Lab #2: Servo System Simulation

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SYSC 3600B

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3.2.2

Case	A	ζ	ω_n	ω_d	Behavior
1	4	2.0700	0.2472	0.4480i	overdamped
2	17	1.0041	0.5096	0.461i	Critically damped
3	35	0.6998	0.7312	0.5223	underdamped
4	300	0.2390	2.1407	2.0786	underdamped

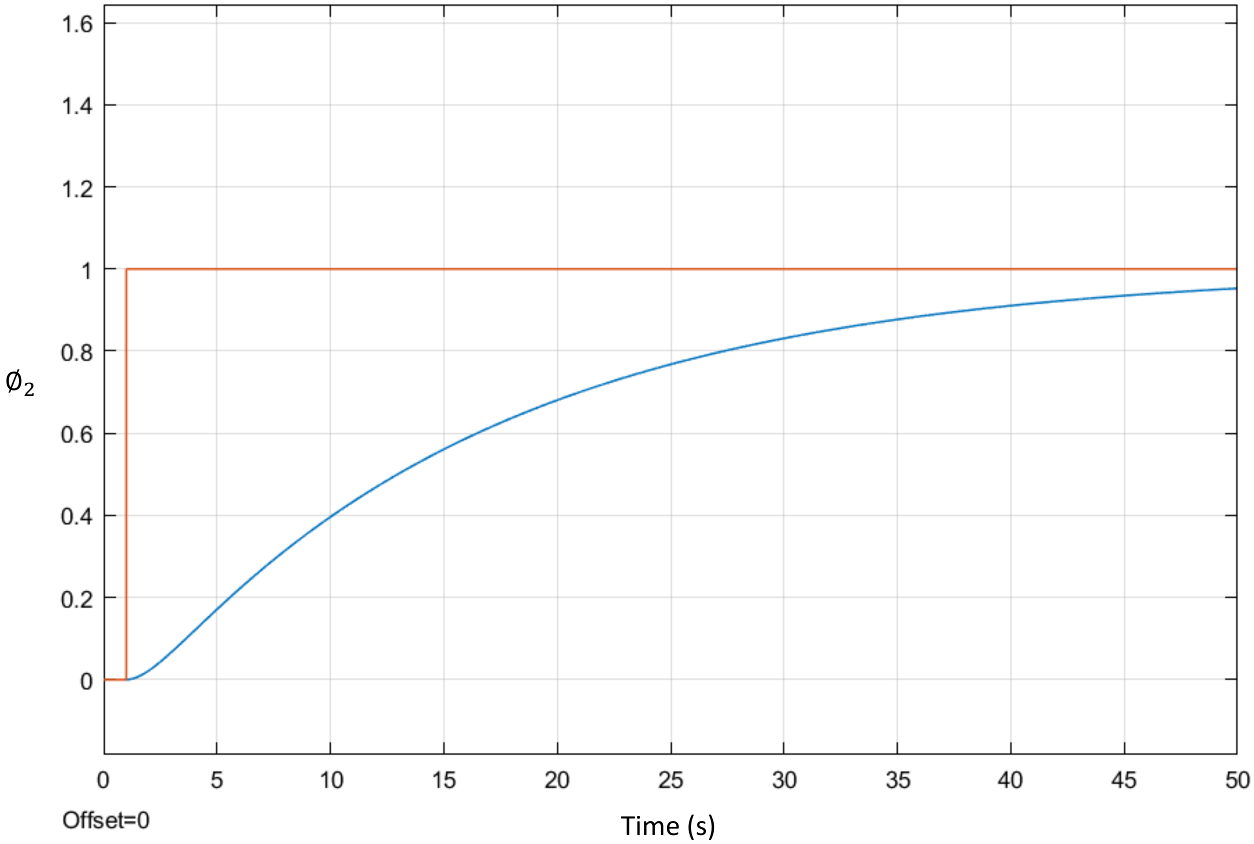
Table 1. Summary of values for different cases of amplifier values.

