• $MSE = \frac{SSE}{\underset{(3)}{(b-1)}\underset{(2)}{(K-1)}} = \frac{0.9933277}{6} = 0.1655546$

• $F_T = \frac{MSTr}{MSE} = \underline{21.523624}$ $F_B = \frac{MSB}{MSE} = \underline{41.188179}$

ANOVA Table

Source of Variation	df	SS	MS	F
Treat.'s	2	7.126667	3.563335	21.523624
Blocks	3	21.9466669	7.315563	44.188179
Error	6	.9933277	.165546	
Tot.	11	30.066667		

$H_0: \mu_1 = \mu_2 = \mu_3$
H_a: @ least one $\mu \neq$; $\alpha = .01$

test stat: $F_T = \frac{MSTr}{MSE} = \underline{21.523624}$

R.R - Reject H_0 if $F_T > F_\alpha(\overset{\text{from MSTr}}{K-1}, \overset{\text{from MSE}}{(b-1)(K-1)}) = F_{.01}(2, 6) = 10.92$

Since $F_T = 21.52 > 10.92$, we reject H_0 and conclude that @ 1% level of significance there is an evidence that the 3 couriers differ in their speed of delivery

- must do a follow-up analysis to find out which couriers differ.

Tukey's h.s.d. Multiple Comparisons

① Calculate $\binom{k}{2} = \binom{3}{2}$ = 3 pairs of $|\bar{y}_i - \bar{y}_j|$
for $H_0: \mu_i = \mu_j$ $i,j = 1,2,3$
$H_a: \mu_i = \mu_j$ $i \neq j$

$\bar{y}_1 = \frac{T_1}{4} = 4.65$ $\bar{y}_3 = \frac{T_3}{4} = 6.5$ $\bar{y}_2 = \frac{T_2}{4} = 5.25$

$|\bar{y}_1 - \bar{y}_2| = .6 \not> 1.2877879 \Rightarrow \mu_1 = \mu_2$
$|\bar{y}_1 - \bar{y}_3| = 1.85 > 1.2877879 \Rightarrow \mu_1 \neq \mu_3$
$|\bar{y}_2 - \bar{y}_3| = 1.25 \not> 1.2877879 \Rightarrow \mu_2 = \mu_3$

i.e. there are differences in speed of delivery b/wn carriers A & C

② $h.s.d = q_\alpha[K, \text{error}]\sqrt{\frac{MSE}{2}\left(\frac{1}{n_i} + \frac{1}{n_j}\right)} = q_\alpha[K, (b-1)(k-1)]\sqrt{\frac{MSE}{b}}$

$= q_{.01}[3,6]\sqrt{\frac{.1655546}{4}} = 6.33\sqrt{\cdots} = \underline{1.2877879}$

If on ass. or test end prob here

Is blocking needed?

② $H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4$ (i.e. blocking is not needed); $\alpha = .01$
H_a: @ least one $\beta \neq$ (i.e. blocking is beneficial)

only have to do if question asks for block test.

test-stat F_B test $= \frac{MSB}{MSE} = \underline{44.188179}$

R.R. We reject H_0 if $F_B > F_\alpha(b-1, (b-1)(k-1)) = F_{.01}(3,6)$
$= 9.78$

Since $F_B = 44.188179 > 9.78$ we reject H_0 and conclude that @ 1% level of significance there is an evidence that blocking is beneficial, hence R.B.D & can be used

Don't have to do unless asked for in the question

Comparing RBD to CRD
(i.e Relative Efficiency (RE)

Let MSE_{RB} and MSE_{CR} denote MSE's under RBD + CRD respectively. One way of measuring the precision is to calc. the variance of $(\bar{y}_i)$

- for RBD the variance of treat. means
$$var(\bar{y}_i) = \frac{MSE_{RB}}{b}$$

- for CRD $var(\bar{y}_i) = \frac{MSE_{CR}}{r}$, where r = # of replications of observ.'s of each treat. that are required to satisfy the following relationship

$$\frac{MSE_{RB}}{b} = \frac{MSE_{CR}}{r}$$

$$\Rightarrow \left(\frac{r}{b}\right) = \frac{MSE_{CR}}{MSE_{RB}}$$

R.E. relative efficiency

$Kb = n$

Computational Form.

$$R.E. = \frac{(b-1)MSB + b(K-1)MSE_{RB}}{(Kb-1)\,MSE_{RB}}$$

Ex cont.'d

$$R.E. = \frac{3(7.3155563) + 4(2)(.1655546)}{(11)(.165546)} = 12.778594 = 13$$

i.e approx. 13 times as many observ.'s in each treat. would be needed if we were to use CRD instead of RBD in order to obtain same precision for comparison of treat. means

