



CVG 2140/2540
Mechanics of Materials / Résistance des Matériaux I
FINAL EXAMINATION / EXAMEN FINAL

Time / Temps: 3 hours

April / Avril 25, 2007, 14:00

Ms. S. Cimilli / Dr. E. Erkmen / Dr. B. Martín-Pérez

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Family Name: _____

Other Names: _____

Student Number: _____

Number of booklets submitted: _____

Signature: _____

CLOSED BOOK EXAM. Attempt all four problems. Calculators are allowed. Marks are as shown for each question. Clearly indicate the coordinates you are using.

LIVRE FERME. Essayez de répondre aux quatre questions. Les calculatrices sont permises. Les points sont indiqués pour chaque question. Identifiez clairement le système de coordonnées que vous utilisez.

If you do not understand a question, clearly state an assumption and proceed. Programmable calculators or other electronic devices are not allowed. At the end of the exam, when time is up:

- Stop working and turn your exam upside down.
- Remain silent.
- Do not move or speak until all exams have been picked up, and a TA or the Professor gives the go-ahead to leave.

Si vous ne comprenez pas une question, mettez clairement une supposition et continuez. Les calculateurs programmables et autres instruments électroniques ne sont pas permis. A la fin de l'examen, quand le temps est fini :

- Cessez travailler et tournez le verso de votre examen.
- Restez silencieux.
- Ne vous déplacez pas ou ne parlez pas jusqu'à ce que tous les examens aient été pris, et un(e) Assistant(e) ou le Professeur donne la permission pour partir.

(En français à la page suivante)

1. The assembly shown in Figure 1 consists of two solid circular bars AC and CB , and is subjected to $T=5\text{kNm}$ torque at C . The sections at A and B are fixed against rotation. The diameters of the bars AC and CB are 100mm and 80mm , respectively. Find the distance x between sections A and C if the maximum shear stress at both points e and f are identical. For both bars $G = 70 \text{ GPa}$. (25 points).

2. Consider the beam illustrated in Figure 2.
 - a. Draw the shear force and bending moment diagrams, clearly indicating the extreme values (7 points).
 - b. Derive the equation for the deflection curve of the beam (9 points).
 - c. Determine the location and the value of the maximum deflection between the supports. $E = 200 \text{ GPa}$, $I = 2.0 \times 10^8 \text{ mm}^4$ (9 points).

3. Two columns subjected to identical axial loads $P=500\text{kN}$ are shown in Figure 3. Both columns have square cross-sections with a_1 and a_2 as dimensions for the simply supported and fixed-fixed columns, respectively. By considering the safety factor, calculate the minimum possible values for a_1 and a_2 to prevent buckling. $E = 210 \text{ GPa}$, $n_{cr} = 2$ (25 points).

4. The beam illustrated in Figure 4 is fixed at section C and is subjected to a horizontal load $P=8\text{kN}$ at the mid-height of section A , thus load P does not produce torsion. Find the principal stresses and their orientations at points d and e on section B (25 points).

(In English on previous page)

1. L'assemblée illustrée à la Fig. 1 se compose de deux barres solides circulaires AC et CB , et elle est soumise à un moment de torsion $T=5\text{kNm}$ à C . Les sections A et B sont fixées contre la rotation. Les diamètres des barres AC et CB sont 100mm et 80mm , respectivement. Trouvez la distance x entre les sections A et C si les contraintes de cisaillement maximum aux points e et f sont identiques. Pour toutes les deux barres, $G = 70 \text{ GPa}$. (25 points).

2. Pour la poutre illustrée à la Fig.2,
 - a. Tracez les diagrammes des efforts tranchants et des moments fléchissant, indiquant clairement les valeurs extrêmes (7 points)
 - b. Ecrivez l'équation de la flèche (9 points)
 - c. Déterminez la location et la valeur de la flèche maximale entre les appuis. $E = 200 \text{ GPa}$, $I = 2.0 \times 10^8 \text{ mm}^4$ (9 points).

3. Deux colonnes sont soumises aux charges axiales identiques $P=500\text{kN}$ sont illustrées à la Fig 3. Les deux colonnes ont des sections carrées avec a_1 et a_2 comme des dimensions pour les colonnes rotule-rotule et encastrée-encastrée, respectivement. En considérant le facteur de sécurité, calculez la plus petite valeur possible pour a_1 et a_2 afin d'empêcher le flambement. $E = 210 \text{ GPa}$, $n_{cr} = 2$ (25 points).

