Poisson Statistics

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PHYS 2604 A1

Supervisor: -

Partner: -

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Data Performed: -

Date Submitted: -

1.) Purpose

In this experiment, the statistics of random, independent events in physical measurements was explored. The random events were counts from a Geiger-Muller detector exposed to gamma rays created from the decay of 137Cs. The purpose of this experiment is to check that for smaller number of counts, the distribution is Poisson and as the number of counts increases, the distribution becomes closer and closer to a Gaussian distribution.

2.) Theory

If a total of N measurements were made, and a total of n bins, the frequency in the k-th bin is f_k and the predicted frequency is NP_k. And assuming Poisson's distribution $\sigma_k^2 = NP_k$, then χ^2 is defined as:

$$\chi^2 = \sum_{k=0}^{\iota} \frac{(f_k - NP_k)^2}{NP_k} \tag{1}$$

χ^2 for Poisson (2)	χ^2 for Gaussian (3)
$\sum_{k=0}^{\iota} \frac{\left(f_k - N\left({k_{av}}^{k} * e^{-\frac{k_{av}}{k!}}\right)\right)^2}{N\left({k_{av}}^{k} * e^{-\frac{k_{av}}{k!}}\right)}$	$\sum_{k=0}^{\iota} \frac{\left(f_k - N\left(\frac{e^{-\frac{(k-k_{av})^2}{2k_{av}}}}{\sqrt{2\pi k_{av}}}\right)\right)^2}{N\left(\frac{e^{-\frac{(k-k_{av})^2}{2k_{av}}}}{\sqrt{2\pi k_{av}}}\right)}$

The χ^2 test tells us which distribution suits the data better. χ^2 is then used as a lower limit for the integral of a Pearson distribution:

$$\frac{1}{\Gamma\left(\frac{\ell-2}{2}\right)2^{\left(\frac{\ell-2}{2}\right)}} \int_{\chi^2}^{\infty} x^{\left(\frac{\ell-2}{2}-1\right)} e^{\left(-\frac{x}{2}\right)} dx \tag{4}$$

Where Γ is defined as:

$$\Gamma(\mathrm{m}) = \int_{0}^{\infty} x^{\left(\frac{\ell-2}{2}\right)} e^{-x} dx \tag{5}$$

So, the Pearson distribution is:

$$\frac{1}{2^{\left(\frac{\ell-2}{2}\right)} \int_0^\infty x^{\left(\frac{\ell-2}{2}\right)} e^{-x} dx} \int_{\chi^2}^{\infty} x^{\left(\frac{\ell-2}{2}-1\right)} e^{\left(-\frac{x}{2}\right)} dx \tag{6}$$

Where χ^2 is the calculated value from equation (2), and ℓ is the number of bins (n). The value of equation (6) serves to compare to the significant level, which is 0.05 in this lab. If the value of the integral above is greater than 0.05, the hypothesis that the underlying distribution is Poisson's cannot be rejected, if the integral is smaller than 0.05 the hypothesis is rejected.

the dead time of the Geiger Muller counter. During the 90 micro seconds it takes to recover, gamma rays could be passing through it undetected. A method of approximating the loss is given by equation:

$$N = \frac{n}{1 - nt} \tag{7}$$

Where N is the approximated correct amount of counts, n is the observed value and t is the dead time of Geiger counter at $9 \times 10^{-5} s$.

3.) Apparatus:

Instrument	Description
Geiger-Muller detector	Detects radiation (explained further)
^{137}Cs	Radioactive material
LoggerPro files	Used to record and analyze data