3.2.3

Case	A	Final Value
1	4	1
2	17	1
3	35	1
4	300	1

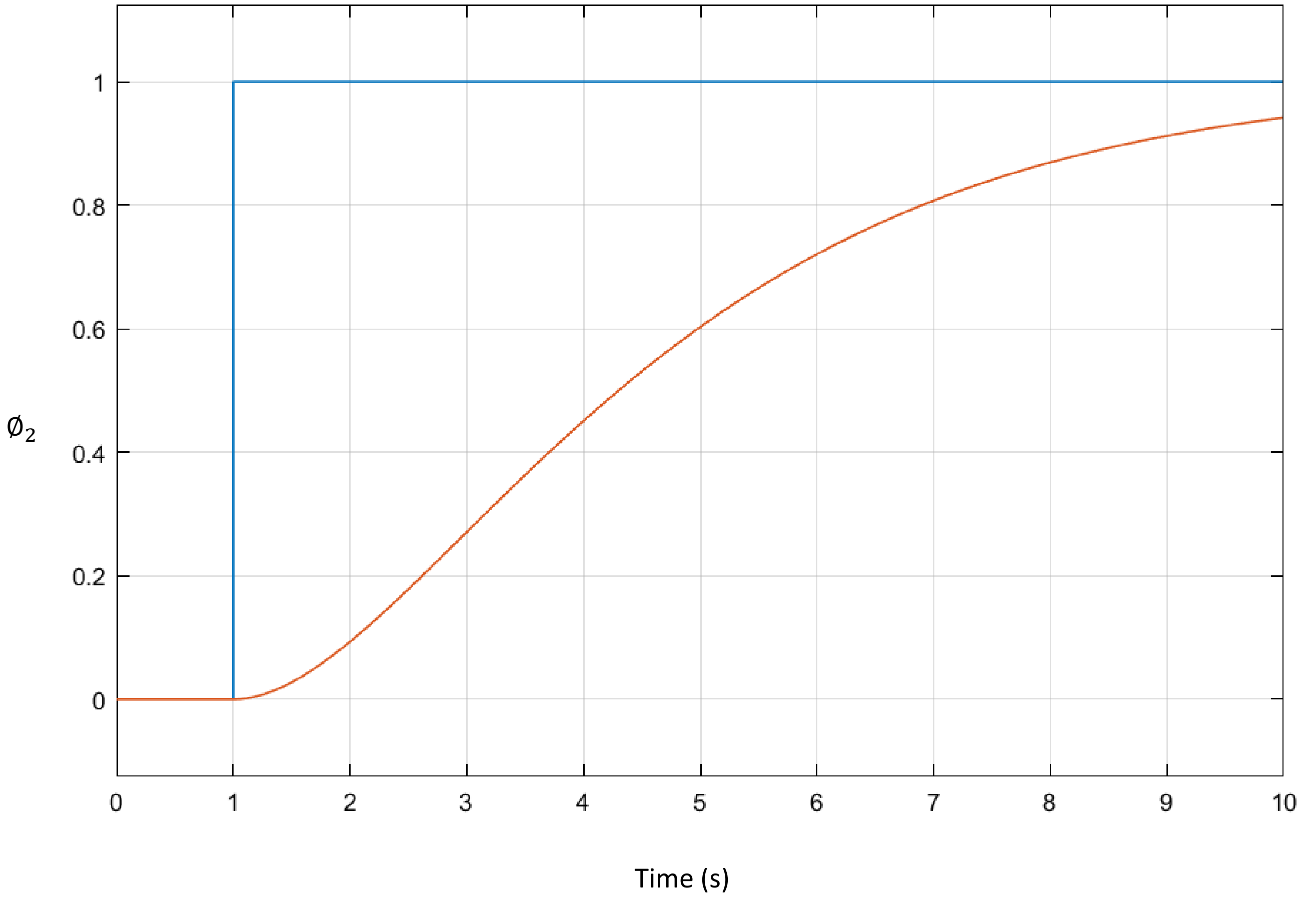
Table 2. Final value theorem calculated approached value

Figure 1. Simulation of servo system when amplifier value is 4. Blue: step response.