4. La poutre illustrée à la Fig 4 est fixée à la section C et est soumise à une charge horizontale $P=8\text{kN}$ à la mi-hauteur de la section A , ainsi la charge P ne produit pas de torsion. Trouvez les contraintes principales et leurs orientations aux points d et e à la section B (25 points).

FIGURES

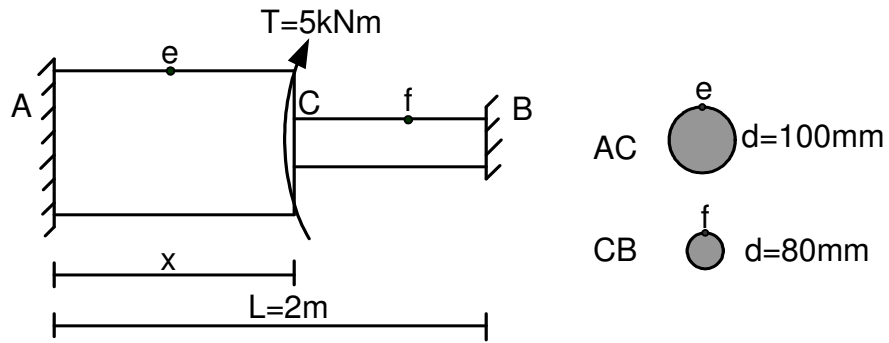


Figure 1

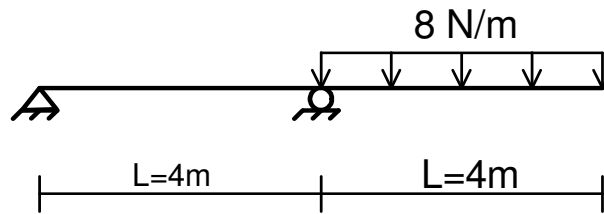


Figure 2

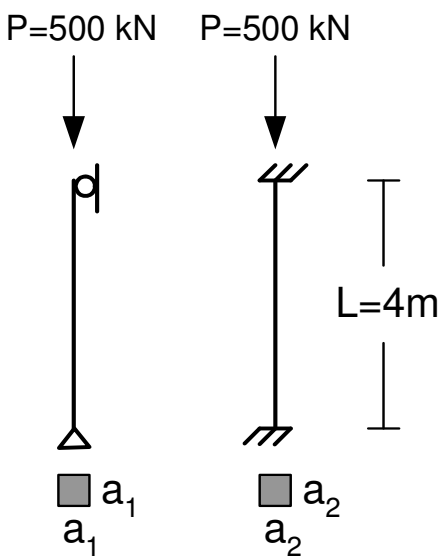


Figure 3

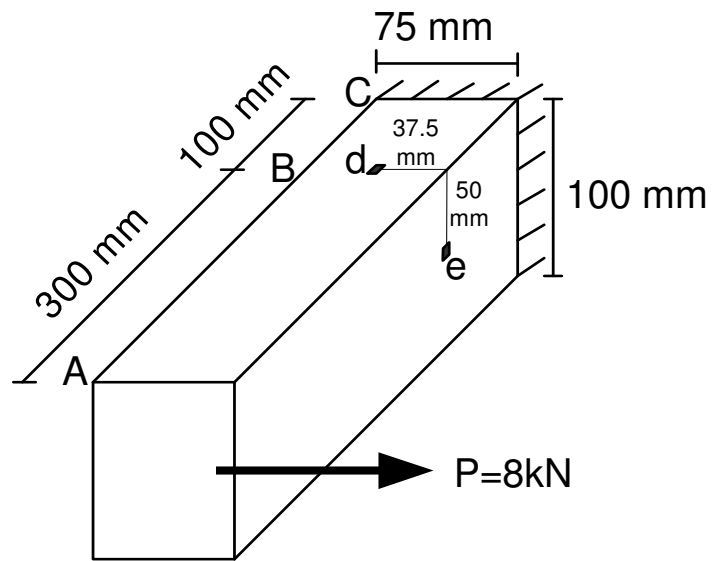


Figure 4

RELATIONS BETWEEN W, V, M / RELATIONS ENTRE W, V, M

$$\frac{dV}{dx} = -w(x), \quad \frac{dM}{dx} = V$$

CENTROID

$$\bar{x} = \frac{Q_y}{A} = \frac{\int x dA}{\int dA}, \quad \bar{y} = \frac{Q_x}{A} = \frac{\int y dA}{\int dA}$$

$$\bar{x} = \frac{Q_y}{A} = \frac{\sum \bar{x}_i A_i}{\sum A_i}, \quad \bar{y} = \frac{Q_x}{A} = \frac{\sum \bar{y}_i A_i}{\sum A_i}$$

