Use trig identity $\sin(2x) = 2\sin x \cos x = \cos x$

$2\sin x \cos x - \cos x = 0$

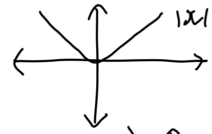
$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ or $2\sin x - 1 = 0$

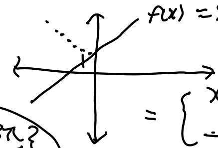
$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad 2\sin x = 1 \quad \sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\rightarrow x \in \{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}\}$

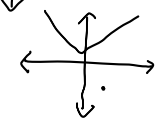
2. (a) $|x| = \begin{cases} x & x \ge 0 \\ -x & x < 0 \end{cases}$



$f(x) = |x|+1$ $\quad f(x) = x+1$



$= \begin{cases} x+1 & x \ge 0 \\ -x+1 & x \le 0 \end{cases}$



DGD 1 problem list.

1. Find all values of x in $[0, 2\pi]$ that satisfy $\sin(2x) = \cos(x)$.

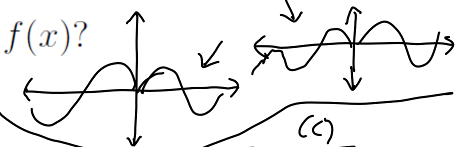
① draw $f(x)$ over all real x.
② Erase the corresponding to negative x.
③ reflect the positive point through the y-axis.

(b) $\sin(|x|) = \begin{cases} \sin x & x \ge 0 \\ \sin(-x) & x < 0 \end{cases}$

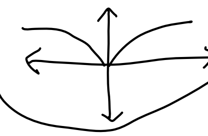
2. (a) How is the graph of $f(|x|)$ related to the graph of $f(x)$?

(b) Sketch the graph of $\sin(|x|)$.

(c) Sketch the graph of $\sqrt{|x|}$.



(c) $\sqrt{|x|} = \begin{cases} \sqrt{x} & x \ge 0 \\ \sqrt{-x} & x < 0 \end{cases}$



4. First, $f(x)$ is one to one function,

$y = 1 + \sqrt{2+3x}$

$y - 1 = \sqrt{2+3x}$ for $y \ge 1$

$(y-1)^2 = 2+3x$

$x = \frac{(y-1)^2 - 2}{3}$

switch $x \leftrightarrow y$

$y = \frac{(x-1)^2 - 2}{3}, x \ge 1$

$f^{-1}(x) = \frac{(x-1)^2 - 2}{3}, x \ge 1$

$e^{1-x^2} = 1$

$1 - x^2 = 0 \quad x^2 = 1$

$x \in (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

3. Find the domain of each function

(a) $\dfrac{1 - e^{x^2}}{1 - e^{1-x^2}}$ $\quad f(x)$ is not defined when $1 - e^{1-x^2} = 0$

(b) $\dfrac{1+x}{e^{\cos x}}$ $\quad g(x)$ is not defined when $e^{\cos x} = 0$

$x \in (-\infty, \infty)$

5. (a) $\log_2(2^5)$

$5\log_2(2) = 5$ $\quad \boxed{\log_a a = 1}$

4. Find a formula for the inverse of $f(x) = 1 + \sqrt{2+3x}$.

6. (b) Let $e^x = y$

$y^2 - 3y + 2 = 0$

$(y-1)(y-2) = 0$

$y = 1 \Leftrightarrow e^x = 1 \Leftrightarrow x = 0$

$y = 2 \Leftrightarrow e^x = 2 \Leftrightarrow \ln e^x = \ln 2$

$\Leftrightarrow x = \ln 2$

5. Find the exact value (without a calculator!)

(a) $\log_2(32)$

(b) $\log_8(2)$

(b) $\log_8(8^{\frac{1}{3}})$ $\boxed{\sqrt[3]{8}} = 2$

$= \frac{1}{3}\log_8(8) = \frac{1}{3}$

6. (a) $x^2 - 1 > 0$

$x < -1$ or $x > 1$

$e^{\ln(x^2-1)} = e^3$

$x^2 - 1 = e^3$

$x^2 = e^3 + 1$

$x = \pm\sqrt{e^3+1}$

6. Solve each equation for x.

(a) $\ln(x^2 - 1) = 3$

(b) $e^{2x} - 3e^x + 2 = 0$

7. Sketch the graph of the function and use it to determine the values a for which $\lim_{x\to a} f(x)$ exists.

$$f(x) = \begin{cases} 1 + \sin(x) & x < 0 \\ \cos(x) & 0 \le x \le \pi \\ \sin(x) & x > \pi \end{cases}$$

each part is continuous So we only need to check $a = 0$ and $a = \pi$.