Table 1. *Includes all apparatus used in lab.*

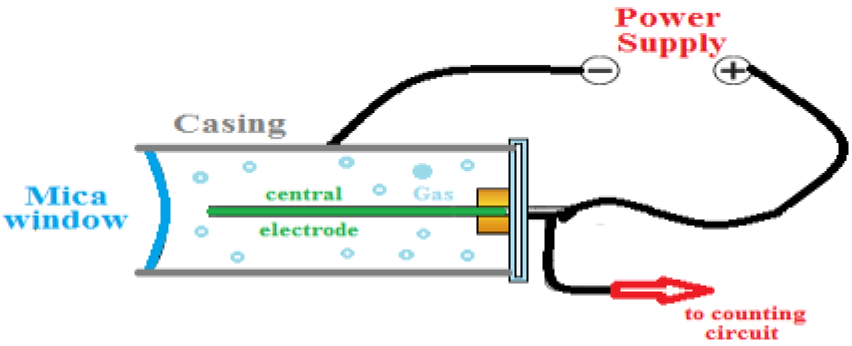


Figure 3. *Geiger-Muller detector*

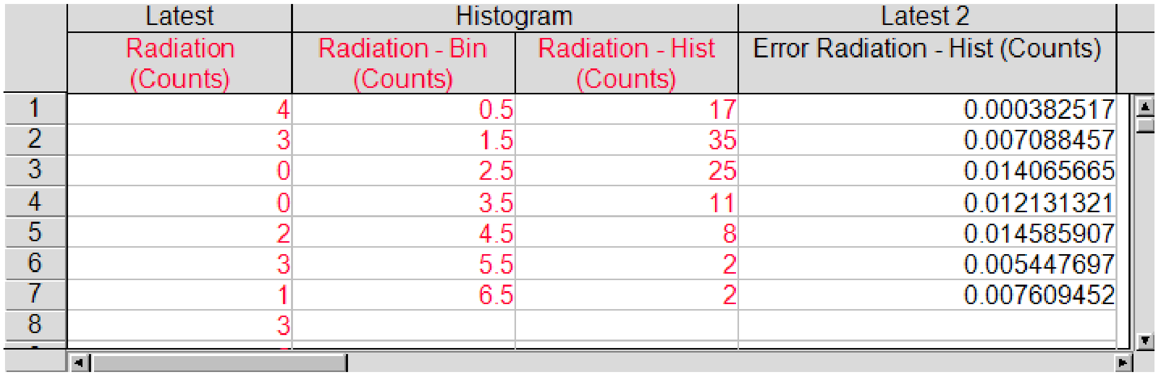
Ionizing particles passing through the detector produce an avalanche of charges which collectively induce a signal. See equation (7) in theory for an explanation on dead time.

4.) Procedure

After setting up the GM counter and interface and opening the LoggerPro file, the software was set to 100 samples of 5s duration for the background radiation. After the data was collected, gloves were worn, and the software was set to an appropriate distance from the 137Cs source to achieve about 3 counts per second and to collect 600 samples. Then the distance was adjusted to achieve about 15 counts per second.

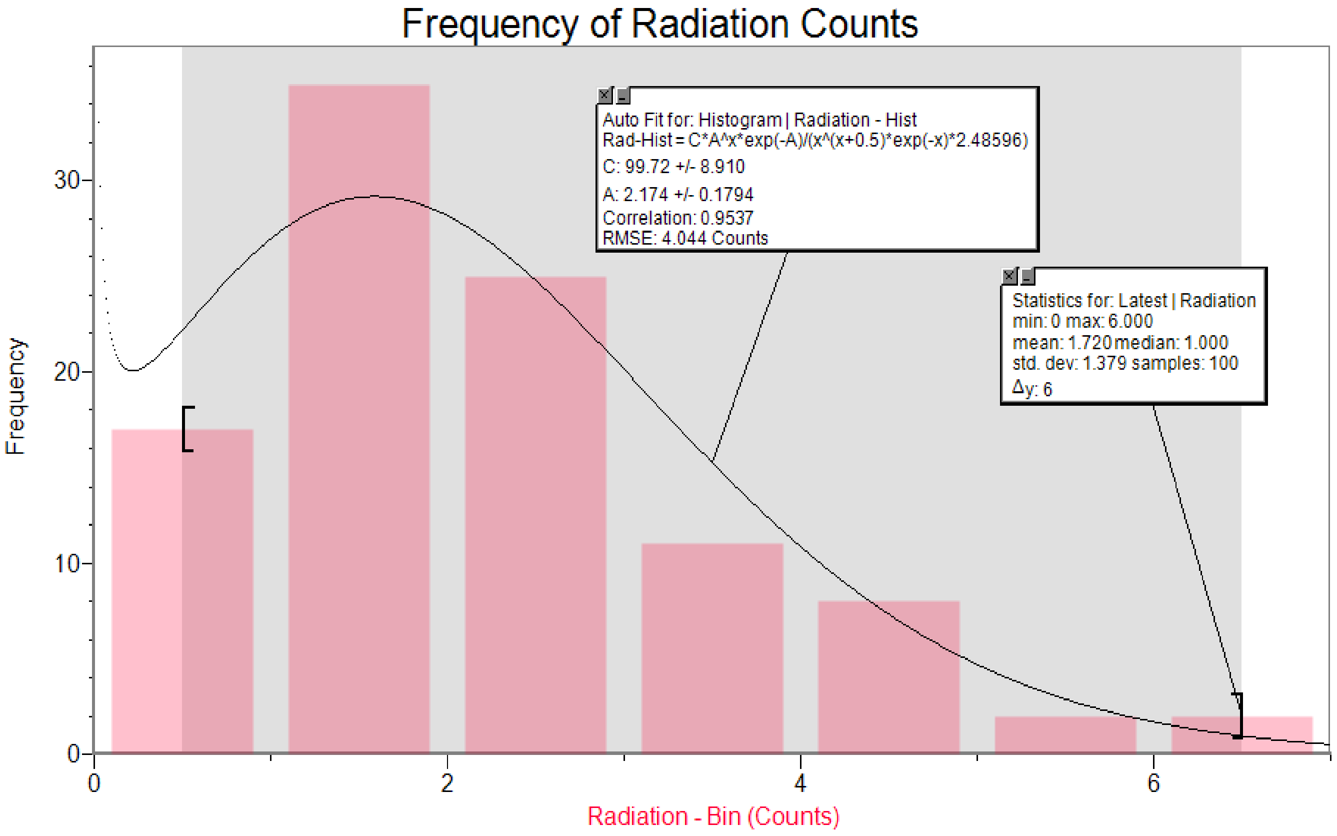
Afterwards, the data was analyzed using the 'helpforCHI2' LoggerPro file by copying the frequency of counts into the relevant columns to compute CHI squared.

