Ex con'd

① Calc. $\binom{K}{2} = \frac{K(K-1)}{2}$ pairs of $|\bar{y}_i - \bar{y}_j|$ for $H_0: \mu_i = \mu_j$
$H_a: \mu_i \neq \mu_j$
$i, j = 1 \ldots K$
$i \neq j$

$\binom{4}{2} = 6$ pairs of $|\bar{y}_i - \bar{y}_j|$

$\mu_1 = \mu_2 \Leftarrow$	$\lvert\bar{y}_1 - \bar{y}_2\rvert$	$= 2.76 \not> 7.633862$	$\bar{y}_1 = \frac{T_1}{n_1} = 75.67$
$\mu_1 = \mu_3 \Leftarrow$	$\lvert\bar{y}_1 - \bar{y}_3\rvert$	$= 4.84 \not> 7.9220518$	$\bar{y}_2 = \frac{T_2}{n_2} = 78.43$
$\mu_1 \neq \mu_4 \Leftarrow$	$\lvert\bar{y}_1 - \bar{y}_4\rvert$	$= 12.08 > 8.8571232$	$\bar{y}_3 = \frac{T_3}{n_3} = 70.83$
$\mu_2 = \mu_3 \Leftarrow$	$\lvert\bar{y}_2 - \bar{y}_3\rvert$	$= 7.6 \not> 7.633862$	$\bar{y}_4 = \frac{T_4}{n_4} = 87.75$
$\mu_2 \neq \mu_4 \Leftarrow$	$\lvert\bar{y}_2 - \bar{y}_4\rvert$	$= 9.32 > 8.6003403$	$\alpha = 10\%$ $K = 4$
$\mu_3 \neq \mu_4 \Leftarrow$	$\lvert\bar{y}_3 - \bar{y}_4\rvert$	$= 16.92 > 8.8571232$	$n = 23$

② $l.s.d = t_{\alpha/2;(n-K)} \sqrt{mse\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$ $n_i + n_j$ chng w/ diff. prs.
$= t_{.05;(19)} \sqrt{72.980576605\left(\frac{1}{6} + \frac{1}{7}\right)}$
1.729

③ Here we reject H_0 if $|\bar{y}_i - \bar{y}_j| > Lsd$

i.e. there's a difference btwn teaching techniques 1 + 4, 2 + 4, and 3 + 4 → (We don't state level of significance.

Use this if not specified on test

Tukey's honest Significant Difference (h.s.d)
- We use as default multiple comparison method
- post-hoc procedure (ie we've seen the data, we just need to find the difference)
- $h.s.d = q_\alpha(K, error) \sqrt{\frac{mse}{2}\left(\frac{1}{n_i} + \frac{1}{n_j}\right)} = q_\alpha(K, n-K) \sqrt{\frac{mse}{2}\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$

critical value from studentized Range dist.
($\alpha = 1\%$ or 5%, any other α from SAS)

3.149

i.e. $hsd = \underbrace{q_{.01}(4,19)}_{\text{from SAS } 3.474} \sqrt{\frac{62.98057605}{2}\underbrace{\left(\frac{1}{} + \frac{1}{}\right)}_{\text{changes w/ pair } (n_i)}}$

* $\not> 10.845589 \Rightarrow \mu_1 = \mu_2$
* $\not> 11.25531 \Rightarrow \mu_1 = \mu_3$
* $\not> 12.58382 \Rightarrow \mu_1 = \mu_4$
* $\not> 10.845889 \Rightarrow \mu_2 = \mu_3$
* $\not> 12.218994 \Rightarrow \mu_2 = \mu_4$
* $> 12.58382 \Rightarrow \underline{\mu_3 \neq \mu_4}$

*must be @ least one difference or there is an error here or in the main test.

Multiple Comparisons

- Fisher's L.S.D
- Tukey's hsd ← default
- Bonferroni

} post-hoc procedures

1. Calculate $\binom{K}{2}$ pairs of $|\bar{y}_i - \bar{y}_j|$ for $H_0: \mu_i = \mu_j$, $(i \neq j) = 1 \ldots K$; $H_a: \mu_i \neq \mu_j$
2. critical region (C.R)
3. Reject $H_0: \mu_i = \mu_j$ if $|\bar{y}_i - \bar{y}_j| > C.R.$

Bonferroni Multiple Comparisons

- here each individual ~~test~~ is done @ some α^* level of signif, $\left(\alpha^* = \frac{\alpha}{\binom{K}{2}}\right) \Rightarrow$ the overall level of significance is $= \alpha$ due to Boole's inequality

