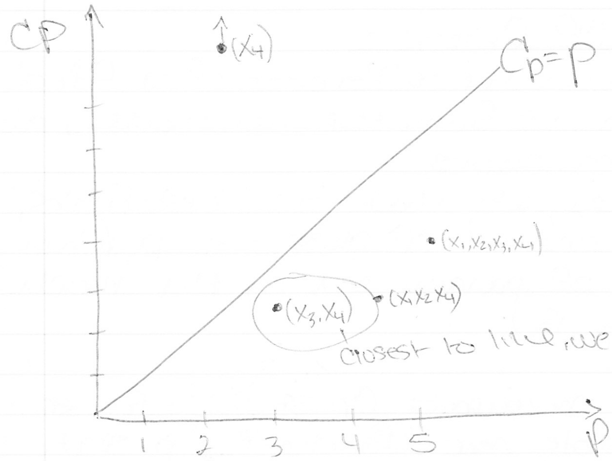
not on exam



- Here "best-fitting" model is w/ var.'s x_3 and x_4

*MLR to (this pnt) Test 2

not on final, but on (card) always given → • Backwards Elimination Procedure

- We are given $X_1, X_2 \dots X_K$ — critical → anything below eliminated
- critical value (to-delete-available) $= t_0^{**}$
- We test contribution of each var. (one @ a time) and we remove the one w/ smallest contribution

① Fit full model: $y = \beta_0 + \beta_1 x_1 + \dots + \beta_K x^K + \varepsilon$

- calc. (abs. val) $\left| t_j = \frac{\hat{\beta}_j}{\sqrt{\hat{var}(\hat{\beta}_j)}} = \frac{\hat{\beta}_j}{\sqrt{V_{jj} MSE}} \right|$ for each $j = 1 \dots K$ (i.e. $H_0: \beta_j = 0$)
- select the smallest ($\min |t_j|$) and then compare $|t_j|$ w/ t_0^{**}
  If $\min |t_j| < t_0^{**}$ Remove x_j from model and go to step 2. If $\min |t_j| \geq t_0^{**}$ can't remove + stop here. Have to stay w/ full model

* not on final, but on ass

- Backwards Elimination Procedure

t, F = F partial - if missing info to calc. t, use F

$X_1, X_2, \ldots, X_K$

- critical value (to delete a var.) $= t_0^{**}$

$\xrightarrow{\text{above}} t_0^{**}$ below $\downarrow X \rightarrow$ deleted

- We eliminate variable/s w/ the min. contribution, one-at-a-time

Step ① Fit the full model: $y = \beta_0 + \beta_1 X_1 + \ldots \beta_K X_K + \varepsilon$

- calc $|t_j| = \left|\frac{\hat{\beta}_j}{\sqrt{Var(\hat{\beta}_j)}}\right| = \left|\frac{\hat{\beta}_j}{\sqrt{V_{jj} \cdot MSE}}\right|$ for $j = 1 \ldots K$ - abs values

- select $\min |t_j|$
- if $\min |t_j| < t_0^{**}$ we delete X_j, $\Rightarrow$ Go to step 2
- if $\min |t_j| \geq t_0^{**} \rightarrow$ STOP $\Rightarrow$ the "best-fitting" model is full model.

Step ② we fit model $y = \beta_0 + \beta_1 X_1 + \ldots + \beta_{K-1} X_{K-1} + \varepsilon$

- calc. $|t_j|$ for $j = 1 \ldots K-1$
- select $\min |t_j|$
- if $\min |t_j| < t_0^{**} \Rightarrow$ delete $X_j \Rightarrow$ go to step 3
- if $\min |t_j| \geq t_0^{**} \Rightarrow$ stop $\Rightarrow$ the "best-fitting" model is $y = \beta_0 + \beta_1 X_1 + \ldots + \beta_{K-1} X_{K-1} + \varepsilon$ (from step 2)

Step ③ continue in this way until you cannot delete any variable any more

NOTE $(t_0^{**})^2 = F_{0\text{-part}}^{**}$

- full mod. is mod. from previous step
- we select $\min F_j$
- if $\min F_j < F_0^{**}$ - we delete X_j

not on exam

* not on final but on Assignment.