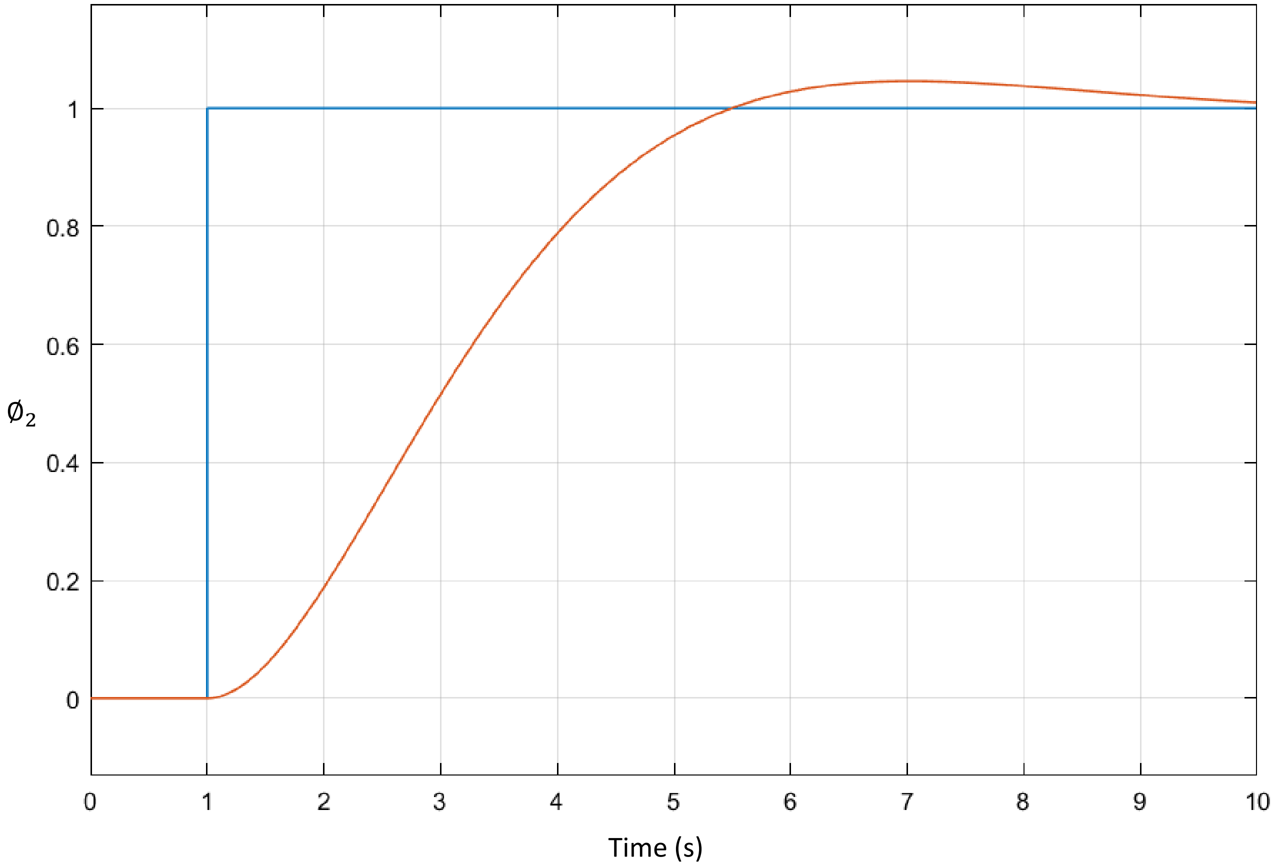
Final value = 1. It agrees with the calculated value from the FVT. The behavior of the step response agrees as damping > 1, so it is overdamped and takes a long to approach the final value of 1. There are no oscillations and damped frequency (ωd) is an imaginary number so it agrees.

Figure 2. Simulation of servo system when amplifier value is 17. Red: step response.



Final value = 1. It agrees with the calculated value from the FVT. The behavior of the step response agrees as damping ~= 1, so it is critically damped and takes the least amount of time to approach the final value of 1 without overshooting and oscillating. There are no oscillations and damped frequency (ωd) is an imaginary number so it agrees.

Figure 3. Simulation of servo system when amplifier value is 35.