- let $A_1 =$ event of rejecting $H_0: \mu_1 = \mu_2$ | (it should not be rejected)
  $A_2 =$ " $H_0: \mu_1 = \mu_3$ | "
  $A_{\binom{K}{2}} =$ " ..... " $H_0: \mu_{K-1} = \mu_K$ | "

- Proba of $A_1 = \alpha^* = \frac{\alpha}{\binom{K}{2}}$

$A_2 = \alpha^*$

$\vdots$

$A_{\binom{K}{2}} = \alpha^*$

not prt of course ※ Boole's Inequality $P\left[\bigcup_{i=1}^{K}\right] \leq \sum_{i=1}^{K} P[A_i]$

$$\underbrace{P\left[A_1 \cup A_2 \cup \ldots \cup A_{\binom{K}{2}}\right]}_{} \leq P(A_1) + P(A_2) + \ldots + P\left(A_{\binom{K}{2}}\right)$$

$$\frac{\alpha}{\binom{K}{2}} \quad \frac{\alpha}{\binom{K}{2}} \quad \ldots \quad \frac{\alpha}{\binom{K}{2}} \rightarrow \text{"K choose 2"}$$

performing simultaneous $\binom{K}{2}$ comparisons

$$\binom{K}{2} \text{ times } \frac{\alpha}{\binom{K}{2}} = \alpha$$

※

$$CR = t_{\frac{\alpha^*}{2}; n-K} \sqrt{mse\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

$$= t_{\frac{\alpha}{2\binom{K}{2}}; n-K} \sqrt{mse\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

Chap. 15.6 Non-parametric Approach to C.R.D

- when we cannot assume normality

$\downarrow$

- Krustal-Wallis Test

- It is based on ranks

i.e: we order the observations from smallest to largest

- we rank the observations w/ the smllst receiving rank(1) + the lrgst one receiving rank (n) "repeats"

- Assign average rank to each tied observ.

e.g. 2, 5, 6, 6, 6, 10, 31   n=7

(1) (2) (4) (4) (4) (6) (7)

(3) (4) (5)

(3+4+5)/3

Assumptions: ① C.R.D. (completely randomized design)

② Treat's come from pop's that are distributed w/ approximately the same shape (distribution) and the same spread (variance)

$H_0: Md_1 = Md_2 = \dots Md_k$; α

H_a: @ least one Md ≠

Test Stat: $H = \frac{12}{n(n+1)} \left[\sum_{i=1}^{k} \frac{Tr_i^2}{n_i} \right] - 3(n+1)$

where Tr_i = sum of the ranks on treatment i

$SST_r = \sum_{i=1}^{k} \frac{T_i^2}{n_i}$ } same

R.R. We Reject H_0 if $H > \chi^2_{\alpha;(k-1)}$ (# of treatments)

check (ranks) $\sum_{i=1}^{k} Tr_i = \frac{n(n+1)}{2}$

Ex. con'd Assume: ① C.R.D. (given in prob) ✓

② teaching techniques have distributions w/ approximately the same shape + spread

- $H_0 = Md_1 = Md_2 = Md_3 = Md_4$ (medians) ; $\alpha = .10$

H_a = @ least one Md ≠

Teaching Techniques $n_1=6$ $n_2=7$ $n_3=6$ $n_4=4$

I	II	III	IV
65(3)	75(9)	59(1)	94(23)
87(19)	69(5.5)	78(11)	89(21)
73(8)	83(17.5)	67(4)	80(14)
79(12.5)	81(15.5)	62(2)	88(20)
81(15.5)	72(7)	83(17.5)	$TR_4=78$
69(5.5)	79(12.5)	76(10)	
$TR_1=63.5$	90(22)	$TR_3=45.5$	
	$TR_2=89$		

$\frac{12+13}{2}=12.5$

$\frac{15+16}{2}=15.5$

$\frac{5+6}{2}=5.5$

276

Check: $\sum_{i=1}^{4} TR_i = \frac{n(n+1)}{2}$ $63.5+89+45.5+78=276$ ✓

$\frac{n(n+1)}{2} = \frac{23(24)}{2} = 276$ ✓

Test Stat: $H = \frac{12}{n(n+1)}\left[\sum_{i=1}^{4}\frac{TR_i^2}{n_i}\right] - 3(n+1) =$

$= \frac{12}{23(24)}\left[\frac{(63.5)^2}{6} + \frac{(89)^2}{7} + \frac{(45.5)^2}{6} + \frac{(78)^2}{4}\right] - 3(24) =$

$= 79.775103 - 72 = \underline{\underline{7.775103}}$

R.R - Reject H_0 if $H > \chi^2_{\alpha,(k-1)} = \chi^2_{.1,3} = 6.251$

Since $H = 7.775103 > 6.251$

We reject H_0 & conclude that @ 10% level of significance there is an evidence that there are differences among 4 teaching techniques.