5.) Observations



	Latest	Histogram		Latest 2
	Radiation (Counts)	Radiation - Bin (Counts)	Radiation - Hist (Counts)	Error Radiation - Hist (Counts)
1	4	0.5	17	0.000382517
2	3	1.5	35	0.007088457
3	0	2.5	25	0.014065665
4	0	3.5	11	0.012131321
5	2	4.5	8	0.014585907
6	3	5.5	2	0.005447697
7	1	6.5	2	0.007609452
8	3			

Table 2. *Contains raw data of measured Radiation counts with background radiation and placed into frequency bins. The error on Radiation – Hist (Counts) is the Radiation – Bin (Counts). The Error Radiation – Hist(Counts) affects the actual value in Radiation – Hist (Counts) due to the dead time of the Geiger Counter.*

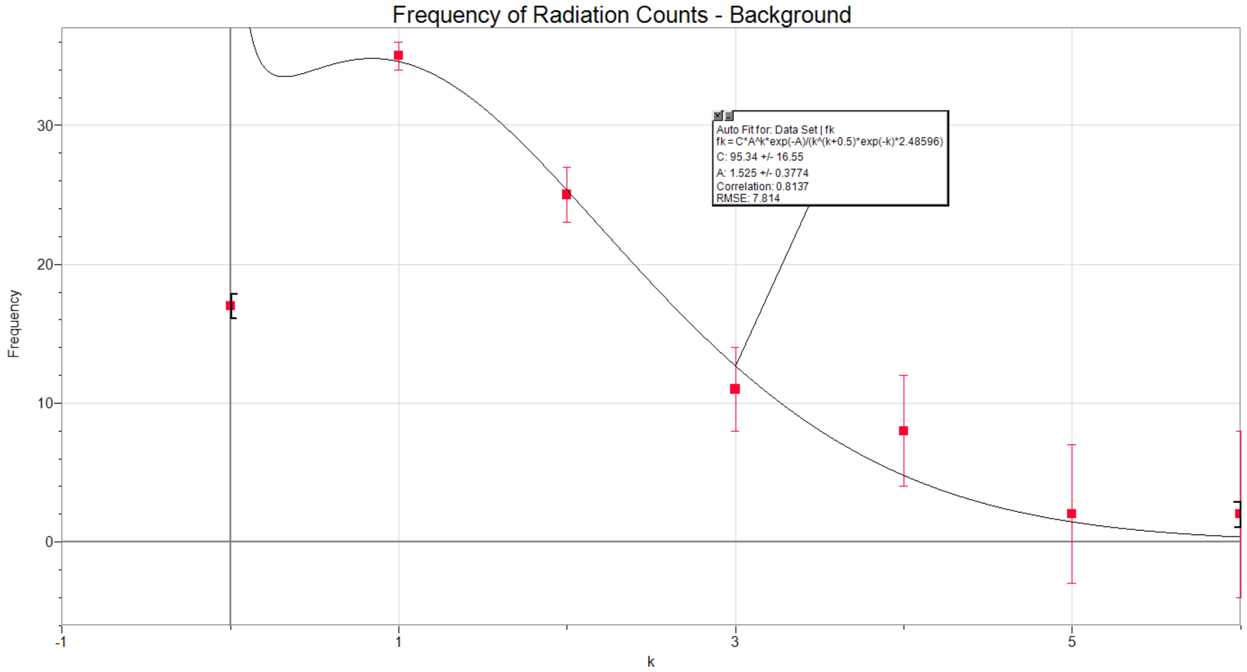


Auto Fit for: Histogram | Radiation - Hist
Rad-Hist = C*A^x*exp(-A)/(x^(x+0.5)*exp(-x)*2.48596)
C: 99.72 +/- 8.910
A: 2.174 +/- 0.1794
Correlation: 0.9537
RMSE: 4.044 Counts

Statistics for: Latest | Radiation
min: 0 max: 6.000
mean: 1.720 median: 1.000
std. dev: 1.379 samples: 100
Δy: 6

