# Carleton University

## Laboratory Report

**Course #:** 1008 **Experiment #:** 1

# DC Circuits and Resistance

**Date Performed:**

**Date Submitted:**

**Lab Period:**

**Partner:**

**Station #:**

**TA:**

## Purpose

The goal of this experiment was to measure various electrical quantities, all revolving around voltage, current, and resistance. The resistance values of resistors placed in both series and parallel were determined using a digital multimeter (DMM). The relationship between voltage and current via Ohm's Law was observed by connecting the circuit to both a power supply and a computer so both the voltage and current were plot on a graph. The final quantity measured in the experiment was the resistivity of a wire, which was also determined using the DMM while taking into account the length of each wire section.

## Theory

A resistor is a device that adds electrical resistance to a circuit, generally used to reduce current flow or divide voltages. Each resistor contains a different amount of resistance however, to specify, each resistor contains a unique colour pattern in order to identify the value of its resistance.


**Figure 1.** A colour code (Right) displaying the value each colour represents, ranging from digits to powers and tolerance. A diagram of a resistor (Left) showing the order in which the colour slots appear and what they signify.

Source:

Resistors implemented in a circuit can be connect in two ways; series or parallel. Finding the total resistance of the circuit depends if the resistors are connected in series or parallel. In the case of a series formation consisting of two resistors, the total resistance can be calculated with the following formula

$$R_{eq} = R_1 + R_2 \quad \textbf{(1)}$$

For a circuit with two resistors set up in a parallel structure, the total resistance is found with

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \quad \textbf{(2)}$$

Where $R_{eq}$ is the total resistance in the circuit and $R_1$ and $R_2$ are the two resistors, all in the units of ohms (Ω).

The uncertainties of each of these values can be found using the following equation assuming only two resistors

$$\textbf{Series: } \sigma_{R_{eq}} = \sqrt{\sigma_{R_1}^2 + \sigma_{R_2}^2} \quad \textbf{(3)}$$

$$\textbf{Parallel: } \sigma_{R_{eq}} = \sqrt{\frac{R_1^4\sigma_{R_2}^2 + R_2^4\sigma_{R_1}^2}{(R_1+R_2)^4}} \quad \textbf{(4)}$$

Ohm's law describes the relationship of a resistor with the current that goes through it and the voltage drop, when plotted on a graph the slope of the voltage vs current line is equal to the resistance. Naturally, an equation emerges from this relationship

$$V = IR \quad \textbf{(5)}$$

Where $V$ is the voltage in volts, $I$ is the current in unit amps, and $R$ is the resistance in ohms (Ω).

Resistivity is a quantity that represents how much a certain material can resist electric current, resistivity varies with each object as it is dependent on the proportions of it as seen in the formula

$$\rho = \frac{RA}{L} \quad \textbf{(6)}$$

Where $\rho$ is the resistivity in ohm-meters ($\Omega \cdot \mathrm{m}$), $L$ is the length of the object in meters, $A$ is the surface area of the object in $\mathrm{m}^2$, and $R$ is the resistance in Ω.

In the case of a cylindrical wire the formula can be manipulated to account for its cross sectional area.

$$R_w = \frac{4\rho}{\pi d^2}L \quad \textbf{(7)}$$

When splitting the wire into different length components, uncertainty propagation was used as some lengths were a collection of smaller lengths, the formula below was used

$$\sigma_f = \sqrt{a^2\sigma_x^2 + b^2\sigma_y^2} \quad \textbf{(8)}$$

where *ax* and *bx* are two variables with constant coefficients.

To determine whether values are in agreement with one another the consistency test is utilized, if the result of the test is less than or equal to 2 the results are consistent with each other. The test is represented with the following formula

$$t = \frac{|x_1 - x_2|}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}} \quad \textbf{(9)}$$

where $x_1$ and $x_2$ are the two measurements in the desired units, along with both of their respective uncertainties with the same desired units.