Forward Selection Procedure

- $X_1, X_2 \dots X_K$
- critical value (to add a variable) = t_0^* (or $(t_0^*)^2 = F_0^*$)
- we add the variables into the model that have maximum contribution (one-@-a-time)

Use F test
can use t for step 1 only

Step ① Fit one variable models $y = \beta_0 + \beta_1 X_j + \varepsilon$ for $j = 1 \dots K$
- calc $SSR(X_j)$ for each j
- select max $SSR(X_j)$ ⟷ for this X_j
- calc $F_j = \frac{MSR(X_j)}{MSE(X_j)} = \frac{SSR(X_j)/df}{MSE(X_j)}$
- if $F_j > F_0^* \Rightarrow$ add var. X_j into the model ⇒ go to Step 2
- If $F_j \le F_0^* \Rightarrow$ stop ⇒ No regression i.e $Y = \beta_0 + \varepsilon$

Step ② Fit all possible 2 variable models (w/ X_j in it, say X_1^*)

X_1^* → 1st variable that was entered.

$y = \beta_0 + \beta_1 X_1^* + \beta_2 X_j + \varepsilon$; $j = 1 \dots, K-1$ → means first variable
- calc. $SSR(X_j | X_1^*) = SSR_F(X_1^*, X_j) - SSR(X_1^*)$ for each j
- select max. $SSR(X_j | X_1^*)$
- calc. $F_j = \frac{MSR(X_j|X_1^*)}{MSE_{(full)}(X_1^*, X_j)} = \frac{SSR(X_j|X_1^*)/\text{app. df}}{MSE(X_1^*, X_j)}$
- if $F_j > F_0^* \Rightarrow$ add X_j (call it X_2^*)

called this in class only
- if $F_j \le F_0^* \Rightarrow$ STOP ⇒ best-fitting model is w/ $\{X_1^*\}$

Step ③ Fit all possible 3 variable models

$y = \beta_0 + \beta_1 X_1^* + \beta_2 X_2^* + \beta_3 X_j + \varepsilon$ $j = 1, \dots, K-2$ (reduced)
- calc. $SSR(X_j | X_1^*, X_2^*) = SSR_{(full)}(X_1^*, X_2^*, X_3^*) - SSR(X_1^*, X_2^*)$ for each
- select max $SSR(X_j | X_1^*, X_2^*)$
- calc. $F_j = \frac{MSR(X_j | X_1^*, X_2^*)}{MSE_{(full)}(X_1^*, X_2^*, X_j)} = \frac{SSR(X_j|X_1^*, X_2^*)/df}{MSE(X_1^*, X_2^*, X_j)}$
- if $F_j > F_0^* \Rightarrow$ add X_j into model (X_3^*) ⇒ go to step 4
- if $F_j \le F_0^* \Rightarrow$ STOP ⇒ Best-fitting model is $\{X_1^*, X_2^*\}$

Not on exam

- Step (4) - continue in this way until you can't add any variable any more.

$X_1, X_2, X_3, X_4, X_5,$

(1) select X_3

(2) X_1, X_3
X_2, X_3 $\Rightarrow X_5$
X_4, X_3
X_5, X_3

(3) X_1, X_3, X_5
X_2, X_3, X_5 $\Rightarrow X_1$
X_4, X_3, X_5

(4) X_1, X_2, X_3, X_5 $\Rightarrow X_4$
X_1, X_3, X_4, X_5

*Not on final, but on ASS

• Stepwise Regression

(1) As in Forward selection
- we fit $y = \beta_0 + \beta_1 x_j + \varepsilon$; $j = 1 \dots K$
- calc. $SSR(x_j)$ for each j
- select max $SSR(x_j)$
- calc. F-test i.e. $\frac{MSR(x_j)}{MSE(x_j)}$
  - if $x_j > F_0^* \Rightarrow$ add x_j into model (call it x_1^*) $\Rightarrow$ step 2
  - if $x_j \leq F_0^* \Rightarrow$ stop (no model)
- given F_0^* (to-add)
  F_0^{**} (to-delete)

