 - Solutions

1. (2 pts.) (multiple-choice question) Find the exact value of the expression

$$\tan\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$$

Method 1

$\theta = \sin^{-1}(1/2)$

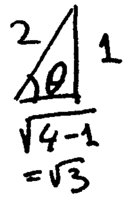
$\Rightarrow \theta = \pi/6$

$\tan \pi/6 = \dfrac{1/2}{\sqrt{3}/2} = \dfrac{1}{\sqrt{3}}$

Method 2:

$\sin\theta = 1/2$

$\tan\theta = ?$



$\tan\theta = \dfrac{1}{\sqrt{3}}$

Select the correct answer.

A) 1/2 **B)** $1/\sqrt{3}$

C) $\sqrt{3}/2$ **D)** 0 **E)** undefined

Write the letter(capital) of the answer in this box $\longrightarrow$ 1.)

2. (3 pts.) Express the following as a single logarithm:

$$\ln(x-1) + \ln(x+1) - 2\ln x$$

$\ln(x^2-1) - \ln(x^2)$

$= \ln\left(\dfrac{x^2-1}{x^2}\right)$

answer: $\boxed{\ln\left(\dfrac{x^2-1}{x^2}\right)}$

3. (4 pts.) Find a formula for the inverse function f^{-1} of $f(x) = \dfrac{x+1}{2x+1}$

$y = \dfrac{x+1}{2x+1}$

$2xy + y = x + 1 \Rightarrow 2xy - x = 1 - y$

$\Rightarrow x = \dfrac{1-y}{2y-1}$

$\Rightarrow f^{-1}(x) = \dfrac{1-x}{2x-1}$

answer: $\boxed{f^{-1}(x) = \dfrac{1-x}{2x-1}}$

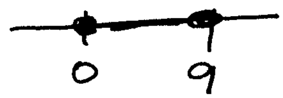
4. (2 pts.) (multiple-choice question) Find the domain of the function

$$f(x) = \sqrt{x} - \sqrt{9-x}$$

$\sqrt{x} \rightarrow x \in [0,\infty)$

$\sqrt{9-x} \rightarrow x \in (-\infty, 9]$

Domain :



$x \in [0,9]$

Select the correct answer.

A) $x \in [0,\infty)$ **B)** $x \in (-\infty, 0]$

C) $x \in (0,9)$ **D)** $x \in [0,9]$ **E)** $x \in (-\infty, 9]$

Write the letter(capital) of the answer in this box ⟶ 4. $\boxed{\ )}$

5. (4 pts.) Find the following limit. If the limit does not exist, explain why.

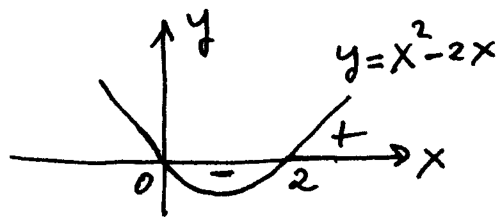
$$\lim_{x\to 2^-} \frac{x^2-4}{|x^2-2x|}$$

Type $0/0$

<u>Method 1</u>: $\displaystyle\lim_{x\to 2^-} \frac{(x-2)(x+2)}{|x||x-2|} = \lim_{x\to 2^-} \frac{(x-2)(x+2)}{x(-(x-2))}$

$\displaystyle= \lim_{x\to 2^-} \frac{x+2}{-x} = \frac{2+2}{-2} = -2.$

<u>Method 2</u>:



$\Rightarrow x^2-2x<0$ as $x\to 2^-$

$\displaystyle\Rightarrow \lim_{x\to 2^-} \frac{(x-2)(x+2)}{-(x^2-2x)} = \lim_{x\to 2^-} \frac{(x-2)(x+2)}{-x(x-2)} = \lim_{x\to 2^-} \frac{x+2}{-x}$

$= -2.$

6. (2 pts.) (multiple-choice question) Find the limit $\lim_{x \to 0^-} \frac{1}{2+e^{1/x}}$

$$\lim_{x \to 0^-} e^{1/x} = \lim_{u \to -\infty} e^{u} = 0$$

$$\Rightarrow \lim_{x \to 0^-} \frac{1}{2+e^{1/x}} = 1/2$$

Select the correct answer.

