Carleton University

Laboratory Report

Course Code : PHYS 1007 Experiment #: 5

Simple Pendulum

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Abstract:

The basis of this experiment involved analyzing a pendulum, more specifically the relationships of a simple pendulum by observing different masses, angles, periods, and heights. By changing these values, graphs involving the relationship between period and constant values can be made. The law of conservation of energy will also be observed in this experiment.

Theory:

The first equation calculates the amplitude of a simple pendulum, shown below.

$$\theta_0 = sin^{-1}(\frac{x}{L}) \tag{1}$$

θ_0 is the amplitude in degrees, x is the horizontal distance of the bob to the pendulum in cm, and L is the length of the pendulum in cm.

The next equation calculates the period when the amplitude is less than 60°.

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{2}$$

T is period in seconds, g is gravitational acceleration, and L is length in cm.

If the amplitude of the oscillations is greater than 60°, the equation below is used.

$$T = 2\pi\sqrt{\frac{L}{g}}\,(1 + \frac{1}{16}\theta_0^2 + \frac{11}{3072}\theta_0^4) \tag{3}$$

T is period in seconds, g is gravitational acceleration, and L is length in cm.

The following equation calculates the vertical displacement of the bob.

$$h = L - \sqrt{L^2 + x^2} \tag{4}$$

H is vertical displacement in cm, x is the horizontal distance of the bob to the pendulum in cm, and L is the length of the pendulum in cm.

The equation to calculate the total mechanical energy is shown below.

$$\Delta K + \Delta U = 0 \quad (5)$$

ΔK is the change in kinetic energy in joules, ΔU is the change in potential energy in joules. The addition of these values should equal 0 according to the law of conservation of energy.

The equation to calculate the uncertainty of the amplitude of oscillation is shown below.

$$\sigma_{\theta_0} = \sqrt{\left(\frac{1}{(L^2-x^2)}\right)^2 \cdot \sigma_x^2 + \left(\frac{1}{(\frac{L^2}{x})^2-L^2}\right)^2 \cdot \sigma_L^2} \quad (6)$$

σ_{θ_0} is the uncertainty of the amplitude in rads, x is the horizontal distance of the bob to the pendulum in cm, and L is the length of the pendulum in cm.

Finally, a consistency test comparing the measured g, and actual g will be performed. This is done by using the formula below.

$$t = \frac{|x_1 - x_2|}{\sqrt{\sigma_{x_1} + \sigma_{x_2}}} \quad (7)$$

Apparatus:

- Aluminum Bob
- Steel Bob
- String
- Meter Stick (Precision: 0.1cm, Range: 0.1 - 100cm)
- Caliper (Precision: 0.01mm, Range: 0.01 - 150mm)
- Vernier Photogate (VPG-BTD)
- Scale (Precision: 0.01g, Range: 0.01 - 400g)

Procedure:
First, all measurements involving the weight of the bobs, lengths of pendulums and lengths of bobs were taken. Next, the experiment was done, by recording the oscillations using the logger pro file and this was done at various heights. Next part of the experiment, the velocity of the pendulum was measured. This was done by keeping

the height constant, but the angles different. The results were observed in the logger pro, and recorded.

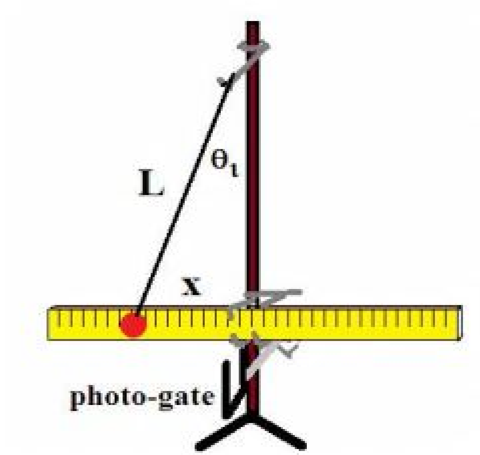

Figure 1: Diagram of the setup of the simple pendulum.

Observations:

Bob	Diameter (mm)	Mass (g)
Steel	19.03mm	42.10g
Aluminum	19.11mm	14.27g

Table 1: Average masses (g) and diameters (mm) of steel and aluminum bobs.

	T (s)	L (cm)	σL (cm)	σT (s)	Sample Size	σmean (s)
1	1.292	41	0.1	0.0004249	9	0.0001
2	1.181	35	0.1	0.0004874	9	0.0002
3	1.091	30	0.1	0.0039	9	0.0013
4	0.9942	25	0.1	0.0006429	9	0.0002
5	0.8284	17	0.1	0.0007495	9	0.0002

Table 2: Values of period (s), length (cm), σL (cm), σT (s), sample size, and σmean (s).

