
$$= -813 \text{ Cm/s} = -8.13 \text{ m/S}$$

Q8 The speed of sound in water is 1500 m/s. A sound wave from an underwater explosion approaches the surface at an angle of incidence of 60°. What is the angle of refraction in air? (ans: 11.3°)

Solution: Refraction of wave (Sound)



refractive index of medium (2) with respect to medium (1) is given } → $n = \frac{\sin i}{\sin r} = \frac{v_1}{v_2}$ (1)

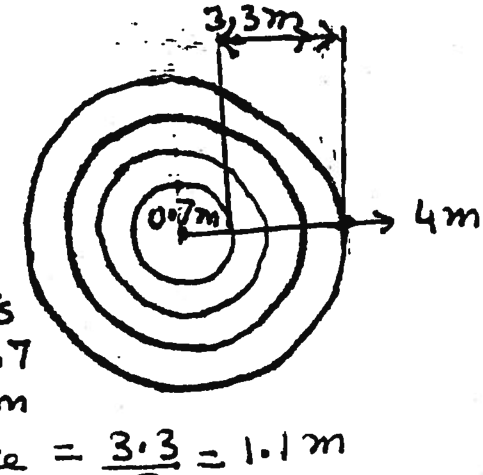
v_1 and v_2 are the speeds in med (1) and med (2)

From (1) $\frac{\sin 60^\circ}{\sin r} = \frac{v_w}{v_{air}} = \frac{1500}{340}$ $\quad v_{air} = 340 \text{ m/s}$

$$\therefore \sin r = (\sin 60^\circ)\left(\frac{340}{1500}\right) = 0.196$$
$$\therefore r = \sin^{-1} 0.196 = 11.3^\circ$$

Q9 A set of circular ripples is produced on the surface of a pond by throwing a stone into water. At a certain instant, the first crest is 4 m from the point where the stone hit the water, and the fourth crest is 70cm from the same point. What is the wavelength of the disturbance? (ans: 1.1 m)

Solution:



No. of crests between the first and fourth crests = 3

$\therefore$ No. of wavelengths between first and fourth crest = 3

Distance between 1st and 4th crests $= 4.0 - 0.7$
$= 3.3$ m

$\therefore$ wavelength of the disturbance $= \frac{3.3}{3} = 1.1$ m

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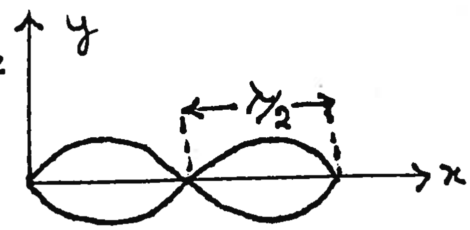
Q10 Standing waves are produced in a stretched rope. If the distance between successive modes is 0.04m, what is the wavelength? If the wave travels with a speed of 84m's, what is the frequency? (ans: λ=.08 m, f=1050Hz)

Solution: Distance between successive modes $= 0.04\text{ m} = \lambda/2$



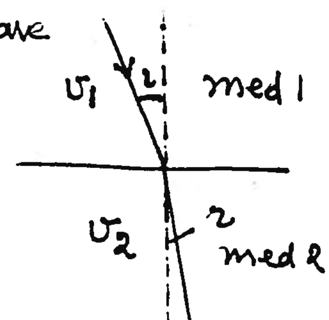
$\therefore$ wavelength $\rightarrow \lambda = 2 \times 0.04$
$= 0.08$ m

speed $\rightarrow v = \lambda f \quad \therefore \quad 84 = (.08) f$

$\therefore f = \frac{84}{0.08} = 1050$ Hz

Q11 When a sound wave is transferred from one medium to another, its frequency does not change. If the wavelength of a disturbance is 17cm in air, find its wavelength in water, steel, and brass if the speed of sound in water is 1450m/s, in steel 5000m/s, and in brass 3500m/s. (ans: 0.725 m, 2.5 m, 1.75 m).

Solution: Law of refraction of sound wave



$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{\nu \lambda_1}{\nu \lambda_2} = \frac{\lambda_1}{\lambda_2} \quad (1)$$

$\lambda_1 = \lambda_{air} = 0.17$ m (given)

$$\therefore \lambda_{water} = \frac{\lambda_{air} \times v_{water}}{v_{air}} = \frac{0.17 \times 1450}{340} = 0.725\text{ m}.$$

$v_{air} = 340$ m/s

Similarly, $\lambda_{steel} = \frac{0.17 \times 5000}{340} = 2.5$ m

$\lambda_{Brass} = \frac{0.17 \times 3500}{340} = 1.75$ m

Q12 .One end of a stretched rope is given a periodic transverse motion with a frequency of 10Hz. The rope is 50 m long, and has mass of 0.5 kg, and is stretched with a tension of 400 N. (a) Find the wave speed and the wavelength. (b) If the tension is doubled, how much the frequency be changed to maintain the same wavelength?
(ans: (a) 200 m/s, 20 m, (b) Increase by $2^{1/2}$)

Solution: (a) $l = 50\,m$ $\quad m = 0.5\,kg$

mass per unit length $\rightarrow \mu = \frac{0.5}{50} = 0.01\,kg/m$

Tension $\rightarrow T = 400\,N$

$\therefore$ wave speed $\rightarrow v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{400}{0.01}} = 200\,m/s$

wavelength $\rightarrow \lambda = \frac{v}{f} = \frac{200}{10} = 20\,m.$

(b) If T is doubled, $T' = 800\,N$

$\therefore v' = \sqrt{\frac{800}{0.01}} = 200\sqrt{2}$

i.e v' increases by $2^{1/2}$

Q13 The equation of a certain traveling transverse wave is $y = \overset{200}{\cancel{2}}\sin 2\pi\left(\frac{t}{0.01} - \frac{x}{30}\right)$, where x and y are in centimetres and t is in seconds. What are (a) the amplitude, (b) the wavelength, (c) the frequency, and (d) the speed of propagation of the wave? (ans: (a) 200 cm, (b) 30.0 cm, (c) 100 Hz, (d) 3000 cm/s).