- must do a follow up analysis using multiple comparisions
  - Dunn's Procedure (not in book)

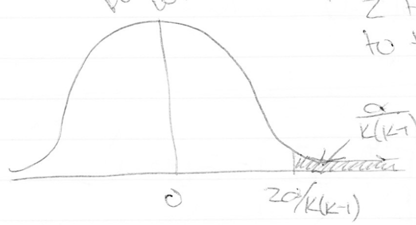
- post-hoc procedures i.e. (after we've seen data & we know there is a difference in treat. i.e.) Now we need to locate that difference.

① Calc $\binom{k}{2} = \frac{k(k-1)}{2}$ pairs of $|\bar{R}_i - \bar{R}_j|$ | for testing $H_0: Md_i = Md_j$
$H_a: Md_i \neq Md_j$
$i = 1 \ldots k$
$i \neq j$

where $\bar{R}_i = \frac{T_{R_i}}{n_i}$ (→ average) ie. is a average rank of treatment.

② Calc. crit region $\underbrace{z_{\alpha/k(k-1)}}_{\text{Not in book posted on the website}} \left(\sqrt{\frac{n(n+1)}{12}} \cdot \left(\frac{1}{n_i} + \frac{1}{n_j}\right)\right)$

crit. pnt. from upper tail z tables w/ the area of $\alpha/k(k-1)$ to the right of the crit pnt.



→ use same as stand. norm table

③ We reject $H_0: Md_i = Md_j$ is $|\bar{R}_i - \bar{R}_j| > C.R.$

Ex. Con'd

$\bar{R}_1 = \frac{T_{R_1}}{n_1} = \frac{63.5}{6} = 10.583333$

$\bar{R}_2 = \frac{T_{R_2}}{n_2} = \frac{89}{7} = 12.714286$

$\bar{R}_3 = \frac{T_{R_3}}{n_3} = \frac{45.5}{6} = 7.583333$

$\bar{R}_4 = \frac{T_{R_4}}{n_4} = \frac{78}{4} = 19.5$

i.e. we calculate $\binom{K}{2} = \binom{4}{2} = 6$ pairs of $|\bar{R}_i - \bar{R}_j|$

for $H_0: md_i = md_j$ $\quad (i \neq j) = 1 \ldots K$

$H_a: md_i \neq md_j$ $\quad (\neq)$

$|\bar{R}_1 - \bar{R}_2| = 2.130953 \not> 9.0371494 \Rightarrow md_1 = md_2$

$|\bar{R}_1 - \bar{R}_3| = 2.\cancel{66}999999 \not> 9.3782932 \Rightarrow \text{"} = md_3$

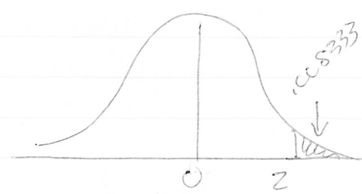
$|\bar{R}_1 - \bar{R}_4| = 8.9166667 \not> 10.485251 \Rightarrow \text{"} = md_4$

$|\bar{R}_2 - \bar{R}_3| = 5.1309527 \not> 9.0371494 \Rightarrow md_2 = md_3$

$|\bar{R}_2 - \bar{R}_4| = 6.785714 \not> 10.181265 \Rightarrow md_2 = md_4$

$|\bar{R}_3 - \bar{R}_4| = 11.916667 > 10.485251 \Rightarrow \underline{md_3 \neq md_4}$

$$C.R = \underbrace{z_{\frac{.10}{4(3)}}}_{z_{0.008333} = 2.395} \sqrt{\frac{23(24)}{12} \cdot \left(\frac{1}{\ } + \frac{1}{\ }\right)}$$



There is a difference btwn teching method 3 and 4

CRD Done

in SAS

Data name;
input treat. result;
Cards;
— — — — —
Run;
Proc ANOVA;
model result=treat ; uses alpha .05 as default
means treat/tukey alpha=.10;
Run;

CRD

model; $y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$

where mean

treat effect for ith treat

error $\varepsilon_i \sim N(0, \sigma^2)$

proc NPAR1WAY WILCOXON;
Class treat;
run;

$H \triangleq$ may not be the same as calc.'d by hand.