MOMENTS OF INERTIA / MOMENTS D'INERTIE

$$I_x = \int y^2 dA, \quad I_y = \int x^2 dA$$

Polar moment of inertia / Moment d'inertie polaire

$$J_o = \int r^2 dA = I_x + I_y$$

Product of inertia / Produit d'inertie

$$I_{xy} = \int xy dA$$

Parallel-axis theorem / Théorème des axes parallèles

$$I_x = I_{x_c} + Ad_1^2 \quad I_y = I_{y_c} + Ad_2^2$$

$$J_o = J_c + Ad^2 \quad I_{xy} = I_{x_c y_c} + Ad_1 d_2$$

Transformation equations / Equations de transformation

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2} \cos 2\theta - I_{xy} \sin 2\theta$$

$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2} \cos 2\theta + I_{xy} \sin 2\theta$$

$$I_{x'y'} = \frac{I_x - I_y}{2} \sin 2\theta + I_{xy} \cos 2\theta$$

Principal moments of inertia / Moments d' inertie principaux

$$\tan 2\theta = -\frac{2I_{xy}}{I_x - I_y}$$

$$I_{1,2} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}, \quad I_{12} = 0$$

AXIAL LOAD/ CHARGE AXIALE

Normal stress & strain / Contrainte et Déformation Normales

$$\sigma = \frac{P}{A}, \quad \varepsilon = \frac{\delta}{L}$$

Displacement / Déplacement

$$\delta = \frac{PL}{AE}, \quad \delta = \sum \frac{N_i L_i}{A_i E_i}, \quad \delta = \int_0^L \frac{N(x) dx}{A(x) E}$$

TORSION

Shear strain in a circular shaft / Déformation de cisaillement dans une section circulaire

$$\gamma = \rho\theta = \frac{\rho\phi}{L}$$

Shear stress in a circular shaft / Contrainte de cisaillement dans une section circulaire

$$\tau = \frac{T\rho}{J}$$

Angle of twist / Angle de rotation

$$\phi = \frac{TL}{JG}, \quad \phi = \sum \frac{T_i L_i}{J_i G_i}, \quad \phi = \int_0^L \frac{T(x) dx}{J(x) G}$$

BENDING / FLEXION

Normal stress / Contrainte normale

$$\sigma = -\frac{My}{I}$$

SHEAR / CISAILLEMENT

Average shear stress / Contrainte de cisaillement moyen (τ_{moyen})

$$\tau_{\text{avg}} = \frac{V}{A}$$

Shear stress in beam of rectangular cross-section /

Contrainte de cisaillement dans les poutres avec des sections rectangulaires

$$\tau = \frac{VQ}{Ib}$$

DEFLECTION / FLÈCHE

$$\frac{d^2v}{dx^2} = \frac{M}{EI}$$

Discontinuity functions for $n \geq 0$ $\langle x-a \rangle^n = \begin{cases} 0 & \text{for } x < a \\ (x-a)^n & \text{for } x \geq a \end{cases}$

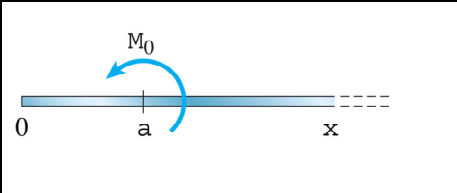
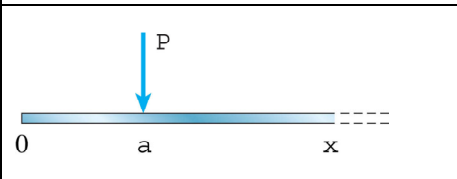
$$\int_{-\infty}^x \langle x-a \rangle^n dx = \frac{\langle x-a \rangle^{n+1}}{n+1}$$