(2) As in Forward selection we fit all possible 2-variable models $y = \beta_0 + \beta_1 x_1^* + \beta_2 x_j + \varepsilon$ for $j = 1 \dots K-1$
- calc. $SSR(x_j | x_1^*) = SSR(x_1^*, x_j) - SSR(x_1^*)$ for each j
- select max. $SSR(x_j | x_1^*)$
- calc $F_j = \frac{SSR(x_j | x_1^*)}{MSE(x_1^* | x_j)}$
  - if $F_j > F_0^* \Rightarrow$ add x_j (call it x_2^*) $\Rightarrow$ go to step 3
  - if $F_j \leq F_0^* \Rightarrow$ STOP $\Rightarrow$ the "best-fitting model" has $\{x^*\}$ (from 1st step)

not on exam

③ Independent variable/s entered into the model @ the previous step(s) may not be needed anymore when new variable/s are entered!

- We have to use backwards elimination to check if they can be removed (one variable @ a time)

e.g. say X_1 X_2 are already in the model

- We add X_3

- Now we need to check for redundancy of X_1 (when $X_2 + X_3$ are in the model)

i.e. $SSR(X_1 | X_2, X_3) \Rightarrow \overset{full}{SSR}(X_1, X_2, X_3) - \overset{red}{SSR}(X_2, X_3)$

calc $F_1^{-one} = \dfrac{SSR(X_1 | X_2, X_3)/df}{MSE(X_1, X_2, X_3)}$

- if $F_1^{one} < F_0$** we delete X_1 and say X_1 is not needed when $X_2 + X_3$ are present)
- if $F_1 \geq F_0$** $\Rightarrow$ we don't delete, and X_1 when $X_2 + X_3$ are present is needed

Check for redundancy of X_2 when $X_1 + X_3$ are in

$SSR(X_2 | X_1, X_3) = \overset{full}{SSR}(X_1, X_2, X_3) - \overset{red.}{SSR}(X_1, X_3)$

- calc. $F_2 = \dfrac{SSR(X_2 | X_1, X_3)/df}{MSE(X_1, X_2, X_3)}$
- if $F_2 < F_0$** $\Rightarrow$ delete X_2 from model, we say X_2 is not needed when $X_1 + X_3$ are present)
- if $F_2 \geq F_0$** $\Rightarrow$ we don't remove X_2 when $X_1 + X_3$ are present, all 3 are needed

④ continue in the way until you can't add more nor delete any variable/s any more

Chap 11 - Design of Experiment or Experimental Design

- Want to find out whether there are differences among K different "treatmnts"
- works under the assumption that the K treatmnts have equal distributions and equal variances

Have to use new F test

$\Rightarrow H_0: \mu_1 = \mu_2 = \dots = \mu_K$

H_a: @ least one the μ_i's $\neq$; α

Ex a)

Pop 1	Pop 2	Pop 3
5.90	5.51	5.01
5.92	5.50	5.00
5.91	5.50	4.99
5.89	5.49	4.98
5.88	5.50	5.02
$\bar{y}_1 = 5.90$	$\bar{y}_2 = 5.50$	$\bar{y}_3 = 5$

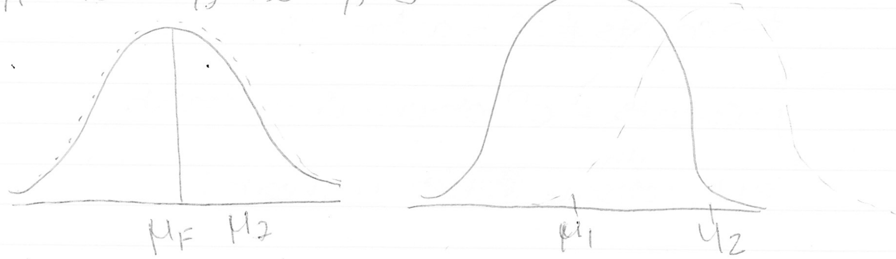
- Here we might say that the 3 treatmnts differed because the variation btwn treatmnts > than variation w/in each sample

b)

