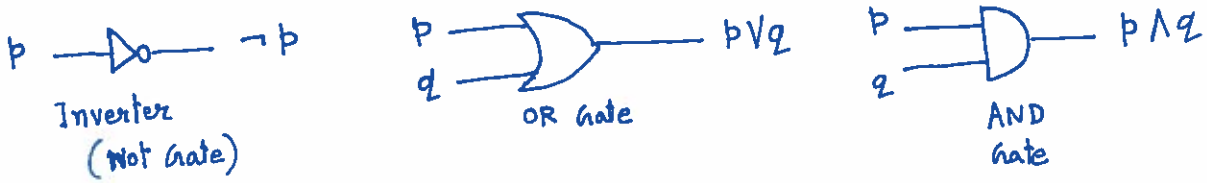
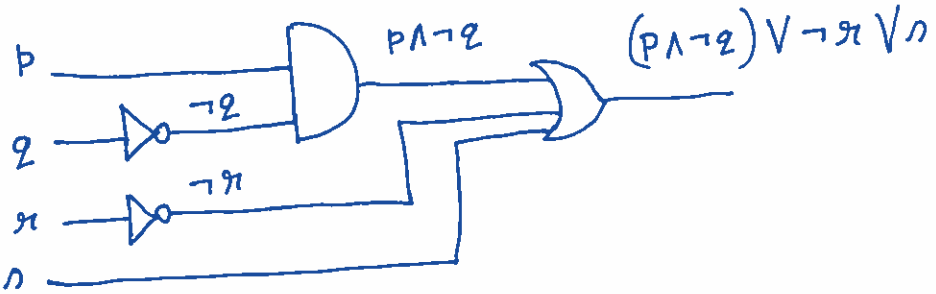
# Logic Circuits

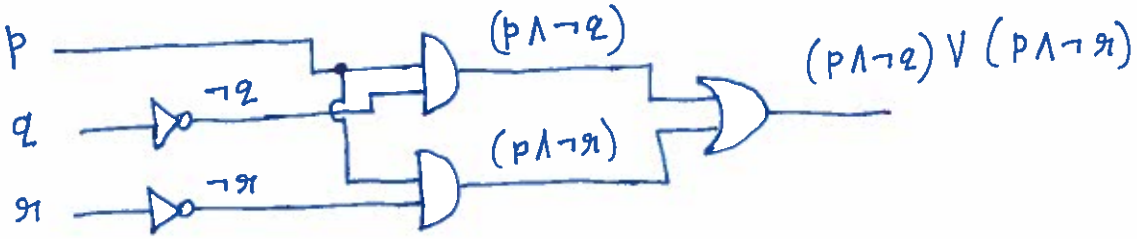



Shannon noticed that propositional logic can be used to design computer hardware.

Determine the output of the following combinatorial circuit




Design a circuit that produces the output $(p \wedge \neg q) \vee (p \wedge \neg r)$.

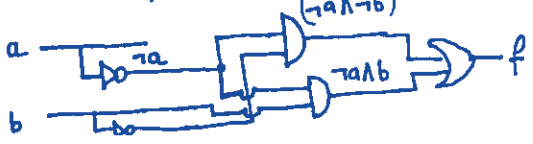



## Any Formula to Circuit

Given a ~~Boolean~~ Boolean function in its truth table representation, how to design a circuit that computes the same function.

Example

| a | b | f(a,b) |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |




step 1 :— Given the truth table identify the rows with $f(a,b) = 1$

step 2 :— Get the ∧ conjunctive clause representation of those rows and connect them with disjunctions. This will give us a DNF representation of the formula f.

step 3 :— Represent the DNF formula as a ∧ circuit.

# Logical Puzzle

- Puzzles that can be solved using logical reasoning are called logical puzzles.

Raymond ~~Sut~~ Smullyan published dozen of books on logical puzzles.