Chap. 15.7 - Non-Parametric Approach to RBD (Friedman Rank Test)

- If assumption of normality is not satisfied
- We work w/ ranks
- We rank the observ.'s from smllest to lrgst. w/in each block
- Tied observ.'s will rec. the average rank

Assumptions:

1. RBD ✓ ~~treat~~
2. Each treat.-block combo. has distribution w/ approx. the same shape + the same spread
3. Blocks + treat.'s do not interact

$H_0: md_1 = md_2 = \dots = md_k$; α

H_a: @ least one md ≠

Test stat: $F_R = \frac{12}{bk(k+1)} \left[\sum_{i=1}^{k} T_{Ri}^2 \right] - 3b(k+1)$

where:

T_{Ri} = sum of ranks in each treat.

✗ Check: $\sum_{i=1}^{k} T_{Ri} = \frac{bk(k+1)}{2}$ # of blocks must be ≥ 5

RR: Reject H_0 if $F_R > \chi^2_{\alpha;(k-1)}$ ← # of treat.'s

Ex: con't

Assume

1. RBD ✓
2. Distributions of each courier + time of day have approx the same shape + the same spread.
3. Couriers + times of day do not interact

Couriers Treatments

Time of Day	A	B	C
9:30am	3.6 (1)	4.2 (2)	5 (3)
11:30am	5.4 (1)	5.8 (2)	7 (3)
12:30 pm	6.1 (1)	7 (2)	9.1 (3)
2:00 pm	3.5 (1)	4 (2)	4.9 (3)
	$T_{R1}=4$	$T_{R2}=8$	$T_{R3}=12$

blocks

Check $\Sigma T_{Ri} = 4+8+12 = 24$

$= \frac{bk(k+1)}{2} = \frac{12(4)}{2} = 24$ ✓

from email

H_0: No difference in speed (i.e. times of delivery for 3 couriers ($Md_A = Md_B = Md_C$)

H_a: not all of them the same; $\alpha = .01$

Test stat: $F_R = \frac{12}{bk(k+1)}\left[\sum_{i=1}^{3} T_{Ri}^2\right] - 3b(k+1) =$

$= \frac{12}{(4\cdot3)(4)}\left[4^2+8^2+12^2\right] - 3(4)(4) =$

$= .25(224) - 48 = 56 - 48 = \underline{8}$

✗ Check: $\sum_{i=1}^{3} T_{Ri} = 4+8+12 = 24$

$\frac{bk(k+1)}{2} = \frac{12(4)}{2} = 24$

- We reject H_0 if $F_R > \chi^2_{(k-1);\alpha} = \chi^2_{(2).01} = 9.210$

Since our $F_R = 8 \not> 9.210$, we do not reject H_0 and conclude that @ 1% level of significance there is no evidence to indicate that the delivery times of 3 different couriers are different.

- If we used $\alpha = .05 \Rightarrow \chi^2_{(2);.05} = 5.991$

$\Rightarrow F_R = 8 > 5.991 \Rightarrow$ Reject H_0 and @ 5% level of significance there are differences among 3 couriers.

- Which couriers are different?

Nemenyi's Procedure

- a post-hoc multiple comparison procedure used after Friedmans test to declare which sample medians are not equal

- have to make $\binom{K}{2} = \frac{K(K-1)}{2}$ comparisons

① compute average rank $\bar{R}_j$ for each of the K groups i.e. $\bar{R}_j = \frac{TR_i}{n_j}$ $(j = 1 \ldots K)$

② critical range $= \underbrace{q_\alpha(K, \infty)}_{\text{upper-tailed critical value from a studentized range dist}} \sqrt{\frac{K(K+1)}{12b}}$

③ if $|\bar{R}_i - \bar{R}_j| >$ critical range $\Rightarrow$

Reject $H_0: Md_i = Md_j$ and conclude that their popul's have different medians. (i.e. they differ)

Ex con'd

$H_0: Md_i = Md_j$ for $i, j = A, B, C$; $\alpha = .05$
$i \neq j$

$K = 3 \Rightarrow \binom{3}{2} = 3$, i.e. 3 comparisons

$b=3$
$m_1=m_2=m_3$
$=4=b$

$\bar{R}_1 = \frac{T_{R1}}{m_1} = \frac{4}{4} = 1$

$\bar{R}_2 = \frac{T_{R2}}{m_2} = \frac{8}{4} = 2$

$\bar{R}_3 = \frac{T_{R3}}{m_3} = \frac{12}{4} = 3$

and $C.R. = \underbrace{q_\alpha(k,\infty)}\sqrt{\frac{k(k+1)}{12b}} = 1.655$

$= \sqrt{\frac{3(4)}{12(4)}} = .05$

$q_{.05}(3,\infty) = 3.31$

$\Rightarrow |\bar{R}_1 - \bar{R}_2| = 1 \not> 1.655 \Rightarrow md_A = md_B$

$|\bar{R}_1 - \bar{R}_3| = 2 > 1.655 \Rightarrow md_A \neq md_C$

$|\bar{R}_2 - \bar{R}_3| = 1 \not> 1.655 \Rightarrow md_B = md_C$

Notes

- Non-parametric tests are more powerful (greater proba of rejecting a false null hypothesis) than the F-test. if the assumptions for F-test violated
- However, if F-test assumptions reasonably satisfied then F-test is more powerful & hence should be used.