1	2	3
5.90	6.31	4.52
4.42	3.54	6.93
7.51	4.73	4.48
7.89	7.20	5.55
3.78	5.72	3.52
$\bar{y}_1 = 5.90$	$\bar{y}_2 = 5.50$	$\bar{y}_3 = 5$

- Here we might say that the treatmnts are the same as the variation btwn treatmnts is < the variation w/in the each sample

03/18



$H_0: \mu_1 = \mu_2 = \dots \mu_K$

H_a: @ least one μ_i's $\neq$

- assume (1) treat's are normally distributed
(2) w/ equal variances

Completely Randomized Design (CRD)
(one-way ANOVA)

- Assume (1) K independent random samples of sizes $n_1, n_2 \ldots n_K$
Assumed → (2) taken from K normally distributed pop's w/ mean $\mu_1, \mu_2 \ldots \mu_K$
Has to be verified → (3) and w/ common variance σ^2
→ use Hartley's test to verify

Def.: CRD - is a random selection of K independent samples from K independent pop's

Samples (treat's)	observ.'s	Treat Tot	Treat Means $\bar{y}_i = \bar{T}_i$
1	$y_{11}, y_{12} \cdots y_{1n_1}$	$T_1 = \sum_{j=1}^{n_1} y_{1j} = y_{1\cdot}$	$\bar{y}_1 = T_1/n_1$
2	$y_{21}, y_{22} \cdots y_{2n_2}$	$T_2 = \sum_{j=1}^{n_2} y_{2j}$	$\bar{y}_2 = T_2/n_2$
⋮	⋮	⋮	⋮
i	$y_{i1}, y_{i2} \cdots y_{in_i}$	$T_i = \sum_{j=1}^{n_i} y_{ij}$	$\bar{y}_i = T_i/n_i$
⋮	⋮	⋮	⋮
K	$y_{K1}, y_{K2} \cdots y_{Kn_K}$	$T_K = \sum_{j=1}^{n_K} y_{Kj}$	$\bar{y}_K = T_K/n_K$

→ Con't on next pg

$i = 1 \ldots K$
→ counts # of treatments

j = counts # of observ.'s in treat. i

→ y_{ij} = j^{th} observ. in treat i

Treat means

$= \dfrac{\sum_{j=1}^{n} y_{ij}}{n_i} = \bar{y}_{1\cdot}$

$= \dfrac{\sum_{j=1}^{n_2} y_{2j}}{n_2} = \bar{y}_{2\cdot}$

. . .

$= \bar{y}_{i\cdot}$

. . .

$\bar{y}_{k\cdot}$

$n = \sum_{i=1}^{k} n_i$ → overall sample size

$G.T. = \sum_{i=1}^{k} T_i = \sum_{i=1} \sum_{j=1}^{n_i} y_{ij}$ → grand total (overall tot.)

$\bar{y}_{\cdot\cdot} = \dfrac{G.T.}{n} = \dfrac{\sum_{i=1}^{k} \sum_{j=1}^{n_i} y_{ij}}{n}$ → overall mean

$\Sigma (y_{ij} - \bar{y}_{\cdot\cdot})^2 = (y_{ij} - \bar{y}_{i\cdot}) + (\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})$ $/^2$ both sides

$(y_{ij} - \bar{y}_{\cdot\cdot})^2 = (y_{ij} - \bar{y}_{i\cdot})^2 + (\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2 + 2(y_{ij} - \bar{y}_{i\cdot})(\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})$ /d

$\sum_i \sum_j$ both sides

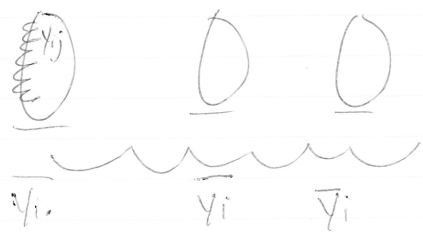
$\sum_{i=1}^{k} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{\cdot\cdot})^2 = \sum_i \sum_j (y_{ij} - \bar{y}_{i\cdot})^2 + \sum_i \sum_j (\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2 + 2 \sum_i \sum_j \cancel{\sum (y_{ij} - \bar{y}_{i})}$