Figure 2. *Histogram of Frequency and Radiation – Bin (Counts) from table 2. Stats is used to calculate the sample mean frequency. A Poisson distribution is fit to the curve because the counts/s is small and a Poisson distribution will fit the data better than a Gauss distribution.*

average=2.17+/-0.18



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Auto Fit for: Data Set | fk
fk = C*A^k*exp(-A)/(k^(k+0.5)*exp(-k)*2.48596)
C: 95.34 +/- 16.55
A: 1.525 +/- 0.3774
Correlation: 0.8137
RMSE: 7.814
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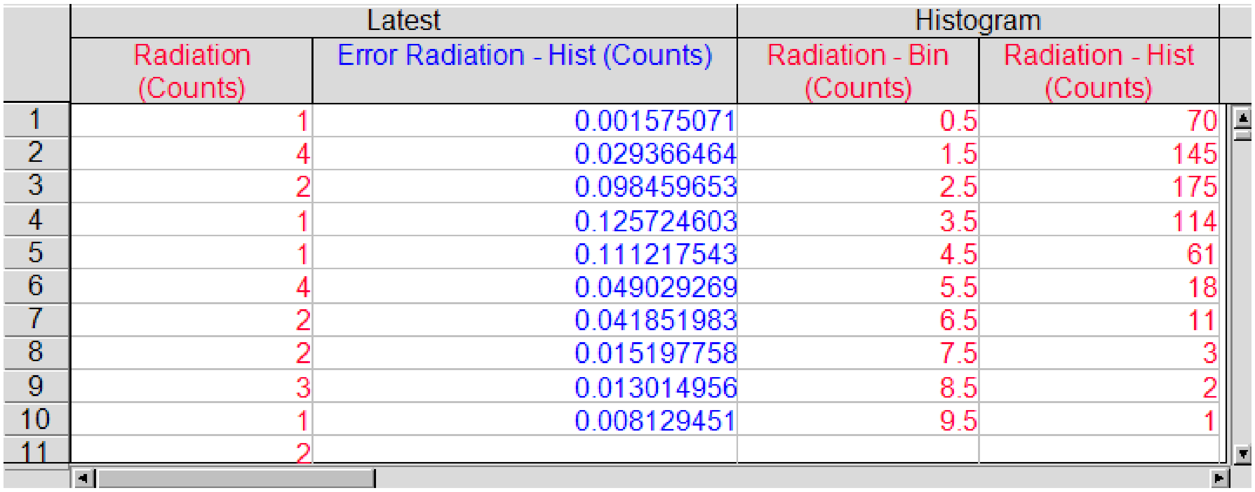
Figure 3. *Graph of Frequency and Radiation – Bin (Counts) with error bars of Radiation Bin – (Counts). The slight difference in coefficients between this Figure and the previous is due to the difference in Radiation – Bin (Counts), which is rounded down to an integer.*



	Latest		Histogram	
	Radiation (Counts)	Error Radiation - Hist (Counts)	Radiation - Bin (Counts)	Radiation - Hist (Counts)
1	1	0.001575071	0.5	70
2	4	0.029366464	1.5	145
3	2	0.098459653	2.5	175
4	1	0.125724603	3.5	114
5	1	0.111217543	4.5	61
6	4	0.049029269	5.5	18
7	2	0.041851983	6.5	11
8	2	0.015197758	7.5	3
9	3	0.013014956	8.5	2
10	1	0.008129451	9.5	1
11	2			

Table 3. *Contains raw data of measured Radiation counts with ~3 counts/s and placed into frequency bins. The error on Radiation – Hist (Counts) is the Radiation – Bin (Counts). The Error Radiation – Hist(Counts) affects the actual value in Radiation – Hist (Counts) due to the dead time of the Geiger Counter.*