	T (s)	Angle (rad)	σT (s)	L (cm)	σL (cm)	x (cm)	σx (cm)	v (m/s)	σv (m/s)	Sample Size	σθ₀ (rad)
1	1.39	0.692	0.001799	47	0.1	30	0.1	1.089	0.008322	3	0.003
2	1.395	0.840	0.002026	47	0.1	35	0.1	1.373	0.02648	3	0.004
3	1.405	1.018	0.003552	47	0.1	40	0.1	1.649	0.01745	3	0.005
4	1.429	1.278	0.004008	47	0.1	45	0.1	1.84	0.02342	3	0.010
5	1.491	1.364	0.01036	47	0.1	46	0.1	2.02	0.02083	3	0.015

Table 3: Values of period (s), angle (rad), x (cm), σx (cm), v (m/s), σv (m/s), length (cm), σL (cm), σT (s), sample size, and $\sigma\theta_0$ (rad).

Calculations:

Sample Amplitude of Oscillation:

x = 30 cm   L = 47 cm

$$\theta_0 = sin^{-1}(\frac{x}{L})$$
$$\theta_0 = sin^{-1}(\frac{30\ cm}{47\ cm})$$
$$\theta_0 = 39.7\ ^\circ$$

Sample Period for $\theta_0 < 60^\circ$:

L = 0.410 m g = 9.81 m/s^2

$$T = 2\pi\sqrt{\frac{L}{g}}$$
$$T = 2\pi\sqrt{\frac{0.410\ m}{9.81\ m/s^2}}$$
$$T = 1.284s$$

Sample Period for $\theta_0 > 60^\circ$:

L = 0.470 m g = 9.81 m/s^2

$\theta_0 = 1.364\ rad$

$$T = 2\pi\sqrt{\frac{L}{g}}\ (1 + \frac{1}{16}\theta_0^2 + \frac{11}{3072}\theta_0^4)$$
$$T = 2\pi\sqrt{\frac{0.470\ m}{9.81\ m/s^2}}\ (1 + \frac{1}{16}(1.364\ rad)^2 + \frac{11}{3072}(1.364\ \ rad)^4)$$
$$T = 1.552\ \text{s}$$

Sample Uncertainty of Amplitude:

L =0.470 m x = 0.40 m
$\sigma_L = 0.001$ m $\sigma_x = 0.001$ m

$$\sigma_{\theta_0} = \sqrt{(\frac{1}{(L^2 - x^2)})^2 \cdot \sigma_x^2 + (\frac{1}{(\frac{L^2}{x})^2 - L^2})^2 \cdot \sigma_L^2}$$

$$\sigma_{\theta_0} = \sqrt{(\frac{1}{((0.470\ m)^2 - (0.4\ m)^2)})^2 \cdot (0.001\ m)^2 + (\frac{1}{(\frac{(0.470\ m)^2}{0.40\ m})^2 - (0.470\ m)^2})^2 \cdot (0.001\ m)^2}$$

$$\sigma_{\theta_0} = 0.0000119 \text{ rad}$$

Discussion:

Overall, the calculated results seem to fall in line with the experimental results. There are multiple things that could change how this lab could have gone however. A good example is if a force sensor could have been used. If a force sensor was used, this can give us a more precise measure to observe the movement of the pendulum, specifically the velocity. It can be used to directly sense how fast the pendulum is moving, and this in turn will give us a more accurate and precise measure of velocity. There are also a multitude of other factors that can skew the results on a minimal level. The first of which is friction. Friction can cause the pendulum to start slowing down slowly over time, but this is not that big of an issue in the experiment, as it does not throw the results off too much. This is because the effect of friction is very weak, and it would take a very long time for the pendulum to completely stop from friction. The next thing that can skew results is if the pendulum was not passing the photogate perpendicularly. The way this throws off the results is because of the force of acceleration acting on multiple directions, since the pendulum is swinging diagonally. This changes the period, and velocity of the pendulum, which in turn skews the results, so it is important to make sure that the pendulum is perpendicular to the photogate.

Conclusion:

The sample amplitude of oscillation was 39.7°, the sample period for $\theta_0 < 60^\circ$ was 1.284 s, the sample period for $\theta_0 > 60^\circ$ was 1.552 s and the sample uncertainty of amplitude was 0.0000119rad.