$(\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2$ (y) sum of cross product terms $= 0$

measures how individual observ's vary around their mean w/in each treatment SSE

$\underbrace{\sum_i \sum (y_{ij} - \bar{y}_{\cdot\cdot})^2}_{\text{tot. variation of y's i.e. } V(y) = TSS} = \sum_i \sum_j (\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2 + \sum_i \sum_j (y_{ij} - \bar{y}_{i\cdot})^2$

→ measures variation of treatment mean about the overall mean. SSTr (sum of squares for treatment)



i.e. $TSS = SST_r + SSE$

<u>Computational formulas</u>

- var. of y's

• $TSS = \sum_{i=1}^{k} \sum_{j=1}^{n_i} y_{ij}^2 - \frac{(\sum\sum y_{ij})^2}{n} = \sum\sum y_{ij}^2 - \frac{(G.T)^2}{n}$

$= \sum\sum y_{ij}^2 - C.M.$

C.M. correction for the mean

- counts as 1 deg. of freedom

• $SST_r = \sum_{i=1}^{k} \frac{T_i^2}{n_i} - C.M. = \sum \frac{T_i^2}{n_i} - \frac{(G.T.)^2}{n} = \sum \frac{T_i^2}{n_i} - \frac{(\sum\sum y_{ij})^2}{n}$

• $SSE = TSS - SST_r$

$df_{TSS} = n-1$
$df_{SST_r} = k-1$ (# of treat's) -1
$df_{SSE} = df_{TSS} - df_{SST} = n-k$

var. btwn treat's

• $MST_r = \frac{SST_r}{k-1}$ } tests equality of treat's $\Rightarrow F_T = \frac{MST_r}{MSE}$

var. w/in treat's

• $MSE = \frac{SSE}{n-k}$

ANOVA Table

Source of Variation	df	SS	MS	F_ϕ
Treat.'s	$K-1$	SSTr	MSTr	MSTr/MSE
Error	$n-K$	SSE	MSE	
Tot	$n-1$	TSS		

$H_0: \mu_1 = \mu_2 = \dots \mu_K$; α (treat.'s same)
H_a: @ least one of the μ_i's $\neq$ (treat.'s differ)

R.R. - Rej. H_0 if $F_T > F_{\alpha(K-1, n-K)}$

Hartley's Test (not in book) - must do before main test

- test for equality of treat.'s variances

$H_0: \sigma_1^2 = \sigma_2^2 = \dots = \sigma_K^2$; α - 1% → if testing @ another level, calc both these, or pick the one that matches other result
H_a: @ least one of the σ^2 $\neq$ — 5%

test stat: $F_{max} = \dfrac{S^2_{max}}{S^2_{min}}$ choose from $S_1^2, S_2^2 \dots S_K^2$

S^2_{max} is the lrgst sample variance
S^2_{min} is the smllst sample variance

table on website, not in book!

R.R.: Reject H_0 is $F_{max} > F_{max(K, [\bar{n}]-1), \alpha}$ for treat.'s
where K = # of treat.'s
$\bar{n}$ = average sample size $\left(\dfrac{n_1 + n_2 + \dots + n_K}{K}\right)$
$[\bar{n}]$ = integer part of average sample size

e.g. if $\bar{n} = 5.47 \Rightarrow$
$[\bar{n}] = 5$
only look @ the integer prt, don't round

- If we don't reject $H_0 \Rightarrow$ i.e. variances are =
$\Rightarrow$ proceed to main test: $H_0: \mu_1 = \mu_2 = \dots = \mu_K$

- If we reject $H_0 \Rightarrow$ i.e. variances differ
$\Rightarrow$ cannot proceed STOP here

Ex Groups of students were randomly assigned to be taught by four different teaching methods. Because of drop-outs in the experimental groups (due to sickness, transfers, ect.) the # of students varied from group to group (i.e. unequal sample sizes) Do the data present sufficient evidence to indicate a difference in the mean achievement for students taught using the 4 teaching methods? $\alpha = 10\%$

Teaching Techniques	observ.'s	Sample size's n_i	Treat tot.'s, T_i	T_i/n_i Treat means $\bar{y}_i$
1	65, 87, 73, 79, 81, 69	6	454	75.67
2	75, 69, 83, 81, 72, 79, 90	7	549	78.43
3	59, 78, 67, 62, 83, 76	6	425	70.83
4	94, 89, 80, 88	4	351	87.75
		23	1,779	
		$\downarrow$	$\downarrow$	
		n	G.T.	