ANOVA
Var.
Model
Error
Tot.

Specific contribution
Var
Treat

3|30 Summary:

CRD - appropriate when we have homogeneous experimental units
- Experimental units are randomly assigned to K different treat.'s
- K treat.'s are independent
- model $y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$; $\varepsilon_{ij} \sim N(0, \sigma^2)$

j^{th} observation on treat. i

error

treat. effect for treat i

overall mean

$H_0: \mu_1 = \mu_2 = \mu_K \Leftrightarrow H_0: \alpha_1 = \alpha_2 = \ldots = \alpha_K$

we test the means but make conclusions about the treatments

- Let μ_i be the mean of the pop. i
so the expected value of $\underbrace{\mathcal{E}(y_{ij})}_{\mu_i} = \mu + \alpha_i$

- Let μ_j = mean of pop. j
$\mu_j = \mathcal{E}(y_{jj}) = \mu = \alpha_j$

$H_0: \mu_i = \mu_j \Rightarrow H_0: \mu_i - \mu_j = 0 \Rightarrow (\mu - \alpha_i) - (\mu + \alpha_j) = 0 \Rightarrow H_0: \alpha_i = \alpha_j$

Randomized Block Design (R.B.D)

K = counts treat.'s.
$i = 1 \dots K$ (treat.)
$j = 1 \dots n_i$ (obser)

- if exp. units are not homogeneous, then we may divid them into homogeneous groups (so-called blocks)
- units w/in each blck are homogeneous, but from blck to blck we have heterogeneity
- All treat.'s are randomly assigned w/in each blck, each blck must contain all treat.'s and treat.'s must appear in each block the same # of times

model: $\underbrace{y_{ij}}_{\text{observ. in treat}_i \text{ \& in block } j} = \underbrace{\mu}_{\text{overall mean}} + \underbrace{\alpha_i}_{\text{treat effect for treat}_i} + \underbrace{\beta_j}_{\text{Block effect for block}_j} + \underbrace{\varepsilon_{ij}}_{\text{errors}}$, $\varepsilon_{ij} \sim N(0, \sigma^2)$

$\alpha_{ij}(\beta_j$ = don't interact i.e. are independ.)

in SAS

proc ANOVA;
model($\boxed{y = \text{block treat;}}$) (blocks come 1st in SAS)

$i = 1 \dots K$ (index which counts treat.'s)
$j = 1 \dots b$ (counts blocks)

Hint: * Questions ask for diff.'s in treat.'s

given this on test →

Treat.'s	Blocks 1	2	... j ...	b	Treat. Tot., T_i	Treat. means $\bar{y}_i = \frac{T_i}{b}$
1	y_{11}	y_{12}	... y_{1j} ...	y_{1b}	$T_1 = \sum_{j=1}^{b} y_{1j}$	$\bar{y}_1 = \frac{T_1}{b}$
2	y_{21}	y_{22}	... y_{2j} ...	y_{2b}	$T_2 = \sum_{j=1}^{b} y_{2j}$	$\bar{y}_2 = \frac{T_2}{b}$
i	y_{i1}	y_{i2}	... y_{ij} ...	y_{ib}	$T_i = \sum_{j=1}^{b} y_{ij}$	$\bar{y}_i = \frac{T_i}{b}$
K	y_{K1}	y_{K2}	... y_{Kj} ...	y_{Kb}	$T_K = \sum_{j=1}^{b} y_{Kj}$	$\bar{y}_K = \frac{T_K}{b}$

Block Totals B_j | $\sum_{i=1}^{k} y_{i1}$ $\quad \sum_{i=1}^{k} y_{i2}$ | $\cdots$ | $B_j = \sum_{i=1}^{k} y_{ij} \cdots B_b = \sum_{i=1}^{k} y_{ib}$ |