The uncertainty of the graphically obtained resistivity can be found with the following formula

$$\sigma_{\rho_g} = \rho_g \sqrt{(\frac{\sigma_m}{m})^2 + (\frac{2\sigma_d}{d})^2} \quad \textbf{(10)}$$

The uncertainty for the calculated resistivity can be found with the following formula

$$\sigma_{\rho_c} = \rho_c \sqrt{(\frac{\sigma_{R_w}}{R_w})^2 + (\frac{2\sigma_d}{d})^2 + (\frac{\sigma_{L_{AF}}}{L_{AF}})^2}$$

## Apparatus

- Voltage Probe (Range: 0-6 V, Precision: 0.002V)
- Digital Multimeter (Range: 0-200 Ω, Precision: 0.02 Ω)
- Meter Stick (Range: 0-100 cm, Precision: 0.1 cm)
- Current Probe (Range: 0-600 mA, Precision: 0.02 mA)
- Resistors (2)
- Banana Plug Wires (4)
- Circuit Board
- Shorting Plugs (2)

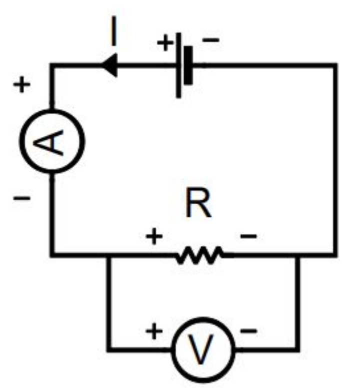


**Figure 2.** DC circuit diagram outlining the position of specific components of the circuit, including the current/voltage probe and the resistor, as well as indicating the direction of the current.

Source:

# Observations

**Table.1** Quantitative data regarding the resistance of each resistor based on its given colour code, along with its set tolerance percentage which can be used to find the range of appropriate values. The approximate resistance from the DMM is recorded as well as if it falls in the tolerance range.

| **Colours** | **Color Code R (Ω)** | **Tolerance** | **Range of Values (Ω)** | **DMM R (Ω)** | **DMM value in range (yes/no)** |
|---|---|---|---|---|---|
| Brown, Black, Brown, Gold | 100 | 5%, 5Ω | 95-105 | $99.54 \pm 0.01$ | Yes |
| Green, Blue, Black, Gold | 56 | 5%, 2.8Ω | 53.2-58.8 | $55.01 \pm 0.01$ | Yes |

**Table 2.** Quantitative data concerning the total resistance of the two resistors in both a series and parallel formation. Each total resistance was determined using a DMM and through basic computation, both values where then compared using the t-Test to establish consistency.

| | **$R_{eq}$ from DMM (Ω)** | **$R_{eq}$ calculated (Ω)** | **t-Test Value** | **Consistent/ Inconsistent** |
|---|---|---|---|---|
| **Series** | $154.65 \pm 0.01$ | $154.55 \pm 0.01$ | 7.07 | Inconsistent |
| **Parallel** | $35.34 \pm 0.01$ | $35.43 \pm 0.004$ | 8.36 | Inconsistent |

**Table 3.** Quantitative data of the length of each individual section of the wire, measured using a standard meter stick.

| **Wire Section** | AB | BC | CD | DE | EF |
|---|---|---|---|---|---|
| **Section Length *m* ($\pm$0.001)** | 0.387 | 0.394 | 0.395 | 0.396 | 0.392 |