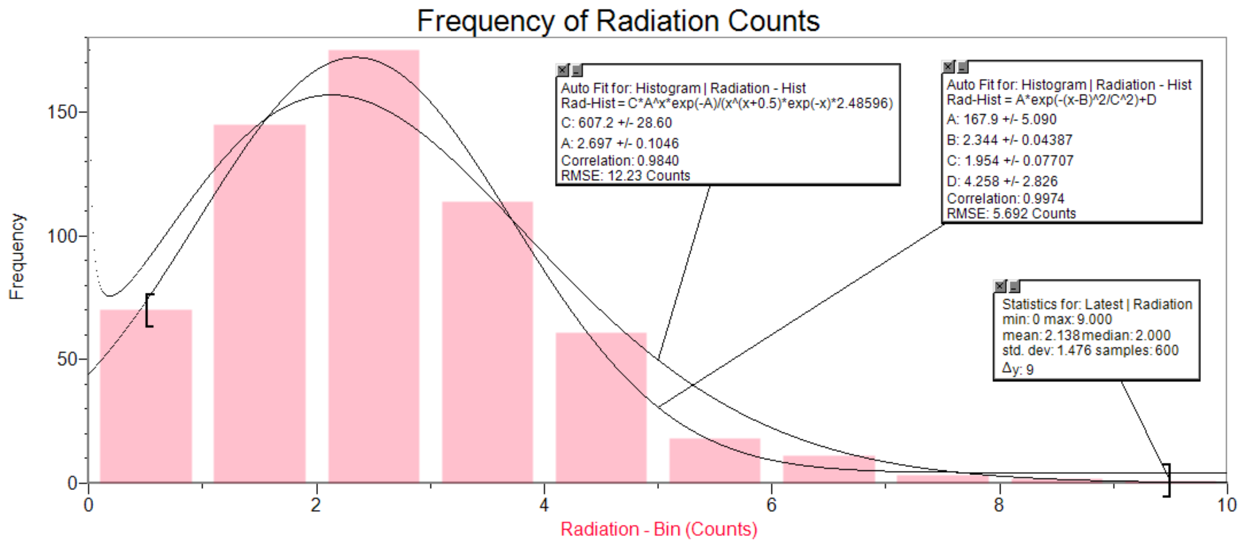


Figure 4. *Histogram of Frequency and Radiation – Bin (Counts) from table 3 with ~3 counts/s. A Poisson and a Gaussian distribution is fit to the curve to compare them to determine which applies better to this count value of ~3counts/s.*

k[avPoisson] =2.70+/-0.10

k[avGauss] = 2.34+/-0.04

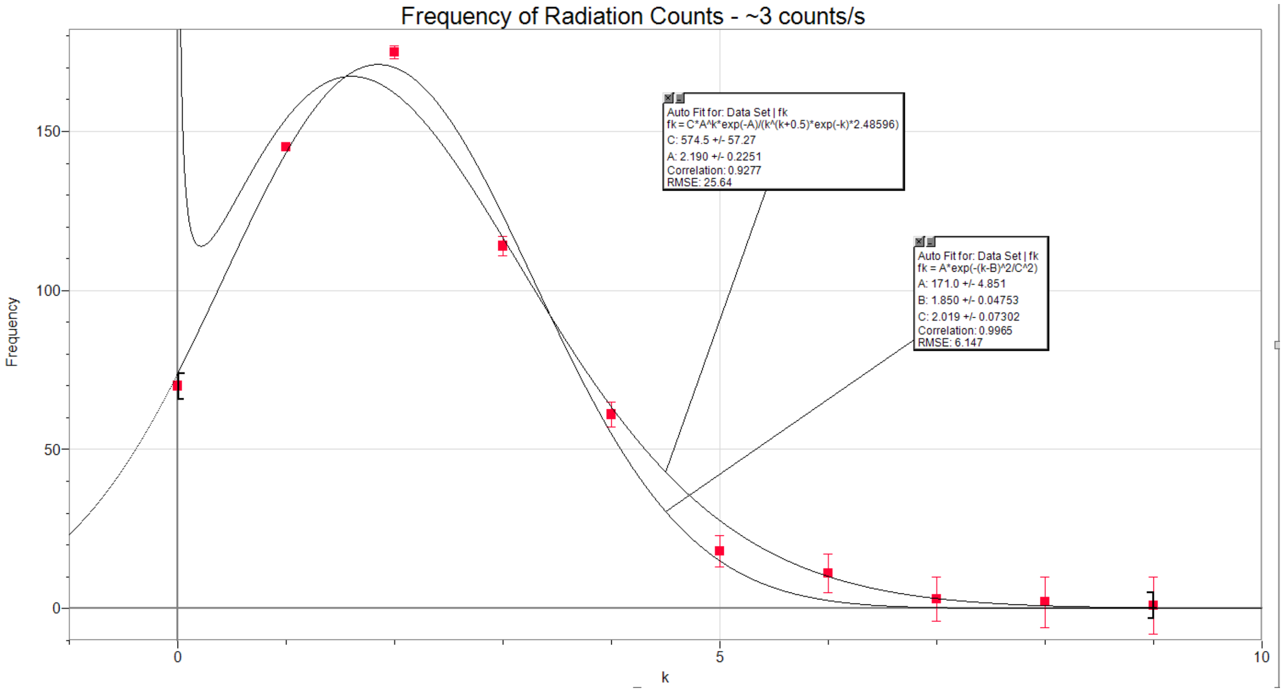
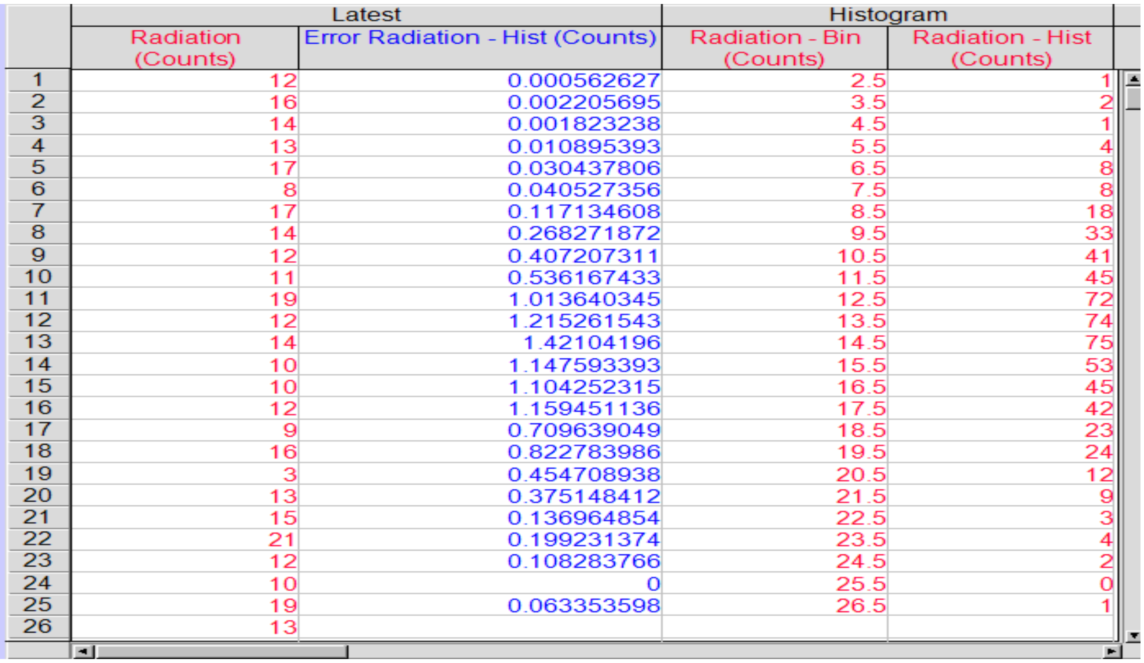


