Question 4. [8] What should the value of K be so that the following piece-wise function is continuous at $x = 1$.

$$f(x) = \begin{cases} -x^4 - 1, & x < 1; \\ K, & x = 1; \\ 3x - 5, & x > 1. \end{cases}$$

We should have $K = \lim_{x \to 1^-} (-x^4 - 1) = \lim_{x \to 1^+} (3x - 5)$

so $K = -1 - 1 = 3 - 5$

$\Rightarrow \boxed{K = -2}$ *

Question 5. [6] Use the Intermediate Value Theorem to show that the function $f(x) = 3x^3 - 2x^2 - 10$ has a root in the interval $[1, 2]$.

Since $f(x)$ is continuous on the interval $[1,2]$

We need to show $f(1)f(2) < 0$.

$f(1) = 3(1)^3 - 2(1)^2 - 10 = -9$

$f(2) = 3(2)^3 - 2(2)^2 - 10 = +6$

$\rightarrow f(1)f(2) < 0$ so f has a root in $[1,2]$

Question 6. [8] Evaluate the following limits:

(a) [4] $\lim_{x \to \infty} \left(2x - \sqrt{4x^2 + 1}\right)$

$$= \lim_{x \to \infty} \frac{(2x - \sqrt{4x^2+1})(2x + \sqrt{4x^2+1})}{2x + \sqrt{4x^2+1}} = \lim_{x \to \infty} \frac{\cancel{4x^2} - \cancel{4x^2} - 1}{2x + \sqrt{4x^2+1}}$$

$$= 0$$

(b) [4] $\lim_{x \to \infty} \frac{x^3 + 2}{x^3 + \sqrt{x^9 + 4}}$

$$= \lim_{x \to \infty} \frac{x^3/x^{4.5} + 2/x^{4.5}}{x^3/x^{4.5} + \sqrt{x^9/x^9 + 4/x^9}}$$

$$= \lim_{x \to \infty} \frac{\frac{1}{x^{1.5}} + \frac{2}{x^{4.5}}}{\frac{1}{x^{4.5}} + \sqrt{1 + \frac{4}{x^9}}} = \frac{0 + 0}{0 + \sqrt{1+0}} = 0$$