## Calculations

**1. Calculation of the range of resistance values for Brown, Black, Brown, Gold resistor:**

R= 100 Ω
Tolerance = 5 Ω
**Refer Table 1**

$$\text{Range} = 100 \pm 5\ \Omega$$

**Therefore the range of the resistor is:**

Range = **95-105 Ω**

**2. Calculations for $R_{eq}$ and $\sigma_{R_{eq}}$ for both series and parallel:**

$R_1$= 99.54 Ω
$R_2$= 55.01 Ω
$\sigma_{R_{1,2}} = 0.01\Omega$
**Refer Table 1**

| Series | Parallel |
|---|---|
| $R_{eq} = R_1 + R_2$<br>$R_{eq} = 99.54\Omega + 55.01\Omega$<br>$R_{eq} = 154.55\,\Omega$<br>$\sigma_{R_{eq}} = \sqrt{\sigma_{R_1}^2 + \sigma_{R_2}^2}$<br>$\sigma_{R_{eq}} = \sqrt{(0.01)^2 + (0.01)^2}$<br>$\sigma_{R_{eq}} = 0.0141\Omega$ | $R_{eq} = \frac{R_1 R_2}{R_1+R_2}$<br>$R_{eq} = \frac{(99.54\Omega)\cdot(55.01\Omega)}{(99.54\Omega) + (55.01\Omega)}$<br>$R_{eq} = 35.43\ \Omega$<br>$\sigma_{R_{eq}} = \sqrt{\frac{R_1^4\sigma_{R_2}^2 + R_2^4\sigma_{R_1}^2}{(R_1+R_2)^4}}$<br>$\sigma_{R_{eq}} = \sqrt{\frac{(99.54\Omega)^4\cdot(0.01\Omega)^2 + (55.01\Omega)^4\cdot(0.01\Omega)^2}{(99.54\Omega + 55.01\Omega)^4}}$<br>$\sigma_{R_{eq}} = 0.004\,\Omega$ |

**Therefore the total resistance for parallel and series is:**

Series $R_{eq}$ = **(154.55 ± 0.01)** Ω
Parallel $R_{eq}$ = **(35.43 ± 0.004)** Ω

**3. Consistency calculation for series and parallel $R_{eq}$:**

Series: $x_1$= 154.65 Ω (DMM), $x_2$= 154.55 Ω, $\sigma_{x_1} = 0.01\,\Omega$, $\sigma_{x_2} = 0.01\,\Omega$
Parallel: $x_1$= 35.43 Ω, $x_2$= 35.34 Ω (DMM), $\sigma_{x_1} = 0.004\,\Omega$, $\sigma_{x_2} = 0.01\Omega$
**Refer Table 2**

| **Series** | **Parallel** |
|---|---|
| $t = \frac{\lvert x_1 - x_2 \rvert}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}}$<br>$t = \frac{\lvert 154.65\Omega - 154.55\Omega \rvert}{\sqrt{(0.01\Omega)^2 + (0.01\Omega)^2}}$<br>$t = 7.01 > 2$ | $t = \frac{\lvert x_1 - x_2 \rvert}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}}$<br>$t = \frac{\lvert 35.43\Omega - 35.34\Omega \rvert}{\sqrt{(0.004\Omega)^2 + (0.01\Omega)^2}}$<br>$t = 8.36 > 2$ |

**Therefore** the calculated and DMM $R_{eq}$ for series and parallel are **not consistent** as the test yielded values greater than 2 indicating that the values are not consistent.

**4. Consistency test for the slope in *V* vs. *I* graph and $R_1$:**

$R_1$= 99.54 Ω, $\sigma_{R_1} = 0.01\,\Omega$ **- Refer Table 1**
$m$ = 99.97 Ω, $\sigma_m = 0.3320\,\Omega$ **- Refer Figure 3**

$$t = \frac{\lvert x_1 - x_2 \rvert}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}}$$

$$t = \frac{\lvert 99.97\Omega - 99.54\Omega \rvert}{\sqrt{(0.3320\Omega)^2 + (0.01\Omega)^2}}$$

$$t = 1.29$$

**Therefore the slope and the resistance are consistent with each other as the test yielded a value that was less than 2.**

**5. AF side length and uncertainty calculation:**

**Refer to Table 3** for Wire Section Lengths/Uncertainties.

| **Length** | **Uncertainty** |
|---|---|
| AF = AB+BC+CD+DE+EF | $\sigma_{AF} = \sqrt{(\sigma_{AB})^2 + (\sigma_{BC})^2 + (\sigma_{CD})^2 + (\sigma_{DE})^2 + (\sigma_{EF})^2}$ |

| | |
|---|---|
| AF= 0.387m+0.394m+0.395+0.396m+0.392m<br><br>AF = 1.96m | $\sigma_{AF} = \sqrt{(0.001m)^2 + (0.001m)^2 + (0.001m)^2 + (0.001m)^2 + (0.001m)^2}$<br><br>$\sigma_{AF} = 0.0022361\text{m}$ |