Figure 5. *Graph of Frequency and Radiation – Bin (Counts) at ~3counts/s with error bars of Radiation Bin – (Counts). The slight difference in coefficients between this Figure and the previous is due to the difference in Radiation – Bin (Counts), which is rounded down to an integer.*



	Latest		Histogram	
	Radiation (Counts)	Error Radiation - Hist (Counts)	Radiation - Bin (Counts)	Radiation - Hist (Counts)
1	12	0.000562627	2.5	1
2	16	0.002205695	3.5	2
3	14	0.001823238	4.5	1
4	13	0.010895393	5.5	4
5	17	0.030437806	6.5	8
6	8	0.040527356	7.5	8
7	17	0.117134608	8.5	18
8	14	0.268271872	9.5	33
9	12	0.407207311	10.5	41
10	11	0.536167433	11.5	45
11	19	1.013640345	12.5	72
12	12	1.215261543	13.5	74
13	14	1.42104196	14.5	75
14	10	1.147593393	15.5	53
15	10	1.104252315	16.5	45
16	12	1.159451136	17.5	42
17	9	0.709639049	18.5	23
18	16	0.822783986	19.5	24
19	3	0.454708938	20.5	12
20	13	0.375148412	21.5	9
21	15	0.136964854	22.5	3
22	21	0.199231374	23.5	4
23	12	0.108283766	24.5	2
24	10	0	25.5	0
25	19	0.063353598	26.5	1
26	13			

Table 4. *Contains raw data of measured Radiation counts with ~15 counts/s and placed into frequency bins. The error on Radiation – Hist (Counts) is the Radiation – Bin (Counts). The Error Radiation – Hist(Counts) affects the actual value in Radiation – Hist (Counts) due to the dead time of the Geiger Counter.*