Assumptions:

given ✓ ① CRD (i.e. Students are randomly assigned to 4 different teaching techniques)

No need to verify < ② Teaching techniques are normally distributed

③ w/ common variance σ^2 (?) $\rightarrow$ Have to test using Hartley's Test

We need $s_1^2, s_2^2, s_3^2, s_4^2$

$$s_i^2 = \frac{\sum_{j=1}^{n_i} y_{ij}^2 - \frac{(\sum y_{ij})^2}{n_i}}{n_i - 1}$$

i.e. $s_1^2 = \dfrac{\sum_{j=1}^{6} y_{1j}^2 - \frac{(\sum_{j=1}^{6} y_{1j})^2}{6}}{5}$

$= \dfrac{34{,}686 - \frac{(454)^2}{6}}{5} = 66.66$

$s_2^2 = \dfrac{\sum_{j=1}^{7} y_{2j}^2 - \frac{(\sum y_{2j})^2}{7}}{6} = \dfrac{43{,}361 - \frac{(549)^2}{7}}{6} = 50.62$

$s_3^2 = \dfrac{\sum_{j=1}^{6} y_{3j}^2 - \frac{(\sum_{j=1}^{6} y_{3j})^2}{6}}{5} = \dfrac{30{,}563 - \frac{(425)^2}{6}}{5} = 91.76 \rightarrow$ lrg.

$s_4^2 = \dfrac{\sum_{j=1}^{4} y_{4j}^2 - \frac{(\sum_{j=1}^{4} y_{4j})^2}{4}}{3} = \dfrac{30{,}901 - \frac{(351)^2}{4}}{3} = 33.58 \rightarrow$ min.

3-13 CRD

K treat's

Assume 1.) same dist'n (normal)

2.) Equal variance σ^2 (?) Hartley's test

- if equal variance $\Rightarrow H_0: \mu_1 = \mu_2 = \dots = \mu_K$

Ex 4 teaching techniques, $\alpha = .10$

Hartley's test

$H_0: \sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_4^2$; $\alpha =$ 1% or 5% either match or use both

H_a: @ least one $\sigma^2 \neq$

$s_1^2 = 66.66$ test stat $F_{max} = \dfrac{s^2_{max}}{s^2_{min}} = \dfrac{91.76}{33.58} = 2.7326$

$s_2^2 = 50.62$

$s_3^2 = 91.76$ lrgst R.R. We reject H_0 if $F_{max} > F_{max}(K, [\bar{n}]-1), \alpha$

$s_4^2 = 33.58$ smst

—OVER

$n_1=6$ $\bar{n}=5.75$, $K=4$ $F_{max(4,4)}$; $.01=49$
$n_2=7$ $[\bar{n}]=5$ $.05=20.6$
$n_3=6$
$n_4=4$

If we do reject Ho we stop here

- Since $F_{max}=2.7326 \not> 49$ (or 20.6), we do not reject H_0 & conclude that @ 1% or 5% level of significance there's not enough evidence to say that the variances differ. That means we have equal variance, so we may proceed w/ the mean test.