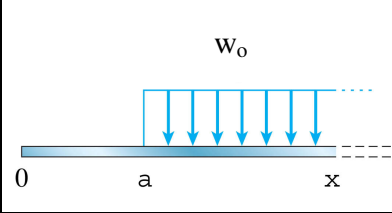
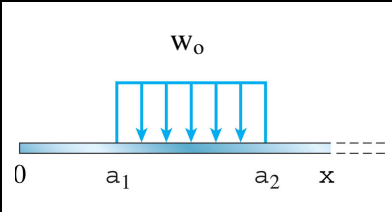
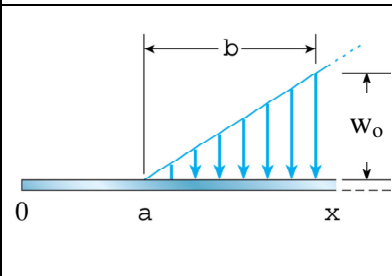
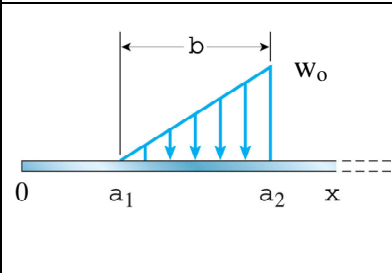
Discontinuity functions for $n < 0$ $\langle x-a \rangle^n = \begin{cases} 0 & \text{for } x \neq a \\ \pm\infty & \text{for } x = a \end{cases}$

$$\int_{-\infty}^x \langle x-a \rangle^n dx = \langle x-a \rangle^{n+1}$$

Loads represented by discontinuity functions /

Charges représentées par les fonctions de discontinuités

	$w(x) = M_o \langle x-a \rangle^{-2}$ $V(x) = -M_o \langle x-a \rangle^{-1}$ $M(x) = -M_o \langle x-a \rangle^0$
	$w(x) = P \langle x-a \rangle^{-1}$ $V(x) = -P \langle x-a \rangle^0$ $M(x) = -P \langle x-a \rangle$

	$w(x) = w_0 \langle x - a \rangle^0$ $V(x) = -w_0 \langle x - a \rangle$ $M(x) = -\frac{1}{2} w_0 \langle x - a \rangle^2$
	$w(x) = w_0 \langle x - a_1 \rangle^0 - w_0 \langle x - a_2 \rangle^0$ $V(x) = -w_0 \langle x - a_1 \rangle + w_0 \langle x - a_2 \rangle$ $M(x) = -\frac{1}{2} w_0 \langle x - a_1 \rangle^2 + \frac{1}{2} w_0 \langle x - a_2 \rangle^2$
	$w(x) = \frac{w_0}{b} \langle x - a \rangle$ $V(x) = -\frac{1}{2} \frac{w_0}{b} \langle x - a \rangle^2$ $M(x) = -\frac{1}{6} \frac{w_0}{b} \langle x - a \rangle^3$
	$w(x) = \frac{w_0}{b} \langle x - a_1 \rangle - \frac{w_0}{b} \langle x - a_2 \rangle - w_0 \langle x - a_2 \rangle^0$ $V(x) = -\frac{1}{2} \frac{w_0}{b} \langle x - a_1 \rangle^2 + \frac{1}{2} \frac{w_0}{b} \langle x - a_2 \rangle^2 + w_0 \langle x - a_2 \rangle$ $M(x) = -\frac{1}{6} \frac{w_0}{b} \langle x - a_1 \rangle^3 + \frac{1}{6} \frac{w_0}{b} \langle x - a_2 \rangle^3 + \frac{1}{2} w_0 \langle x - a_2 \rangle^2$

STRESS TRANSFORMATION EQUATIONS / EQUATIONS DE TRANSFORMATIONS POUR LES CONTRAINTES

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

Principal stresses / Contraintes principales

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}, \quad \tau_{12} = 0$$

Maximum shear stress / Contrainte de cisaillement maximum

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}, \quad \theta_s = \theta_p \pm 45^\circ$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_1 - \sigma_2}{2}, \quad \sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2}$$

BUCKLING / FLAMBEMENT

<i>Critical axial load / Charge critique</i>	Boundary conditions Conditions aux rives	K
$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$	Pinned-pinned / Rotule-rotule	1
<i>Critical stress / Contrainte critique</i>	Fixed-free / Encastré-libre	2
$\sigma_{cr} = \frac{\pi^2 E}{(KL/r)^2}, \quad r = \sqrt{\frac{I}{A}}$	Fixed-fixed / Encastré- Encastré	0.5
	Fixed-pinned / Encastré-rotule	0.7