Final value = 1 and it agrees with the calculated value from the FVT. The behavior of the step response agrees as damping < 1, so it is underdamped. There are oscillations and damped frequency (ωd) is a real number so they agree. Converting ωd to time between peaks:

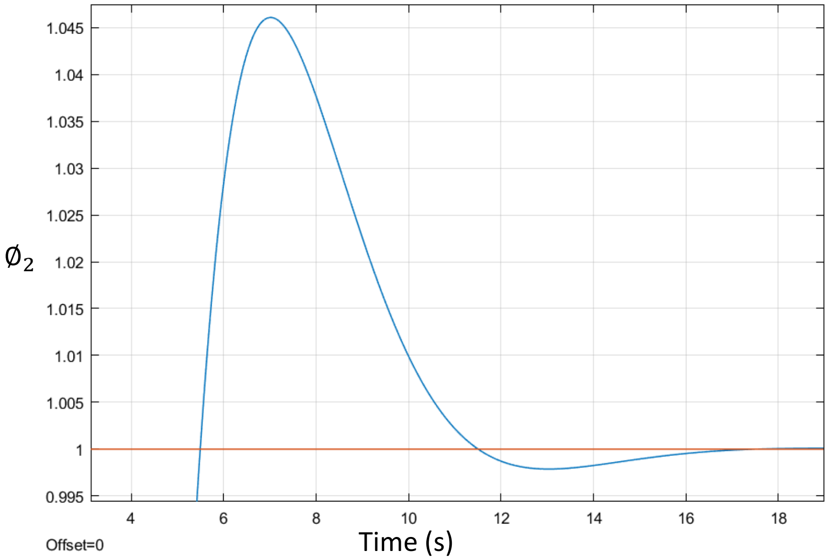


Figure 4. Simulation of servo system when amplifier value is 35. Zoomed in to view peaks.

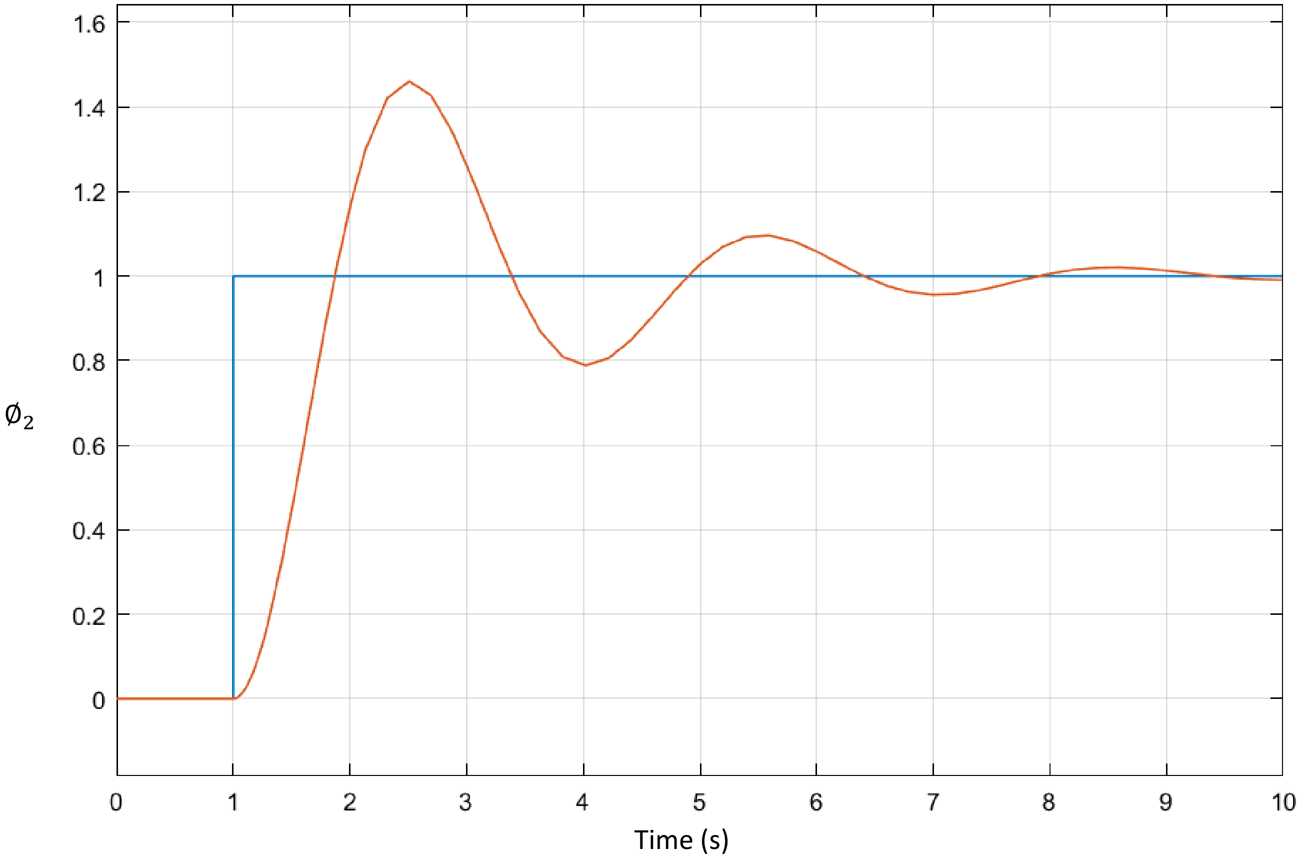
$$\textit{time between peaks} = \frac{2\pi}{\omega_d}$$