G.T. $= \sum_{i=1}^{k} T_i = \sum_{j=1}^{b} B_j$

Block mean $\bar{B}_j = \frac{B_j}{k}$ | $\bar{B}_1 = \frac{B_1}{k}$

$\bar{B}_2 = \frac{B_2}{k}$

$\bar{B}_j = \frac{B_j}{k}$

$\bar{B}_b = \frac{B_b}{k}$

i.e. $y_{ij} - \bar{y}_{..} = (y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..}) + (\bar{y}_{i.} - \bar{y}_{..}) + (\bar{y}_{.j} - \bar{y}_{..})$ $\quad \bar{y}_{..} = \frac{\sum_{i=1}^{k} \sum_{j=1}^{b} y_{ij}}{n}$

$$\underbrace{\sum_{i=1}^{k} \sum_{j=1}^{b} (y_{ij} - \bar{y}_{..})^2}_{\text{Tot. sum of squares or tot. variation of y's}} = \left(\sum\sum \bar{y}_{i.} - \bar{y}_{..}\right)^2 + \left(\sum\sum \bar{y}_{.j} - \bar{y}_{..}\right)^2 + \left(\sum\sum y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..}\right)^2$$

$+ \sum$ of cross product terms $\to 0$

Squaring both sides + summing both sides

TSS = var(y)

$SST_r = \sum\sum (\bar{y}_{i.} - \bar{y}_{..})^2$ = measures how the treat. means vary from the overall mean

sum of squares for blocks $SSB = \sum\sum (\bar{y}_{.j} - \bar{y}_{..})^2$ = measures how the block means vary from the overall mean

$SSE = \sum (y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..})^2$

$TSS = SST_r + SSB + SSE$

<u>Computational Formulas</u>

- $TSS = \sum\sum y_{ij}^2 - \frac{(\sum\sum y_{ij})^2}{bK} = \sum\sum y_{ij}^2 - \frac{(G.T)^2}{bK} = \sum\sum y_{ij}^2 - C.m$

- $SST_r = \sum_{i=1}^{K} \frac{T_i^2}{b} - C.m = \sum \frac{T_i^2}{b} - \frac{(G.T)^2}{bK} = \sum \frac{T_i^2}{b} - \frac{(\sum\sum y_{ij})^2}{bK}$

- $SSB = \sum_{j=1}^{b} \frac{B_j^2}{K} - \underbrace{C.m}_{\text{=1 deg of free.}} = \sum \frac{B_j^2}{K} - \frac{(G.T)^2}{bK} = \sum \frac{B_j^2}{K} \; \frac{(\sum\sum y_{ij})^2}{bK}$

- $SSE = TSS - SST_r - SSB$

$df_{TSS} = n-1 = bK-1$
$df_{SST_r} = K-1$
$df_{SSB} = b-1$
$df_{SSE} = df_{TSS} - df_{SST_r} - df_{SSB} = (b-1)(K-1)$

- $MST_r = \frac{SST_r}{K-1}$ diff in treat.'s $F_T = \frac{MST_r}{MSE}$
- $MSB = \frac{SSB}{b-1}$ diff in blocks $F_B = \frac{MSB}{MSE}$
- $MSE = \frac{SSE}{(b-1)(K-1)}$

ANOVA Table

Source of Variation	df	SS	MS	F
Treat.'s	$K-1$	SST_r	MST_r	F_T
Blocks	$b-1$	SSB	MSB	F_B
Error	$(b-1)(K-1)$	SSE	MSE	
Tot.	$n-1$	TSS		

main test

① To check if treat.'s are same

$H_0: \mu_1 = \mu_2 = \dots = \mu_K$

H_a: @ least one $\mu \neq$; α

Test stat: $F_T = \frac{MSTr}{MSE}$

R.R. - We reject H_0 if $F_T > F_\alpha(K-1, (b-1)(K-1))$

-(If diff must follow up w/ Tukey (or other) to find diff.)

Not on test ass

- If diff is in Blocks, this is a gd test

$H_0: \beta_1 = \beta_2 = \dots = \beta_b$ (=) (blocks do not differ)

H_a: @ least one $\beta \neq$; α (blocks differ, i.e. RBD is needed)

Test stat: $F_b = \frac{MSB}{MSE}$

R.R. - Reject H_0 if $F_b > F_\alpha(b-1, (b-1)(K-1))$

Caution on blocking

- blocking is not always beneficial because it reduces the df of error + hence increases variance

CRD: $TSS = SSTr + SSE$

df $\quad V_1 \quad V_2$

RBD: $TSS = SSTr + SSB + SSE$

df = $V_1 \quad V_{21} \quad V_{22}$

Assumptions:

① RBD ✓ - K treat.'s randomly assigned w/in each block, such that each blck contains all treat.'s + every treat. appears exactly the same # of times in each block