A) 0 **B)** $\frac{1}{2}$

C) $\frac{1}{2+e}$ **D)** $\frac{1}{3}$ **E)** ∞

Write the letter(capital) of the answer in this box ⟶ 6. $\boxed{)}$

7. (4 pts.) Find the following limit.

$$\lim_{x \to 1} \left(\frac{1}{x-1} - \frac{2x}{x^2-1} \right)$$

$\frac{1}{x^2-1} = \frac{1}{(x+1)(x-1)}$ has same sign as $\frac{1}{x-1}$ as $x \to 1$

so $\lim_{x \to 1} \left(\frac{1}{x-1} - \frac{2x}{x^2-1} \right)$ is of type $\infty - \infty$

$$\lim_{x \to 1} \left(\frac{x+1-2x}{(x-1)(x+1)} = \lim_{x \to 1} \frac{1-x}{(x-1)(x+1)} = \lim_{x \to 1} \frac{-1}{x+1} = -1/2$$

answer: $\boxed{-1/2}$

8. (2 pts.)(multiple-choice question) Determine where $f(x)$ is discontinuous.

$$f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3 - x & \text{if } 0 \le x < 3 \\ (3-x)^2 & \text{if } x > 3 \end{cases}$$

$f(3)$ DNE $\to$ disc.

$$\left.\begin{array}{l} \lim_{x \to 0^-} f(x) = 0 \\ \lim_{x \to 0^+} f(x) = 3 \end{array}\right\} \begin{array}{l} \lim_{x \to 0} f(x) \text{ DNE} \\ \to f \text{ disc.} \end{array}$$

Select the correct answer.

A) 0 and 3 **B)** 0 only

C) 3 only **D)** 0 and −3 **E)** −3 only

so f is disc. at 0 and 3.

Write the letter(capital) of the answer in this box ⟶ 8. $\boxed{\quad)}$

9. (6 pts.) Find the following limit.

$$\lim_{x \to -\infty} \left(\sqrt{9x^2 - x} + 3x \right)$$

Type $\infty - \infty$

$$\lim_{x \to -\infty} \frac{9x^2 - x - 9x^2}{\sqrt{9x^2 - x} - 3x} = \lim_{x \to -\infty} \frac{-x}{\sqrt{9x^2 - x} - 3x} = \lim_{x \to -\infty} \frac{-x}{\sqrt{x^2(9 - 1/x)} - 3x}$$

$$= \lim_{x \to -\infty} \frac{-x}{-x\sqrt{9 - 1/x} - 3x} = \lim_{x \to -\infty} \frac{-1}{-\sqrt{9 - 1/x} - 3} = \frac{-1}{-\sqrt{9} - 3} = \frac{1}{6}$$

answer: $\boxed{1/6}$

10. (2 pts.)(multiple-choice question) Consider the curve $y = \dfrac{x^2-1}{x+1}$. Does this curve have any vertical asymptotes? If yes, indicate the vertical asymptotes.

$$\lim_{x\to -1} \frac{x^2-1}{x+1} \overset{0/0}{=} \lim_{x\to -1}(x-1) = -2$$

no no VA.

Select the correct answer.

A) No. **B)** Yes. $x = 1$ only.

C) Yes. $x = -1$ only. **D)** Yes. $x = 1$ and $x = -1$

Write the letter(capital) of the answer in this box ⟶ 10. $\boxed{\quad)}$

11. (6 pts.) Find the following limit.

$$\lim_{x\to 0} \frac{(x+3)^3 - 27}{x}$$

Type 0/0

Method 1:

$$\lim_{x\to 0} \frac{(x+3-3)\left[(x+3)^2+3(x+3)+9\right]}{x}$$

$$= \lim_{x\to 0} \left[(x+3)^2+3(x+3)+9\right] = 3^2+3^2+9 = 27.$$

Method 2:

$$\lim_{x\to 0} \frac{x^3+3(3)x^2+3(3)^2x+3^3-27}{x}$$

$$= \lim_{x\to 0} \frac{x^3+9x^2+27x}{x} = \lim_{x\to 0}(x^2+9x+27) = 27$$

answer: $\boxed{27}$