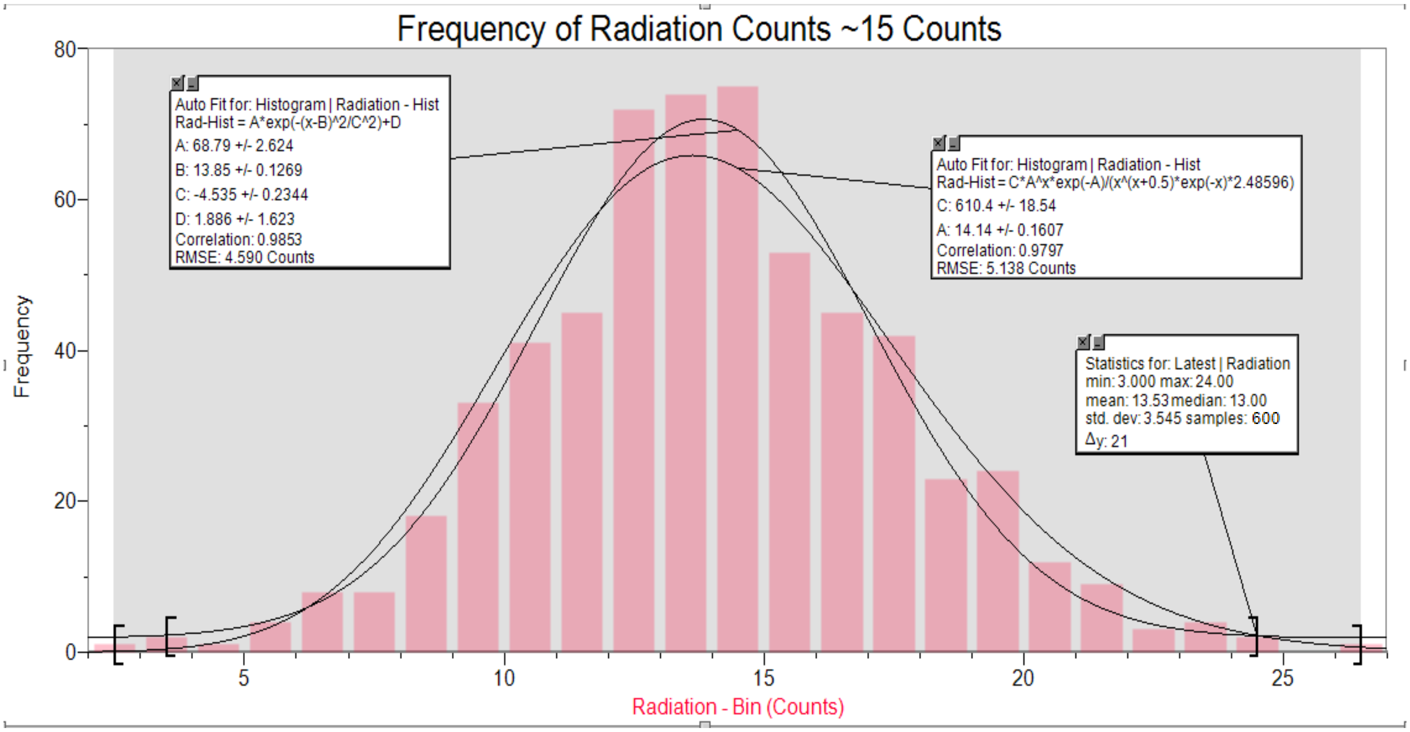


Figure 6. *Histogram of Frequency and Radiation – Bin (Counts) from table 3 with ~15 counts/s. A Poisson and a Gaussian distribution is fit to the curve to compare them to determine which applies better to this count value of ~15 counts/s.*

k[avGauss]=13.85+/-0.13

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k[avPoiss]=14.14+/-0.16

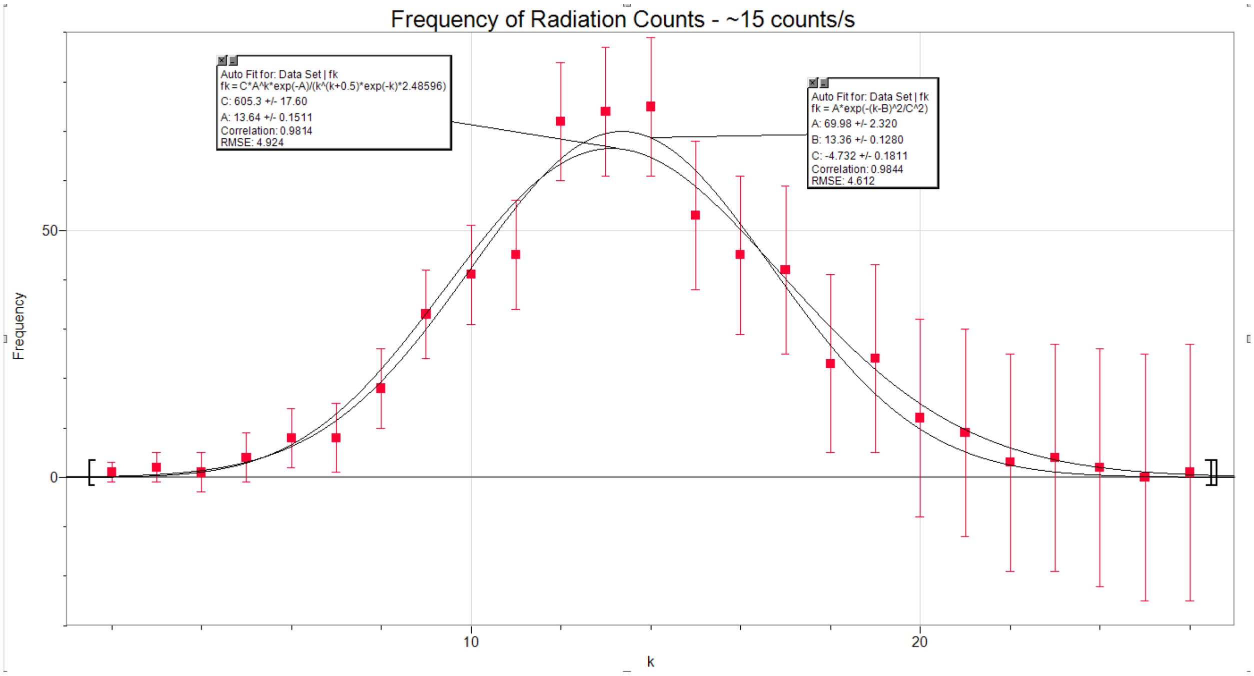

Figure 7. *Graph of Frequency and Radiation – Bin (Counts) at ~15counts/s with error bars of Radiation Bin – (Counts). The slight difference in coefficients between this Figure and the previous is due to the difference in Radiation – Bin (Counts), which is rounded down to an integer.*