**Therefore the value of AF side length is:**

$$\mathbf{AF = (1.96 \pm 0.002)m}$$

**6. Calculation of $\rho_g$ and $\sigma_{\rho_g}$ :**

$d = (0.405 \pm 0.002) \times 10^{-3}$ m,

$m$= $(8.159 \pm 0.09)\ \Omega$/m **-Refer Figure 4**

| Value | Uncertainty |
|---|---|
| $R_w = \frac{4\rho}{\pi d^2}L$<br>$mL + b = \frac{4\rho}{\pi d^2}L$<br>$m = \frac{4\rho}{\pi d^2}$<br>$\rho = \frac{m(\pi d^2)}{4}$<br>$\rho = \frac{8.159\Omega/m(\pi \cdot (4.05\times10^{-4}m)^2)}{4}$<br>$\rho = 1.05 \times 10^{-6}\ \Omega \cdot m$ | $\sigma_{\rho_g} = \rho_g\sqrt{(\frac{\sigma_m}{m})^2 + (\frac{2\sigma_d}{d})^2}$<br>$\sigma_{\rho_g} = 1.05 \times 10^{-6}\Omega \cdot m\sqrt{(\frac{0.09\Omega/m}{8.159\Omega/m})^2 + (\frac{2\cdot(0.002\times10^{-3}m)}{4.05\times10^{-4}m})^2}$<br>$\sigma_{\rho_g} = 1.55 \times 10^{-8}\ \Omega \cdot m$ |

**Therefore the final value of $\rho_g$ is:**

$$\rho_g = \mathbf{(1.05 \pm 0.02) \times 10^{-6}\ \Omega \cdot m}$$

**7. Calculation of $\rho_c$ and $\sigma_{\rho_c}$ :**

$L_{AF}$= 1.96 m, $R_w$= 16.36 $\Omega$ **-Refer Table 5**

$d = (0.405 \pm 0.002) \times 10^{-3}$ m

| Value | Uncertainty |
|---|---|
| $(\frac{\pi d^2}{4L})R_w = \frac{4\rho_c}{\pi d^2}L\ (\frac{\pi d^2}{4L})$<br>$\rho_c = \frac{R_w \pi d^2}{4L}$<br>$\rho_c = \frac{(16.36\Omega)\pi(4.05\times10^{-4}m)^2}{4(1.96m)}$<br>$\rho_c = 3.42 \times 10^{-7}\Omega\cdot m$ | $\sigma_{\rho_c} = \rho_c \sqrt{(\frac{\sigma_{R_w}}{R_w})^2 + (\frac{2\sigma_d}{d})^2 + (\frac{\sigma_{L_{AF}}}{L_{AF}})^2}$<br>$\sigma_{\rho_c} = 3.42 \times 10^{-7}\Omega\cdot m \sqrt{(\frac{0.01\Omega}{16.36\Omega})^2 + (\frac{2\cdot(0.002\times10^{-3}m)}{(0.405\times10^{-3}m)})^2 + (\frac{0.002m}{1.96m})^2}$<br>$\sigma_{\rho_c} = 3.40 \times 10^{-9}\Omega\cdot m$ |

**Therefore the final value of $\rho_c$ is:**

$$\rho_c = \mathbf{(3.42 \pm 0.034) \times 10^{-7}\ \Omega \cdot m}$$

**8. Calculation for average resistivity, $\rho_{av}$ value and $\sigma_{\rho_{av}}$ :**

| Value | Uncertainty |
|---|---|
| $\rho_{av} = \frac{\rho_g + \rho_c}{2}$<br>$\rho_{av} = \frac{(1.05\times10^{-6}\Omega\cdot m) + (3.42\times10^{-7}\Omega\cdot m)}{2}$<br>$\rho_{av} = 6.96 \times 10^{-7}\ \Omega\cdot m$ | $\sigma_{\rho_{av}} = \sigma_{\rho_c} + \sigma_{\rho_g}$<br>$\sigma_{\rho_{av}} = 3.40 \times 10^{-9}\Omega\cdot m + 1.55 \times 10^{-8}\Omega\cdot m$<br>$\sigma_{\rho_{av}} = 1.89 \times 10^{-8}\Omega\cdot m$ |