① Case 1 : $a = 0$

$\lim_{x\to 0^-}(1 + \sin x) = 1 = \lim_{x\to 0^+}(\cos x)$

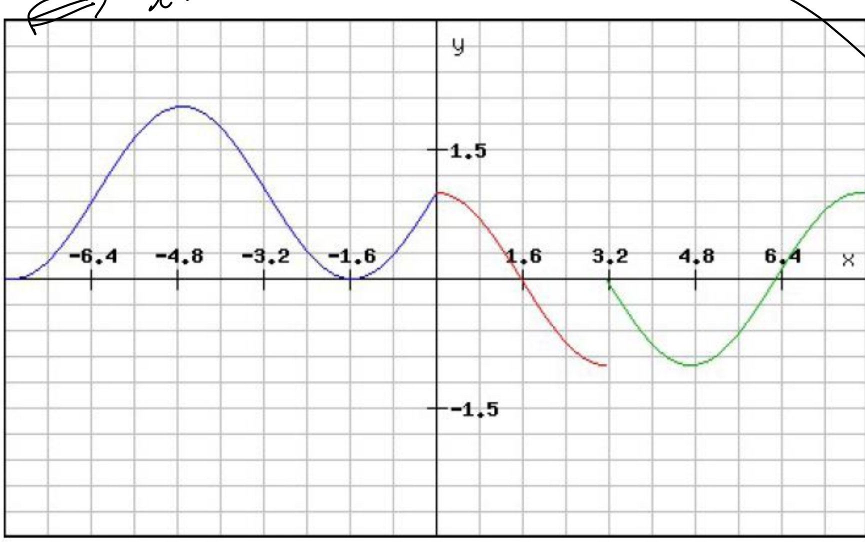
② Case 2 : $a = \pi$

$\lim_{x\to\pi^-}(\cos x) = -1$

$\lim_{x\to\pi^+}(\sin x) = 0$

$\lim_{x\to\pi} f(x)$ DNE

We say that $\lim_{x\to a} f(x)$ exists if $\lim_{x\to a^-} f(x) = L = \lim_{x\to a^+} f(x)$



8. ① $x \to 3^+$, $x^2-9 \Rightarrow 0^+$
② Let $t = x^2-9$
③ $\lim_{x\to 3^+} (\ln(x^2-9)) = \lim_{t \to 0^+} (\ln(t)) = -\infty$

clearly, $\ln t \Rightarrow -\infty$ when $t \to 0^+$
$\lim_{x\to 0^+} (\ln(x^2-4)) = -\infty$
This means the limit does not exist.

8. Determine the limit $\lim_{x\to 3^+} \ln\left(x^2-9\right)$.

9. Evaluate the limit $\lim_{x\to -4} \dfrac{\frac{1}{4}+\frac{1}{x}}{4+x}$

10. Evaluate the limit $\lim_{t\to 0}\left(\dfrac{1}{t\sqrt{1+t}}-\dfrac{1}{t}\right)$

11. Find the limit $\lim_{x\to 3}(2x+|x-3|)$

9. ① simplify the function
$\dfrac{\frac{1}{4}+\frac{1}{2}}{4+x} = \dfrac{\frac{x+4}{4x}}{4+x} = \dfrac{\cancel{4+x}}{4x(\cancel{4+x})} = \dfrac{1}{4x}$
② $\lim_{x\to -4} \dfrac{\frac{1}{4}+\frac{1}{x}}{4+x} = \lim_{x\to -4} \left(\frac{1}{(4x)}\right)$ for $x \neq -4$
$= -\frac{1}{16}$

10. ① simplify the function
$\dfrac{1}{t\sqrt{1+t}} - \dfrac{1}{t} = \dfrac{1-t\sqrt{1+t}}{t\sqrt{1+t}} \left(\dfrac{1-t\sqrt{1+t}}{t\sqrt{1+t}}\right) \left(\dfrac{1+t\sqrt{1+t}}{1+t\sqrt{1+t}}\right)$
$= \dfrac{1-1+t}{}$