Problem: An island has two kinds of inhabitants
- Knights (who always tell truth)
- Knaves (who always lie)
- But they are indistinguishable by sight.

You encounter two people A and B.

1) What are A and B if A says "B is a Knight".
and B says "Two of us are of different types"?

## Solution by Reasoning :-

A says 'B is Knight'. So if A is a Knight A's statement that 'B is a Knight' must also be true and B will have to be has to be a Knight as well. But if B is Knight his statement 'Two of us are of different types' must also be true. But this statement contradicts with the fact that both are Knights.

So A's statement cannot be true and hence A is a Knave. and A's statement 'B is a Knight' is false. So B is a Knave as well.

Furthmore note that since B is a Knave the statement 'Two of us are of different types' is false. This agrees with the fact that both of them are Knaves.

1. A says "I am Knave and B isn't".

steps : (1) Define the following propositional variables

$p$ : "A is Knight".

$q$ : "B is Knight".

step (2) : Write any statements made by the Islanders as compound propositions in $p$ & $q$.

A says $\neg p \wedge q$

Step (3) : Observe that truth value of $p$ must coincide with what A says. Similarly the truth value of $q$ must coincide with what B says.

step (4) : Construct a truth table to determine which truth assignments for $p$ and $q$ will make

(i) truth value of $p$ equal to truth values of what A says

and (ii) truth value of $q$ equal to truth values of what B says.

| $p$ | $q$ | $\neg p \wedge q$ | Observation |
|---|---|---|---|
| F | F | F | $p$ & $(\neg p \wedge q)$ coincides |
| F | T | T | does not coincide |
| T | F | F | does not coincide |
| T | T | F | does not coincide |

(of $p$ and $(\neg p \wedge q)$)

Answer, as the truth value only coincides as when $p$ and $q$ both are False, A and B both must be Knaves.

2. A says "If I am Knight, then so is B".

p: "A is Knight".
q: "B is Knight".

A says $(p \to q)$

| p q | $p \to q$ | p and $(p \to q)$ coincide or not ? |
|---|---|---|
| F F | T | NO |
| F T | T | NO |
| T F | F | NO |
| T T | T | Yes |

So A & B both are Knights.

3. Suppose we know that A & B are either both Knights or both Knaves.
What do you make of A's statement
"If B is Knight, then I am Knave"?

A says "$q \to \neg p$"

| p q | $\neg p$ | $q \to \neg p$ |
|---|---|---|
| F F | T | T |
| T T | F | F |

~~In both~~

In both cases, p, q are both ~~same~~ do not coincide with $(q \to \neg p)$.

So A ~~they~~ cannot say such thing.

4. A says "We are both Knights" and
B says "Either A is Knight or I am a Knight," but not both."

A says '$p \wedge q$'

B says '$p \oplus q$'

| $p$ | $q$ | $p \wedge q$ | $p \oplus q$ | Do $p$ coincide with $(p \wedge q)$ and $q$ coincide with $p \oplus q$ |
|---|---|---|---|---|
| F | F | F | F | Yes, Yes |
| F | T | F | T | Yes, Yes |
| T | F | F | T | No, No |
| T | T | T | F | Yes, No |

So $p$ and $(p \wedge q)$ agree on row 1, 2 and 4;
and $q$ and $(p \oplus q)$ agree on row 1 and 2.

Hence, both pair agrees on row 1 & 2.

Thus we conclude that A must be Knave, while we can't determine B's identity.

## Alternate solution of the question in page 5.

A says $q$

B says $p \oplus q$

| $p$ | $q$ | A says $q$ | B says $p \oplus q$ | $p$ coincides with $q$ | $q$ coincides with $p \oplus q$ |
|---|---|---|---|---|---|
| F | F | F | F | Yes | Yes |
| F | T | T | T | No | No |
| T | F | F | T | No | No |
| T | T | T | F | Yes | |

Hence both coincides when $p := \text{False}$ & $q := \text{False}$.

Thus, A and B both are Knaves.