i.e.

if the assumptions are satisfied then the F test is better @ detecting small differences b/wn the popu^n means than the non-parametric test.

Chap 14 - Categorical Data Analysis

- Variable of interest are of qualitative (i.e. categorical) type.

eg.1: pop. classified by sex (M,F) ← Binomial Distr.

eg.2: Pop. classified by marital status (S,M,D,W)
← multinomial dist.

eg.3 Pop. classified by:
income (<\$20K, \$20-\$50K, \$50-\$100K, >\$100K)
nationality (English, French, etc.)

r x c contingency tables

(I) Binomial Dist. (eg1)

1. fixed # of trials
2. trials are indep. of each other
3. only 2 possible outcomes "success" or "failure"
4. P["success"] = p and it remains constant from trial to trial, same for P["failure"] = q (w/ q = 1 - p)
5. if X is a r.v., such that X = counts # of "successes" in n trials

$\Rightarrow X \sim Bin(n; p)$
with $E(X) = np$
$V(X) = npq$

(II) Multinomial Dist. (eg2)

→ is an extension of Binomial ~~dist~~ distr. w/ having K possible outcomes

O_1	O_2	. . . . .	O_K	→ counts
1	2	. . . .	K	→ category
p_1	p_2		p_K	→ prob.

① fixed # of trials (n)
② trials are indep. of each other
③ trials result in K possible outcomes
(when $K=2$ ⇒ Binomial distr.)
④ P[falling in category i] $= p_i$ remains constant
from trial to trial. and $\sum_{i=1}^{K} p_i = 1$
i.e. laws of prob apply
⑤ let $O_1, O_2 \ldots O_K$ are random variables,
where O_i = counts # of observ.'s in category i
@ the end of n trials
and $\sum_{i=1}^{K} O_i = n$

⇒ if these 5 conditions are @ least
reasonably satisfied, we have a
multinomial distr.

- here we are interested in testing whether
the proportions p_i in each category differ
from a specific value or not.

i.e. $H_0: p_1 = p_{10}, p_2 = p_{20}, \ldots, p_K = p_{K0}$, α
H_a: @ least one $p_i = p_{i0}$

test stat $X^2 = \sum_{i=1}^{K} \frac{(O_i - E_i)^2}{E_i}$ → "Pearson Chi-square test-statistic"

where O_i = observed counts (i.e observed # of
observations in category i @ the end
of n-trials)

E_i = expected counts (i.e. # of observations
in category i we would expect to have
@ the end of n trials)

R.R. - We reject H_0 if $X^2 > X^2_{\alpha;(k-1)}$

Assumptions:

① A random sample taken from

② multinomial distr.

③ n is lrg enough for X^2 to apply (since X^2 is very sensitive to departures from normality we need $n > 30$ so that C.L.T. applies; i.e we have approx. normality)

Ex1 Handout - Heating Gas Payments

	Accnts				
	Fully paid	1 mnth behind	2 mnth behind	>2mnths behind	Total
Observed	287	49	30	34	400
Expected np_{io}	$np_{10}=400(.8)$ $=320$	$np_{20}=400(.1)$ $=40$	$np_{30}=400(.06)$ $=24$	$np_{40}=400(.04)$ $=16$	400

H_0: $p_1 = .8,\ p_2 = .1,\ p_3 = .06,\ p_4 = .04$; $\alpha = .01$

H_a: @ least one $p_i \neq p_{io}$

Assume: ① random sample ✓ (given)

② from multinomial dist.

③ n lrg ($n=400$) for X^2 to apply

Test stat. $X^2 = \sum_{i=1}^{4} \frac{(O_i - E_i)^2}{E_i} = \frac{(287-320)^2}{320} + \frac{(49-40)^2}{40} + \frac{(30-24)^2}{24} + \frac{(34-16)^2}{16} = \underline{27.178}$

R.R. We reject H_0 if $X^2 > X^2_{\alpha;(k-1)} = X^2_{.01(3)} = 11.34$

Since $X^2 = 27.178 > 11.34$, we reject H_0 and conclude that @ 1% significance level there is an evidence that this winters pattern of pay.s differs from the historical norm.

end of email