University of Ottawa
MAT1300-E. Fall 2018, Midterm Exam 2
Monday, November 19^{th}, 2018

First Name **Version A** ____________________________

Family Name **Solution** ____________________________

Do **not** write your student ID number on this front page. Please write your student ID number in the space provided on the second page.

Take your time to read the entire paper before you begin to write, and read each question carefully. Make note of the questions that you feel confident you can do, and then do those first: you do not have to proceed through the paper in the order given.

- You have 80 minutes to complete this exam.
- This is a closed book exam, and no notes of any kind are allowed.
- No calculators are allowed.
- This exam consists of 8 questions: 5 are multiple choice and 3 are long answer.
  - For the 5 multiple choice questions, only the chosen answer entered in the grid on the second page will be marked.
  - For the 3 long answer questions, the correct answer requires justification written legibly and logically: you must convince me that you know your solution is correct. Answer these questions in the space provided. Use the backs of pages if necessary.
- Cellular phones, unauthorized electronic devices or course notes (unless an open-book exam) are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam, academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam.

  By signing below, you acknowledge that you have ensured that you are complying with the above statement.

  Signature :____________________________

- Good Luck!

Student Number: ______________________, Total marks: _______________ out of 25

Grid to enter your multiple choice answers:

Question	1	2	3	4	5
Answer	D	D	A	E	C

Grid for the marker:

	Multiple Choice (10 marks)	Q6 (5 marks)	Q7 (5 marks)	Q8 (5 marks)	Total (25 marks)
Marks					

QUESTION 1. (2 points) Evaluate the definite integral

$$\int_4^9 \left(\frac{1}{\sqrt{x}} + 2 \right) \, dx.$$

Among the following answers, enter the correct one's letter into the grid on page 2.

A) 2 B) 4 C) 10

D) 12 E) 16

$$\begin{aligned}
\int_4^9 \left(\frac{1}{\sqrt{x}} + 2 \right) \, dx &= \int_4^9 (x^{-1/2} + 2) \, dx \\
&= (2x^{1/2} + 2x) \Big|_4^9 \\
&= (2 \cdot 3 + 18) - (2 \cdot 2 + 8) \\
&= 24 - 12 \\
&= 12.
\end{aligned}$$

The answer is D.

QUESTION 2. (2 points) Suppose $f'(x) = 3x^2 + 8x - 4$, and that $f(1) = 2$. Find $f(-1)$.

Among the following answers, enter the correct one's letter into the grid on page 2.

A) 0 B) 2 C) 4

D) 8 E) 10

$$\int (3x^2 + 8x - 4)\, dx = x^3 + 4x^2 - 4x + C.$$

Since $f(1) = 2$, we have

$$1^3 + 4 - 4 + C = 2,$$

or $C = 1$. In other words, $f(x) = x^3 + 4x^2 - 4x + 1$. Therefore

$$f(-1) = (-1)^3 + 4 + 4 + 1 = 8.$$

The answer is D.

QUESTION 3. (2 points) Suppose

$$F(x) = \int_0^x (e^t + 2t)\, dt.$$

Find $F'(1)$.

Among the following answers, enter the correct one's letter into the grid on page 2.

A) $e + 2$ B) 2 C) $e + 1$

D) 1 E) 0

By the Fundamental Theorem of Calculus (part 2), $F'(x) = e^x + 2x$. Therefore $F'(1) = e + 2$. The answer is A.

Question 4. (2 points) Compute the definite integral

$$\int_0^1 (x+1)e^{(x^2+2x)}\,dx.$$

Among the following answers, enter the correct one's letter into the grid on page 2.

A) $2e^3 - 1$ B) $e^3 - 1$ C) $e - 1$

D) $\frac{1}{2}(e-1)$ E) $\frac{1}{2}(e^3-1)$

Set $u = x^2 + 2x$. Then $du = (2x+2)dx = 2(x+1)dx$. Therefore $\frac{1}{2}du = (x+1)dx$.

$$\begin{aligned}\int (x+1)e^{(x^2+2x)}\,dx &= \int e^{(x^2+2x)} \cdot (x+1)dx \\ &= \frac{1}{2}\int e^u\,du \\ &= \frac{1}{2}e^u + C = \frac{1}{2}e^{(x^2+2x)} + C.\end{aligned}$$

So

$$\int_0^1 (x+1)e^{(x^2+2x)}\,dx = \frac{1}{2}e^{(x^2+2x)}\Big|_0^1 = \frac{1}{2}(e^3-1).$$

The answer is E.