- $TSS = \sum_{i=1}^{4} \sum_{j=1}^{n_i} y_{ij}^2 - \frac{(G.T)^2}{n} = \sum\sum y_{ij}^2 - \frac{\left(\sum_{i=1}^{4}\sum_{j=1}^{n_i} y_{ij}\right)^2}{n} =$

$= 139511 - \frac{(1779)^2}{23} = 139511 - 137601.7826 = \underline{1909.217392}$

- $SST_r = \sum_{i=1}^{4} \frac{T_i^2}{n_i} - \frac{(GT)^2}{n} = \left[\frac{(454)^2}{6} + \frac{(549)^2}{7} + \frac{(425)^2}{6} + \frac{(351)^2}{4}\right]$

$- \frac{(1779)^2}{23} = 138{,}314.369 - 137601.7826$

$= \underline{712.586447}$

- $SSE = TSS - SST_r = \underline{1196.630945}$

- $MST_r = \frac{SST_r}{K-1} = \frac{712.586447}{3} = \underline{237.5288157}$

- $MSE = \frac{SSE}{n-K} = \frac{1{,}196.630945}{23-4} = \underline{62.98057605}$

$F_T = \frac{MST_r}{MSE} = \underline{3.771461466}$

ANOVA Table

Source of Variation	df	SS	MS	F
Treat.'s	3	712.59	237.53	3.772
Error	19	1,196.63	62.98	
Tot. (n-1)	22	1,909.22		

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$; $\alpha = .10$
H_a: @ least one of μ_i's $\neq$

Test stat: $F_T = \frac{MSTr}{MSE} = \underline{3.772}$

RR - We reject H_0 if $F_T > F_\alpha(k-1, n-k) = F_{.10}(3,19) = 2.40$

Since $F_T = 3.772 > 2.40$ we reject H_0 and conclude that @ 10% level of significance there is an evidence that the four teaching techniques differ

- Now we need to do a follow up analysis to find out which treat.'s differ

- Multiple comparisions (Not in Textbook)

- We are interested in locating the differences among treat.'s after ~~case~~ H_0 was rejected.

- "Post hoc" procedures i.e. after we've seen the data & know there's a difference in treat.'s we just don't know which treat.'s differ (i.e. we need to locate the difference or differences)

Fisher's Least Significant ~~ce~~ Difference (L.S.d)

$H_0: \mu_i = \mu_j \Rightarrow H_0\ \mu_i - \mu_j = 0$ | - 2 independent pop.'s $\sim N$
$H_a: \mu_i \neq \mu_j \quad H_a: \mu_i - \mu_j \neq 0$ | $- X_1 \sim N(\mu_1, \sigma_1^2)$
$X_2 \sim N(\mu_2, \sigma_2^2)$

must know σ^2, if not use t test

if σ_1^2 & σ_2^2 are known we can use Z test

$$Z = \frac{(\bar{X}_i - \bar{X}_j) - (\mu_i - \mu_j)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}} = \frac{\bar{X}_1 - \bar{X}_j}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

- if σ_1^2 + σ_2^2 unknown, but 2 pop is all normal

$$t = \frac{\bar{X}_i - \bar{X}_j}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$ only if $\sigma_1^2 \neq \sigma_2^2$ (even though they are unknown)

if $\frac{S^2_{max}}{S^2_{min}} < 3$ then we can assume that $\sigma_1^2 = \sigma_2^2$ (even though unknown)

σ^2 (say)

- σ^2 must be estimated using $S_E^2 = \frac{S_i^2(n_i - 1) + S_j^2(n_j - 1)}{n_i + n_j - 2}$

pooled var. estimator

$$\Rightarrow t = \frac{\bar{X}_i - \bar{X}_j}{\sqrt{S_p^2\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}}$$

In all case we have K pop.'s (normally distrib'd)
$S_1^2 = S_k^2$ (Hartley's test)

$$t = \frac{\bar{X}_i - \bar{X}_j}{\sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}}$$ → for doing simultaneous pair wise comparisons

$H_0: \mu_i = \mu_j$
$H_a: \mu_i \neq \mu_j$
$i, j = 1 \ldots K$
$(i \neq j)$

R.R. We reject H_0 if $t > t_{\alpha/2; n-K}$
or $t < -t_{\alpha/2; n-K}$

$\Leftrightarrow |t| > t_{\alpha/2; n-K}$

- if $|t| = \left| \frac{\bar{X}_i - \bar{X}_j}{\sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}} \right| > t_{\alpha/2; n-K}$

$\bar{Y}_i$ $\bar{Y}_j$

or $|\bar{Y}_i - \bar{Y}_j| > t_{\alpha/2; n-K} \cdot \sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$