Solution

The wave equation is

$$y = 200 \sin 2\pi\left[\frac{t}{0.01} - \frac{x}{30}\right]$$

$$= 200 \sin\left[\left(\frac{2\pi}{0.01}\right)t - \left(\frac{2\pi}{30}\right)x\right]$$

Comparing it with $y = A\sin[\omega t - kx]$, we get

(a) amplitude $\rightarrow A = 200\,cm.$

(c) $\omega = \cancel{2\pi} f = \frac{\cancel{2\pi}}{0.01}$, $\therefore f = 100\,Hz$

(b) $k = \frac{\cancel{2\pi}}{\lambda} = \frac{\cancel{2\pi}}{30}$ or $\lambda = 30\,cm.$

(d) speed $\rightarrow v = \lambda f = (30)(100) = 3000\,cm/s$

Q14 A steel wire 6 m long has a mass of 60 g and is stretched with a tension of 1000 N. What is the speed of propagation of a transverse wave in the wire? (ans: 316 m/s)

Solution:

Length of wire → $l = 6 m$

Mass of the wire → $m = 0.06 kg$

∴ mass per unit length → $\mu = \frac{0.06}{6} = 0.01 \frac{kg}{m}$

speed of propagation → $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{1000}{0.01}} = 316 m/s$

Q.15 The speed of sound in water is about 1480 m/s. Find the frequency of a sound wave such that its wavelength in water is the same as the wavelength in air at 20° C of a sound wave of frequency 1000 Hz. (ans: 4302 Hz)

Solution: Note:- The speed of sound at 0°c is 331.4 m/s. The speed of sound varies with temperature. If v_t is the speed of sound at temperature t°c and v_0 the speed at 0°c, then

$$v_t = v_0\sqrt{1 + \frac{t}{273}} \quad (1)$$

If T is the absolute temperature, then $T = t + 273$, and (1) can be written as

$$v_t = v_0\sqrt{\frac{T}{273}} \quad (2)$$

Solution:. spee of sound at 20°c → $v_{20}\sqrt{\frac{273+20}{273}}$

$$\therefore v_{20} = (331.4)\sqrt{\frac{293}{273}} = 343.3 m/s$$

For $f = 1000 Hz$ frequency, $\lambda_{20} = \frac{343.3}{1000}$ (3)

$\therefore \lambda_{water} = \frac{1480}{f}$ (4) If λ_{20} is to be equal to λ_{water}

$$\frac{1480}{f} = \frac{343.3}{1000} \therefore f = \frac{1480 \times 1000}{343.3} = 4311 Hz.$$

Q 16 A sinusoidal wave on a string is described by the equation: $y =$ (0.15m) sin($10\pi t - 3\pi x + \pi/4$), where x is in meters and t in seconds. If the mass per unit length of this string is 12.0 g/m, determine (a) the speed of the wave, (b) the wavelength, (c) the frequency, and (d) the power transmitted to the wave.

(ans: 3.33 m/s, 0.667m, 5.00 Hz, 0.444W)

<u>Solution:</u> The equation of wave is:

$$y = (0.15\,m)\,\mathrm{Sin}\left(10\pi t - 3\pi x + \frac{\pi}{4}\right) \quad (1)$$

Comparing this with the general wave equation,

$$y = A\,\mathrm{Sin}\,(\omega t - kx + \phi) \quad (2),$$

we get: $\left.\begin{array}{l}\omega = 10\pi \\ \text{or } 2\pi f = 10\pi\end{array}\right\}$, $\left.\begin{array}{l}k = 3\pi \\ \frac{2\pi}{\lambda} = 3\pi\end{array}\right\}$, $\phi = \pi/4$

$\therefore f = 5$ $\qquad \therefore \lambda = \frac{2}{3}$

$\therefore$ (a) Speed of wave $\rightarrow v = f\lambda = 5 \times \frac{2}{3} = 3.33$ m/s

$\therefore$ (b) wavelength $\rightarrow \lambda = 2/3 = 0.667$ m

(c) Frequency $\rightarrow f = 5$ Hz

(d) Power transmitted $\rightarrow P = \frac{1}{2}\mu\omega^2 A^2 v$

$$= \frac{1}{2}(0.012)(10\pi)^2(0.15)^2\left(\frac{10}{3}\right)$$

$$= 0.444\ W$$

Q 17 (a) Determine the speed of transverse waves on a string under tension of 80.0 N if the string has a length of 2.00 m and a mass of 5.00 g. (b) Calculate the power required to generate these waves if they have a wavelength of 16.0 cm, and an amplitude of 4.00 cm.

(ans: 179 m/s, 17.7 kW)

<u>Solution:</u> (a) $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{80}{(0.005/2.0)}} = 179$ m/s.

(b) $P = \frac{1}{2}\mu\omega^2 A^2 v = \frac{1}{2}\left(\frac{0.005}{2.0}\right)\left(2\pi \times \frac{v}{\lambda}\right)^2 (0.04)^2 (179)$

$$= \frac{1}{2}\left(\frac{0.005}{2.0}\right)\left(2\pi \times \frac{179}{0.16}\right)^2 (0.04)^2 (179)$$

$$= 1.77 \times 10^4\ W = 17.7\ kW$$