UNIVERSITY OF OTTAWA
Principles of Physics
PHY1321/31 Fall 2018
Dr. A. Czajkowski

STUDENT #: ____________________

NAME: ____________________

ASSIGNMENT 4:
Maxwell Boltzmann
Distribution Heat Engines,

Released: Oct 5, Due: Oct 12 6PM Sharp!

1 Fill the table below:

Molecule	Degrees of Freedom	AVG Energy of single molecule	Cv	Cp	Gamma
A	1	1/2kT	1/2R	3/2R	3
B	5	5/2kT	5/2R	7/2 R	7/5
C	3	3/2kT	3/2R	5/2R	5/3
D	11	11/2 kT	11/2R	13/2R	13/11
E	8kT	4 kT	4R	5R	5/4

2 Given is 1mole of oxygen molecules at atmospheric pressure and temperature of 20°C.

a) Write down the correct expression for N(400,402) , the number of molecules having speeds in the interval (400m/s 402/m/s).

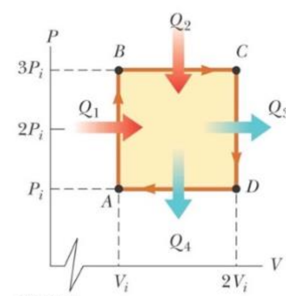
b) Calculate the number of molecules having their speed between 400m/s and 402m/s. (provide the number corresponding to part a (do not show your calculations!)

SOLUTION:

a) $P_v = \frac{N_v}{N} = 4\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}} dv 4\pi\left(\frac{M}{2\pi RT}\right)^{\frac{3}{2}} v^2 e^{-\frac{Mv^2}{2RT}} dv = 4\pi\left(\frac{0.032}{2\pi(8.31)293}\right)^{\frac{3}{2}} (401)^2 e^{-\frac{0.032401^2}{2(8.31)293}} (2)$

b) P_v=0.00425 $N_v \Longrightarrow P_v N = 0.0425 \cdot 6.022 \cdot 10^{23}$ =2.56$\cdot 10^{23}$

3 A 1.00-mol sample of a monatomic ideal gas is taken through the cycle shown. At point *A,* the pressure, volume, and temperature are P_i, V_i, and T_i, respectively. In terms of *R* and T_i, find (a) the total energy entering the system by heat per cycle, (b) the total energy leaving the system by heat per cycle, (c) the efficiency of an engine operating in this cycle,
(HINT: efficiency of the engine= |Work performed|/|Heat absorbed|
same temperature extremes.



At point *A*, $P_iV_i = nRT_i$ and $n = 1.00$ mol
At point *B*, $3P_iV_i = nRT_B$ so $T_B = 3T_i$
At point *C*, $(3P_i)(2V_i) = nRT_C$ and $T_C = 6T_i$
At point *D*, $P_i(2V_i) = nRT_D$ so $T_D = 2T_i$

The heat for each step in the cycle is found using $C_V = \frac{3R}{2}$ and $C_P = \frac{5R}{2}$

$$Q_{AB} = nC_V(3T_i - T_i) = 3nRT_i$$
$$Q_{BC} = nC_P(6T_i - 3T_i) = 7.50nRT_i$$
$$Q_{CD} = nC_V(2T_i - 6T_i) = -6nRT_i$$
$$Q_{DA} = nC_P(T_i - 2T_i) = -2.50nRT_i$$

(a)Therefore, $Q_{entering} = |Q_h| = Q_{AB} + Q_{BC} = \boxed{10.5nRT_i}$ (b) $Q_{leaving} = |Q_c| = |Q_{CD} + Q_{DA}| = \boxed{8.50nRT_i}$

(c) Actual efficiency, $e = \frac{|Q_h| - |Q_c|}{|Q_h|} = \boxed{0.190}$

4 Using Maxwell=Boltzmann Distribution of speeds for Ideal Gas obtain the Boltzmann Distribution of Energies for Ideal Gas. (Follow Lecture Discussions)
/Present your work on the opposite side of this page/

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ASSIGNMENT 4: CONT
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5 A 4 liter sample of a diatomic gas with γ =1.4 confined to a cylinder, is carried through a closed cycle. The gas is initially at 1.00atm. and 300K. First, its pressure is tripled under constant volume. Then it expands isothermally to its original pressure. Finally the gas is compressed isobarically to its original volume.

a) draw pV diagram of this cycle.
b) determine the volume of the end of the isothermal expansion.
c) find the temperature of the gas at the start of the isothermal expansion.
d) find the temperature at the end of the cycle.
e) what was the net work done on the gas for this cycle.

b) $p_1 = p_i = 101300Pa \quad V_1 = V_i = 0.004m^3 \quad T_1 = T_i = 300K$

$p_2 = 3p_i = 303900Pa \quad V_2 = V_i = 0.004m^3 \quad \underline{T_2 = 3T_i = 900K}$ as $p_2V_2 = nRT_2 \quad (3p_iV_i = nRT_2)$

$p_3 = p_i = 101300Pa \quad V_3 = 3V_i = 0.012m^3 \quad \underline{T_3 = 3T_i = 900K}$ as $p_2V_2 = nRT_2 \quad (3p_iV_i = nRT_2)$

c) Temperature does onto change when system goes from 2→ 3 so $T_3 = T_2 = 900K$

d) $T_f = T_i = 300K$

e) $W = W_{2\to3} + W_{3\to1}$

$$W = -nRT_2 ln\frac{V_3}{V_2} - p_3(V_1 - V_2) = -p_2V_2 ln\frac{V_3}{V_2} - p_3(V_1 - V_2) = -3p_iV_i ln\frac{V_3}{V_2} - p_3(V_1 - V_2)$$

$$W = -3p_iV_i ln\frac{V_3}{V_2} - p_3(V_1 - V_2) = -3(101300)(0.004)ln3 - 101300(0.004 - 0.012) = -525J$$

6 A refrigerator has a coefficient of performance of 4.00. The ice tray compartment is at –20.0°C, and the room temperature is 22.0°C. The refrigerator can convert 30.0 g of water at 22.0°C to 30.0 g of ice at -20.0°C each minute. What input power is required? Give your answer in watts.

The Power required is equal to $P_W = \frac{W}{\Delta t}$

For the refrigerator, the COP=$\frac{|Q_c|}{|W|}$

The total heat Q_c removed by the refrigerator is given by:

+

$$Q_c = Q_1 + Q_2 + Q_3 = mc_{ice}(-20 - 0) - mL + mc_{water}(0 - 22))$$
$$Q_c = -0.03 \cdot (20 \cdot 2090 + 333000 + 22 \cdot 4186) = -14007J$$
$$COP = \frac{|Q_c|}{|W|} \Longrightarrow |W| = \frac{14007J}{4} = 3502J \Longrightarrow P_W = \frac{W}{\Delta t} = \frac{3502J}{60s}$$

ANS: To accomplish this task the required power is 58.4 W