$$\textit{time between peaks} = \frac{2\pi}{0.5223}$$

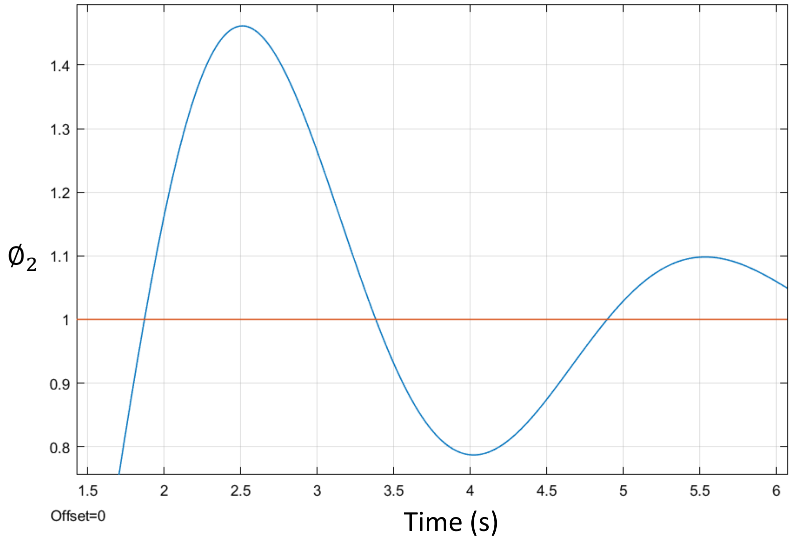
$$\textit{time between peaks} = 12.029 \textit{ seconds}$$

There is a delay of 1 second for the input step. That is the first minimum of the curve. The second minimum is at ~13 seconds, and the difference between the two agrees with the above value.

Figure 5. Simulation of servo system when amplifier value is 300.



Final value = 1 and it agrees with the calculated value from the FVT. The behavior of the step response agrees as damping < 1, so it is underdamped. There are oscillations and damped frequency (ωd) is a real number so they agree. Converting ωd to time between peaks:



$$\mathit{time\ between\ peaks} = \frac{2\pi}{\omega_d}$$

$$\mathit{time\ between\ peaks} = \frac{2\pi}{2.0786}$$

$$\mathit{time\ between\ peaks} = 3.022\ \mathit{seconds}$$

The first peak is at ~2.5 seconds, the second peak is at ~5.5 seconds and the difference between the two agrees with the above value.

Figure 6. Simulation of servo system when amplifier value is 300. Zoomed in to view peaks.