**Therefore the final value of $\rho_{av}$ is:**

$$\rho_{av} = \mathbf{(6.96 \pm 0.2) \times 10^{-7}\ \Omega \cdot m}$$

**9. T-test between graphical and calculation resistivity:**

$$t = \frac{|x_1 - x_2|}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}}$$

$$t = \frac{|1.05\times10^{-6}\Omega\cdot m - 3.42\times10^{-7}\Omega\cdot m|}{\sqrt{(0.02\times10^{-6}\Omega\cdot m)^2 + (0.034\times10^{-7}\Omega\cdot m)^2}}$$

$$t = 34.9 > 2$$

**Therefore the graphical and calculation resistivity are not consistent with each other as the t-test produced a value that is greater than 2.**

## Results

The total resistance for the series circuit was $(154.55 \pm 0.01)\,\Omega$, for the parallel circuit the total resistance was $(35.43 \pm 0.004)\,\Omega$, each had t-tests values of 7.07 and 8.36 respectively when compared to their DMM value counterparts. The resistance obtained from the slope of the *V* vs. *I* graph was consistent with the resistance from the DMM with a t-test value of 1.29. The average resistivity was determined to be $(6.96 \pm 0.2) \times 10^{-7}\,\Omega \cdot m$ and the graphical and calculation resistivity where not consistent with one another, having a t-test value of 34.9.

## Discussion

Throughout the experiment the objective was to find quantities regarding circuits and electricity, this was done through three different methods which where then all compared to one another to test consistency. Overall the quantities of this lab were not consistent with one another, with the exception of the test between the slope resistance and the DMM resistance, every t-test turned out to be inconsistent. Regarding the inconsistency for the resistance in the series and parallel circuits, this is mainly due to the relatively high uncertainty on both values which worked to raise the result of the t-test. There were many sources of error through the experiment, one of the main component would have to be the uncertainty on the DMM which was only $0.01\,\Omega$ this would then go on to cause inconsistencies in future calculations. A device with a smaller uncertainty is recommended to fix this issue. Another prominent source of uncertainty is the resistor itself, each coming with a certain tolerance percentage makes the true value of the resistance to be undocumented, considering this is the base of the experiment it could very likely affect quantities determined later. Resistors with set values are recommended to prevent any inaccuracies in their values. When looking at measurements of the same quantities taken during different experiments there are some similarities, for example a wire of nichrome has resistivity of $1.10 \times 10^{-6}\,\Omega\,\text{m}$. The wire had a length of 1500mm and a diameter of 0.05mm, based on the similar dimensions it makes sense that the resistance are in range with each other. Resistors are very important when it comes to the flow of the circuit, they have the power to limit the flow of the current. Other applications that it has include lowering voltage, testing loads in a specific circuit for safety reasons or otherwise, and they can be used as a fuse with their low power rating. Looking at other instruments such as the voltmeter and the ammeter, they had restrictions of the voltmeter being connected in parallel and the ammeter connected in series. The voltmeter must be connected in parallel due to the simple fact that objects have the same potential difference in parallel. The same notion applies to the ammeter connected in series, objects experience the same current in series, in order to not disrupt the measurements the devices must be in accordance with these restrictions. The method used to find the resistor was good in its precision however there are better alternatives, this mainly due to the flaw of the colour code where the tolerance value disperses the value range and makes the method more inconsistent. The better method was in part two of the experiment where the current and voltage where first found to then find the resistance through the linear relationship. This is overall a more natural method and it disregards the colour code on the resistor and only looks at its current and voltage.