Solution:

MATH 1007 E, TEST 1
Time: 50 Minutes, Total Mark: 40

Name: .... **Student #:** ....

Question 1. [6] Find the natural *domain* of each one of the following functions and express your answer using intervals.

(a) [2] $f(x) = \dfrac{x+3}{x^2-9}$

$D(x+3) = (-\infty,\infty)$, $D(x^2-9) = (-\infty,\infty)$

$D(f) = D(x+3) \cap D(x^2-9) - \{x \mid x^2-9=0\} = (-\infty,-3) \cup (-3,3) \cup (3,\infty)$

(b) [4] $g(x) = \sqrt{x+5} + \frac{1}{x^2-9}$

$D(g) = [-5,\infty) \cap \{(-\infty,-3) \cup (-3,3) \cup (3,\infty)\}$

$= [-5,-3) \cup (-3,3) \cup (3,\infty)$

Question 2. [6] Which ones of the following functions are Odd and which ones are Even? Also identify which one is symmetric across the y-axix and which one is symmetric across the origin $(0,0)$.

(a) [4] $f(x) = \dfrac{\sin x}{x^3}$

$f(-x) = \dfrac{\sin(-x)}{(-x)^3} = \dfrac{-\sin x}{-x^3} = \dfrac{\sin x}{x^3} = f(x)$, so $f(x)$ is even

and it is symmetrical accross y-axis.

(b) [2] $g(x) = x^{-3}$

$g(-x) = (-x)^{-3} = \dfrac{1}{(-x)^3} = \dfrac{-1}{x^3} = -g(x)$. Hence $g(x)$ is Odd

and it is symmetrical across the origin

Question 3. [8] Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = \frac{1}{x}$.

(a) [4] Find $(fog)(x)$. Simplify your answer as much as possible.

$(fog)(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^2 + \dfrac{1}{(1/x)^2}$

$= \dfrac{1}{x^2} + x^2.$

(b) [4] Find $(gof)(x)$. Simplify your answer as much as possible.

$(gof)(x) = g(f(x)) = g\left(x^2 + \frac{1}{x^2}\right) = \dfrac{1}{x^2 + \frac{1}{x^2}} = \dfrac{1}{\frac{x^4+1}{x^2}}$

$= \dfrac{x^2}{x^4+1}.$

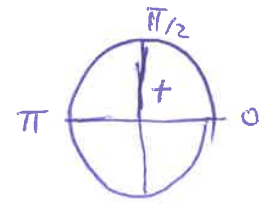
Question 4. [12]

(a) [4] If $\cos x = \frac{-\sqrt{2}}{2}$ on $[0, \pi]$ then what is $\sin x$? What is $\tan x$?
Hint: Remember that $\sin^2 x + \cos^2 x = 1$.

$\sin^2 x = 1 - \left(-\frac{\sqrt{2}}{2}\right)^2 = 1 - \frac{1}{2} = \frac{1}{2}$. So

$\sin x = \pm \frac{\sqrt{2}}{2}$. Since $x \in [0, \pi]$,

the only answer is $\sin x = + \frac{\sqrt{2}}{2}$.



(b) [4] Find the value of $\tan(x)$ such that

$$\tan^3(x + \pi) + 8 = 0$$

Hint: Remember that $\tan(x)$ is periodic.

$\tan^3(x+\pi) = -8 \Rightarrow \tan^3(x) = -8$

$\Rightarrow \tan x = -2$.

(c) [4] Find the value of $\tan(x)$ such that

$$\cos(x + 2\pi) - \sin(x) = 0$$

$\cos(x+2\pi) = \sin x \Rightarrow \cos x = \sin x$ and so

$\tan x = \frac{\sin x}{\cos x} = 1$.

Question 5. [8] By using the exponent rules simplify the below terms as much as possible. (Write some of the steps of your calculations)

(a) [4] $\dfrac{(x-1)\sqrt{x-1}}{\sqrt{(x-1)^3}}$

$$\frac{(x-1)\sqrt{x-1}}{\sqrt{(x-1)^3}} = \frac{(x-1)^{3/2}}{(x-1)^{3/2}} = 1.$$

(b) [4] $\left(x\sqrt{\sin x}\right)^4 \left(x^{-2}\sqrt{1-\cos^2 x}\right)^2$
Hint: Remember that $\sin^2 x + \cos^2 x = 1$.

$$\left(x\sqrt{\sin x}\right)^4 \left(x^{-2}\sqrt{1-\cos^2 x}\right)^2$$

$$= (x^4 \sin^2 x)\big(x^{-4}\underbrace{(1-\cos^2 x)}_{\sin^2 x}\big) = (x^4 x^{-4})(\sin^2 x)(\sin^2 x)$$

$$= \sin^4 x.$$