3.3.2

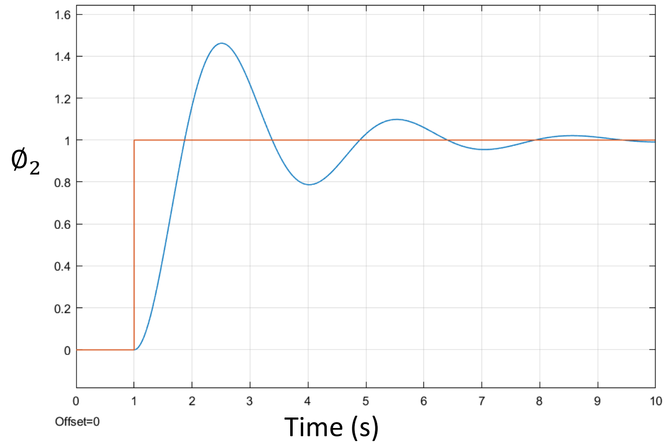


Figure 7. Simulation of servo system when amplifier value is 300 and $k_r = 0$. Same as Figure 5.

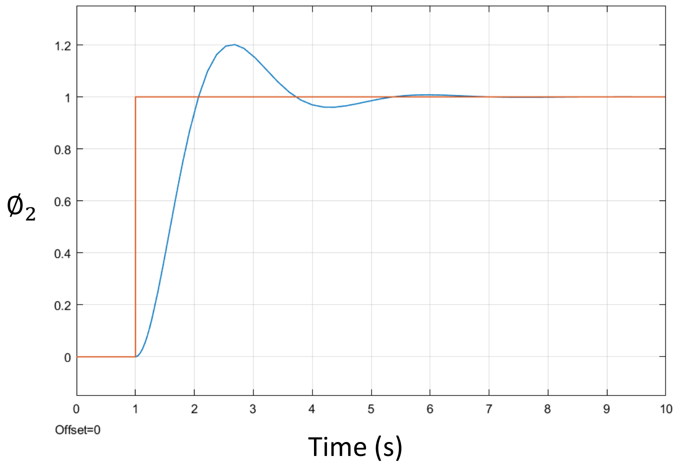


Figure 8. Simulation of servo system when amplifier value is 300 and $k_r = 0.2$

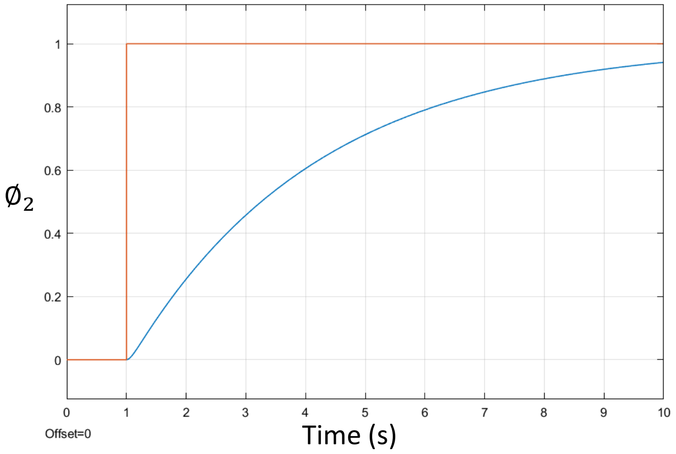


Figure 9. Simulation of servo system when amplifier value is 300 and $k_r = 3$

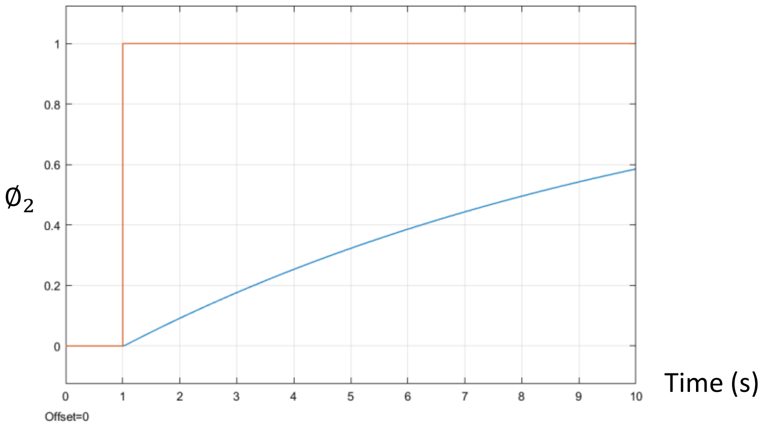


Figure 10. Simulation of servo system when amplifier value is 300 and $k_r = 10$

Increasing k_r increases the damping factor and thus the damped frequency. This is easily mathematically proved by looking at the transfer function:

$$H_{rate} = \frac{k_1 AB/N}{s^2 + (C + ABk_r)s + k_2 AB/N}$$

As the damping factor is equal to the second coefficient of the denominator which is changed by a factor of k_r. This is called a rate feedback loop (or velocity feedback for this system) and allows adjustments to ζ without affecting ω_n.

3.3.3

Rearranging damping equation to solve for k_r:

$$k_r = \frac{R_a J}{A k_m}\left(2\zeta\sqrt{\frac{k_2 A b}{N}} - \frac{R_a b + {k_m}^2}{R_a J}\right)$$

And setting damping factor to 1:

k_r = 0.711

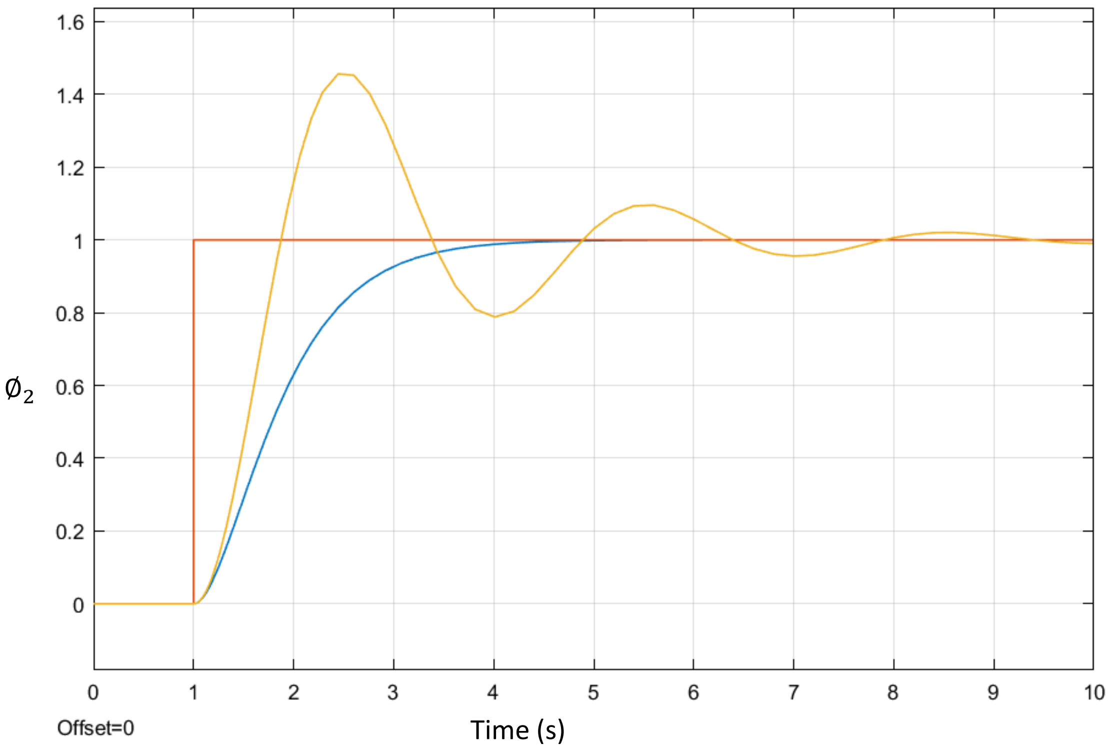


Figure 11. Simulation of servo system when amplifier value is 300. Red: step input. Yellow: $k_r = 0.$ Blue $k_r = 0.711.$

When k_r is decreased by a small amount: oscillations begin to occur and the value of ω_d converts from an imaginary number to a real number. It becomes underdamped as opposed to critically damped as ζ becomes less than 1.