② Populations corresponding to each treat. block combination are normally distributed

③ w/ = variance σ^2 (?) must verify using Hartley's Test

④ Treat.'s + blocks do not interact

Posted on website

Ex In order to help select one of three possible couriers to use, a large co. wanted to determine whether there were any differences in speed of delivery btwn the 3 couriers. An exp. was performed where letters were sent using each of the 3 different couriers @ each of 4 different times of day to a delivery pnt. across town. The # of hrs taken for each delivery was recorded as follows. ($\alpha = .01$)

Couriers

Time of Day	A	B	C	Block Tot. B_j	Block means $\bar{B}_j$
9:30am	3.6	4.2	5	12.8	4.26666667
11:30am	5.4	5.8	7	18.2	6.06666667
12:30 pm	6.1	7	9.1	22.2	7.4
2:00 pm	3.5	4	4.9	12.4	4.1333333
Treat. tot, T_i	18.6	21	26	GT = 65.6	
Treat means $\bar{T}_i$	4.65	5.25	6.5	$n = bk = 4(3) = 12$	

Assumptions:

① RBD ✓ i.e. Each of the 3 couriers is randomly assigned to each of the 4 times of the day

② ~~Times~~ Speed of delivery for each courier - time of day combo. is normalley distributed

③ w/ equal variance σ^2 (?) → Hartley's

④ No interactions btwn couriers + times of day (eg. courier A is not faster in the morning + slower in the afternoon while courier C, say, is the reverse)

3/22 Hartley Test

$k = 3$ (couriers)

$b = 4$ (times of day)

$H_0: \sigma_1^2 = \sigma_2^2 = \sigma_3^2$; $\alpha = .01$

H_a: @ least one $\sigma^2 \neq$

-test stat: $F_{max} = \frac{s^2_{max}}{s^2_{min}} = \frac{3.94}{1.6966667} = \underline{2.3222003}$

where $s_1^2 = \frac{\sum_{j=1}^{4} y_{1j}^2 - \frac{\left(\sum_{j=1}^{4} y_{1j}\right)^2}{4}}{4-1} = \frac{91.58 - \frac{(18.6)^2}{4}}{3} = \underline{1.6966667}$ → min

$s_2^2 = \frac{\sum_{j=1}^{4} y_{2j}^2 - \frac{\left(\sum y_{1j}\right)^2}{4}}{3} = \frac{116.28 - \frac{(21)^2}{4}}{3} = \underline{2.01}$

$s_3^2 = \frac{\sum_{j=1}^{4} y_{3j}^2 - \frac{\left(\sum y_{1j}\right)^2}{4}}{3} = \frac{180.82 - \frac{(26)^2}{4}}{3} = \underline{3.94}$ ← max

B.R - We reject H_0 if $F_{max} > F_{max}(K, [\bar{n}]-1); \alpha =$
$= F_{max}(3,3); .01 = 85$

Since $F_{max} = 2.37 \not> 85$, we do not reject H_0, and we conclude that @ 1% level of significance there is not enough evidence that variances are not equal, i.e. we have equal variance
$\Rightarrow$ may proceed w/ main test.

• $TSS = \sum_{i=1}^{3}\sum_{j=1}^{4} y_{ij}^2 - \frac{\left(\sum_i \sum_j y_{ij}\right)^2}{bK} = 388.68 - \frac{(65.6)^2}{12} =$

$= 388.68 - 358.61333 = \underline{30.066667}$

bk=n

• $SSTr = \sum_{i=1}^{3} \frac{T_i^2}{b} - \frac{(G.T)^2}{bK} = \left[\frac{18.6^2}{4} + \frac{21^2}{4} + \frac{26^2}{4}\right] - \frac{65.6^2}{12} =$
$= 365.74 - 358.61333$
$= \underline{7.12667}$

• $SSB = \sum_{j=1}^{4} \frac{B_j^2}{K} - \frac{(G.T)^2}{bK} = \left[\frac{12.8^2}{3} + \frac{18.2^2}{3} + \frac{22.2^2}{3} + \frac{12.4^2}{3}\right] - \frac{65.6^2}{12} =$
$= 380.56 - 358.61333$
$= 21.946669$

• $SSE = TSS - SSTr - SSB = \underline{.9933277}$

$MSTr = \frac{SSTr}{K-1} = \frac{7.126667}{2} = 3.563335$

• $MSB = \frac{SSB}{b-1} = \frac{21.246669}{3} = \underline{7.3155563}$