DESIGN FOR SAFETY / DIMENSIONNEMENT SECURITAIRE

Allowable stresses / Contraintes permises

$$\sigma_{\text{allow}} = \frac{\sigma_y}{n_y}, \quad \sigma_{\text{allow}} = \frac{\sigma_u}{n_u}, \quad \sigma_{\text{allow}} = \frac{\sigma_{cr}}{n_{cr}}$$

Allowable loads / Charges permises

$$P_{\text{allow}} = \sigma_{\text{allow}} \times A$$

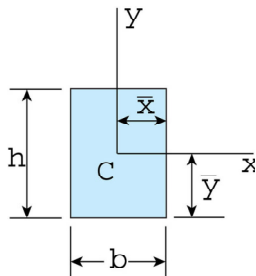
SIGN CONVENTIONS / CONVENTIONS DE SIGNE

	+	-
Axial force/ Effort axial		
Distributed loading/ Charge répartie		
Shear force/ Effort tranchant		
Bending moment/ Moment fléchissant		
Torsion		
Deflection /Flèche, v		
Slope/Pente, $\theta = \frac{dv}{dx}$		
Shear stress /Contrainte de cisaillement		

GEOMETRIC PROPERTIES OF SOME PLANE AREAS / PROPRIÉTÉS GÉOMÉTRIQUES

Rectangle (origin of axes at centroid)

(l'origine des axes est au centroid)



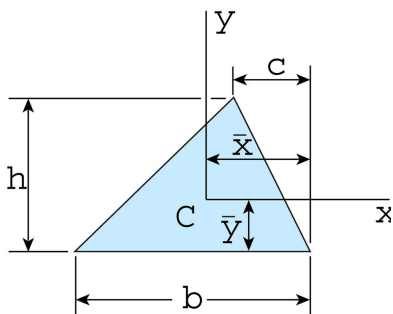
$$A = bh$$

$$\bar{x} = \frac{b}{2}, \quad \bar{y} = \frac{h}{2}$$

$$I_x = \frac{bh^3}{12}, \quad I_y = \frac{hb^3}{12}, \quad I_{xy} = 0$$

$$J_o = \frac{bh}{12}(h^2 + b^2)$$

Triangle (origin of axes at centroid)

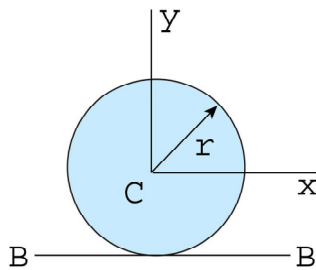


$$A = \frac{bh}{2}, \quad \bar{x} = \frac{b+c}{3}, \quad \bar{y} = \frac{h}{3}$$

$$I_x = \frac{bh^3}{36}, \quad I_y = \frac{bh}{36}(b^2 - bc + c^2), \quad I_{xy} = \frac{bh^2}{72}(b - 2c)$$

$$J_c = \frac{bh}{36}(h^2 + b^2 - bc + c^2)$$

Circle /Cercle (origin of axes at centroid)



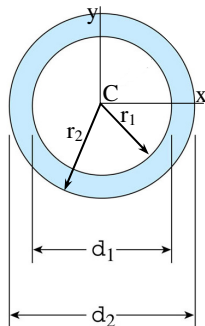
$$A = \pi r^2 = \frac{\pi d^2}{4}, \quad d = 2r$$

$$I_x = I_y = \frac{\pi r^4}{4} = \frac{\pi d^4}{64}, \quad I_{xy} = 0$$

$$J_o = \frac{\pi r^4}{2} = \frac{\pi d^4}{32}$$

Circular tube/Tube circulaire

(origin of axes at centroid)



$$A = \pi(r_2^2 - r_1^2) = \frac{\pi}{4}(d_2^2 - d_1^2)$$

$$I_x = I_y = \frac{\pi}{4}(r_2^4 - r_1^4) = \frac{\pi}{64}(d_2^4 - d_1^4), \quad I_{xy} = 0$$

$$J_o = \frac{\pi}{2}(r_2^4 - r_1^4) = \frac{\pi}{32}(d_2^4 - d_1^4)$$