Question 5. (2 points) A robot is programmed to travel at the speed of the velocity of

$$v(t) = 3t^2 + 2t + 1$$

(meters per second). How far does it travel during its first 3 seconds?

Among the following answers, enter the correct one's letter into the grid on page 2.

A) 29 meters B) 34 meters C) 39 meters

D) 44 meters E) 49 meters

The distance is given by

$$\int_0^3 (3t^2+2t+1)\,dt = (t^3+t^2+t)\Big|_0^3 = 27+9+3 = 39.$$

The answer is C.

Question 6. (5 points) Suppose the function $y = f(x)$ is defined implicitly by

$$x^2 + 2xy - 3y = 5.$$

Find the equation of the tangent line to the graph determined by the above equation at $(2, 1)$

$$\begin{aligned} 2x + 2\frac{d}{dx}(xy) - 3\frac{dy}{dx} &= 0 \\ 2x + 2(y + x\frac{dy}{dx}) - 3\frac{dy}{dx} &= 0 \\ 2x + 2y + 2x\frac{dy}{dx} - 3\frac{dy}{dx} &= 0 \\ (2x-3)\frac{dy}{dx} + (2x + 2y) &= 0. \end{aligned}$$

Solving for $\frac{dy}{dx}$, we get

$$\frac{dy}{dx} = -\frac{2x+2y}{2x-3}. \qquad \text{(3 points up to here)}$$

At the point $(2, 1)$, we have

$$\frac{dy}{dx} = -\frac{4+2}{4-3} = -6. \qquad \text{(1 more point here)}$$

The tangent line is given by

$$y = -6x + b.$$

Plugging in $(2, 1)$, we get $b = 13$. Therefore

$$y = -6x + 13.$$

QUESTION 7. (5 points)

a) (3 points) Find the antiderivative

$$\int \frac{x^2}{x^3+5}\,dx.$$

b) (2 points) Find the antiderivative

$$\int \frac{x^3+5}{x^2}\,dx.$$

a) Set $u = x^3 + 5$. Then $du = 3x^2dx$, or $\frac{1}{3}du = x^2dx$.

$$\begin{aligned}\int \frac{x^2}{x^3+5}\,dx &= \int \frac{1}{x^3+5}\cdot x^2dx = \frac{1}{3}\int \frac{1}{u}\,du \\ &= \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|x^3+5| + C.\end{aligned}$$

b)

$$\int \frac{x^3+5}{x^2}\,dx = \int \left(x + \frac{5}{x^2}\right)\,dx = \frac{x^2}{2} - \frac{5}{x} + C.$$

QUESTION 8. (5 points) Consider a rectangle with length 12 meters and width 8 meters. The length is increasing at a rate of 4 meters per minute, and the width is decreasing at a rate of 3 meters per minute.

a) (2 points) Is the perimeter of the rectangle increasing or decreasing? How fast is the perimeter changing? (*the perimeter is the total length of all four sides of the rectangle.)

b) (3 points) Is the area of the rectangle increasing or decreasing? How fast is the area changing?

Write your answer in the form "**the perimeter (or area) is increasing (or decreasing) at the rate of ...**"

Let L be the length and W be the width. These are functiosn of time t. We are given $\frac{dL}{dt} = 4$ and $\frac{dW}{dt} = -3$.

a) Write P for the perimeter. Then $P = 2L + 2W$.

$$\frac{dP}{dt} = 2\frac{dL}{dt} + 2\frac{dW}{dt} = 2 \cdot 4 + 2 \cdot (-3) = 2.$$

The perimeter is increasing at the rate of 2 meters per minute.

b) Write A for the area. Then $A = LW$.

$$\frac{dA}{dt} = \frac{dL}{dt} \cdot W + L \cdot \frac{dW}{dt} = 4 \cdot 8 + 12 \cdot (-3) = -4.$$

The area is decreasing at the rate of 4 square meters per minute.

Space for additional work