6.) Calculations

	Sample mean k_{av}	Variance σ^2
Background	1.720	1.902
$\sim 3 counts/s$	2.138	2.179
$\sim 15 counts/s$	13.53	12.56

Table 5. *Summary of sample mean and variance of the different count values. Taken from Figures (2)(4)(6).*

Sample calculation for calculating χ^2 for a Poisson distribution (equation 2) given k (table 2), N = 100 and $k_{av} \; = \; 1.720$:

$$\chi^2 = \sum_{k=0}^{\iota=6} \frac{\left(f_k \; - \; N\left({k_{av}}^{k} \; * \; e^{-\frac{k_{av}}{k!}}\right)\right)^2}{N\left({k_{av}}^{k} \; * \; e^{-\frac{k_{av}}{k!}}\right)}$$

$$\chi^2 \; = \; 5.07$$

Sample calculation for calculating χ^2 for a Gaussian distribution (equation 3) given k $(table\ 2)$, N = 100 and $k_{av}\ =\ 1.720$:

$$\chi^2 = \sum_{k=0}^{\iota = 6} \frac{\left(f_k - N\left(\frac{e^{-\frac{(k-k_{av})^2}{2k_{av}}}}{\sqrt{2\pi k_{av}}}\right)\right)^2}{N\left(\frac{e^{-\frac{(k-k_{av})^2}{2k_{av}}}}{\sqrt{2\pi k_{av}}}\right)}$$

$$\chi^2 = 32.101$$

Sample calculation for integral value using χ^2 = 5.070 and L = 6 for background radiation (equation 6):

$$\frac{1}{2^{\left(\frac{\ell-2}{2}\right)} \int_0^\infty x^{\left(\frac{\ell-2}{2}\right)} e^{-x} dx} \int_{\chi^2}^{\infty} x^{\left(\frac{\ell-2}{2}-1\right)} e^{\left(-\frac{x}{2}\right)} dx$$

$$\frac{1}{(4) * (2)}(1.1207)$$

$$0.140$$

7.) Results:

	Type of distribution	χ^2 (Chi squared)	L (bin count)	Integral Value	
Background	Poisson	5.070	6	0.140	Unknown
	Gaussian	32.101		$9.12\ \times\ 10^{-7}$	Rejected
$\sim 3 counts/s$	Poisson	9.971	9	0.0544	Unknown
	Gaussian	471.960		0	Rejected
$\sim 15 counts/s$	Poisson	36.434	24	0.00247	Rejected
	Gaussian	21.743		0.0432	Rejected

Table 6. *Summary of results of* χ^2*values and the integral values for each count value and for the two different types of distribution.*

8.) Discussion:

The results demonstrate the success of the experiment. It is clear that for small count values, the Poisson distribution is accepted. The 0.05 confidence is representative of 3σ or lying within 99.73% of the distribution[1]. In a Poisson distribution the variance is equal to the sample mean[2] as demonstrated by Table5. So it is not quite the same as a Gaussian distribution, as for Gaussian distribution the variance is not at all similar to the sample mean.

The ranges of σ control the distribution of samples. 68.27% of the samples will lie within $\pm 1\sigma$ of the sample mean, 95.45% of the samples will lie within $\pm 2\sigma$ of the sample mean and 99.73% of the samples will lie within $\pm 3\sigma$ of the sample mean[3]. The observed frequencies visually correlate to these figures as they seem to lie under the line.

Essentially, a Poisson distribution with a high enough mean approximates a Gaussian distribution. This is clear in Table 6, as Gaussian is rejected at small/medium count values whereas Poisson is not. On the other end, at high count values, both Poisson and Gaussian distributions were both rejected.

As with all experiments, errors do exist. One such systematic error is the dead time of the Geiger Muller counter. During the 90 micro seconds it takes to recover, gamma rays could be passing through it undetected. A method of approximating the loss is given by equation (7). The cumulated errors were calculated and put in each table. The largest error existing in the center of the distributions where the frequency of counts is higher. The highest value was 1.41. Divided by the count frequency 75 yields a value of 0.0188. Meaning each of those counts in the bin 14.5, should've been increased by 0.0188. If this value was accounted for, the entire distribution would've translated horizontally a little bit to the right as more radiation counts would've passed into higher bins. However, since the value is so small, this would've barely affected the results.

Since this is an experiment on random events, random error is also a factor as each event is independent of each other. Ideally, an infinite number of samples could be taken to increase the precision of the experiment.

9.) Bibliography

1: http://hyperphysics.phy-astr.gsu.edu/hbase/Math/gaufcn.html [Accessed Nov. 14, 2018]

2: https://www.theanalysisfactor.com/differences-between-normal-and-poisson-distributions/ [Accessed Nov. 14, 2018]

3: https://www.statisticshowto.datasciencecentral.com/probability-and-statistics/normal-distributions/ [Accessed Nov. 14, 2018]