MAT 1348(B)

Conditional Statement (Hypothesis → conclusion)

If p, then q
If p, q
p implies q
p is sufficient for q
a necessary condition for p is q.
p only if q

q if p
q when p
q follows from p
q is necessary for p
a sufficient condition for q is p

p	q	p → q
F	F	T
F	T	T
T	F	F
T	T	T

$p \to q$

It's converse : $q \to p$
It's contrapositive : $\neg q \to \neg p$
It's Inverse : $\neg p \to \neg q$

p	q	¬p	¬q	p→q	q→p	¬q→¬p	¬p→¬q
F	F	T	T	T	T	T	T
F	T	T	F	T	F	T	F
T	F	F	T	F	T	F	T
T	T	F	F	T	T	T	T

Biconditional

$p \leftrightarrow q$ p is necessary and sufficient for q
p iff q
q iff p

p	q	$p \leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

Example: You can take the flight iff you buy a ticket.

TRANSLATION FROM ENGLISH TO PROPOSITION

The automated reply cannot be sent when the file system is full.

The automated reply can be sent

p: The automated reply can be sent

q: File system is full.

$q \to \neg p$

p	q	¬p	[illegible]
F	F	T	T
F	T	T	T
T	F	F	T
T	T	F	F

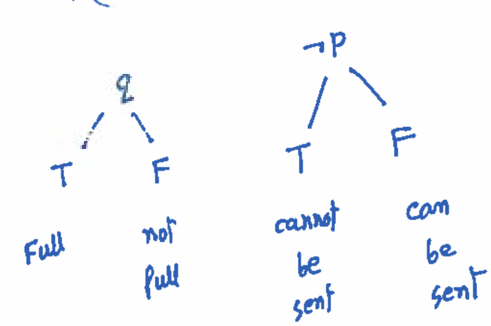
$q \to \neg p$

Alternatively, one can also write

$\neg(q \land p)$

p	q	¬p	
~~F F~~			
F	F	T	T
F	T	T	T
T	F	F	T
T	T	F	F

$q \to \neg p$



$A \cdot 1 \equiv A$ $\qquad$ $A + 1 \equiv 1$

$A \cdot 0 \equiv 0$ $\qquad$ $A + 0 \equiv A$

$\overline{\overline{A}} \equiv A$

$A \cdot A \equiv A$

$A + A \equiv A$

$A \cdot \overline{A} \equiv 0$

$A + \overline{A} \equiv 1$

commutative law

$A + B \equiv B + A$

$A \cdot B \equiv B \cdot A$

associative law

$A \cdot (B \cdot C) \equiv (A \cdot B) \cdot C$

$A + (B + C) \equiv (A + B) + C$

distributive law

$A + BC \equiv (A + B) \cdot (A + C)$

$A \cdot (B + C) \equiv A \cdot B + A \cdot C$

De Morgan's law

$\overline{A \cdot B} \equiv \overline{A} + \overline{B}$

$\overline{A + B} \equiv \overline{A} \cdot \overline{B}$

$A + AB \equiv A(1 + B) \equiv A \cdot 1 \equiv A$

$A \cdot (A + B) \equiv (A + 0) \cdot (A + B) \equiv A + 0 \cdot B \equiv A + 0 \equiv A$

Proving Logical Equivalence

$p \equiv q$ ← not connectives

$p \equiv q$

if $p \Leftrightarrow q$ is a tautology.

compound propositions that have same truth values in all possible cases are logically equivalent.

Tautology :- TRUE for all values of the propositional variables in a compound propositions.

Contradiction :- False for all values of the propositional variables in a compound proposition.

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
F	T	T	F
T	F	T	F

Another example of Tautology :

$(p \wedge q) \rightarrow (p \vee q)$

Logical Equivalence

[example : De Morgan's Theorem]

$\neg(A \wedge B) \equiv \neg A \vee \neg B$

A	B	$\bar{A}$	$\bar{B}$	$A \cdot B$	$\overline{A \cdot B}$	$\bar{A} + \bar{B}$
F	F	T	T	F	T	T
F	T	T	F	F	T	T
T	F	F	T	F	T	T
T	T	F	F	T	F	F

Example 2 :- show that $p \rightarrow q$ and $\neg p \vee q$ are equivalent.

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$
F	F	T	T	T
F	T	T	T	T
T	F	F	F	F
T	T	F	T	T

1. Show that $(p \to r) \vee (q \to r)$ and $(p \wedge q) \to r$ are logically equivalent

$$(p \to r) \vee (q \to r)$$
$$\equiv (\neg p \vee r) \vee (\neg q \vee r)$$
$$\equiv (\neg p \vee \neg q) \vee (r \vee r)$$
$$\equiv (\neg p \vee \neg q) \vee r \qquad [\text{as } a \vee a \equiv a]$$
$$\equiv \neg(\neg(\neg p \vee \neg q)) \vee r \qquad [\text{as } \neg(\neg a) \equiv a]$$
$$\equiv \neg(p \wedge q) \vee r \qquad [\text{Using De Morgan's Theorem}]$$
$$\equiv (p \wedge q) \to r$$

2. Show that $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology.

$$(p \to q) \wedge (q \to r) \to (p \to r)$$
$$\equiv (\neg p \vee q) \wedge (\neg q \vee r) \to (\neg p \vee r)$$
$$\equiv \neg((\neg p \vee q) \wedge (\neg q \vee r)) \vee (\neg p \vee r)$$
$$\equiv \neg(\neg p \vee q) \vee \neg(\neg q \vee r) \vee (\neg p \vee r)$$
$$\equiv (p \wedge \neg q) \vee (q \wedge \neg r) \vee (\neg p \vee r)$$
$$\equiv (p \wedge \neg q) \vee (q \wedge \neg r) \vee \neg p \vee r$$
$$\equiv (p \wedge \neg q) \vee \neg p \vee (q \wedge \neg r) \vee r$$
$$\equiv ((p \vee \neg p) \wedge (\neg q \vee \neg p)) \vee ((q \vee r) \wedge (\neg r \vee r))$$
$$\equiv (T \wedge (\neg q \vee \neg p)) \vee ((q \vee r) \wedge T)$$
$$\equiv (\neg q \vee \neg p) \vee (q \vee r)$$
$$\equiv (\neg q \vee q) \vee (\neg p \vee r)$$
$$\equiv T \vee (\neg p \vee r) \equiv T$$