**1. Factoring formula**

$$(x-a)(x+a) = x^2 - a^2$$

$$(x \pm a)^2 = x^2 \pm 2ax + a^2$$

$$(x \pm a)^3 = x^3 \pm 3ax^2 + 3a^2x \pm a^3$$

$$x^3 \pm a^3 = (x \pm a)(x^2 \mp ax + a^2)$$

**2. Factoring the form** $x^2 + Bx + C$

step 1: find the 2 numbers a and b such that $a + b = B$ and $a \cdot b = C$
step 2: write $x^2 + Bx + C = (x+a)(x+b)$

**2. Factoring the form** $Ax^2 + Bx + C$

step 1: find the 2 numbers a and b such that $a + b = B$ and $a \cdot b = A \cdot C$
step 2: write $x^2 + Bx + C = x^2 + ax + bx + C$
step 3: using the groupping and factoring common terms to obtain the result.

**3. Completing a square:** $x^2 \pm bx$

step 1: add and subtract $\left(\frac{b}{2}\right)^2$ : $x^2 \pm bx + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$

step 2: group the first three terms and obtain the result: $\left(x \pm \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$

**4. Synthetic division**
**5. Fraction operations**

$$\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}; \quad \frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}; \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}; \quad \frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$$

**6. Exponent operations**

$$a^0 = 1; \quad a^{-1} = \frac{1}{a}; \quad a^{-n} = \frac{1}{a^n} : \quad a^n \cdot a^m = a^{n+m}; \quad \frac{a^n}{a^m} = a^{n-m}; \quad (a^n)^m = a^{n \cdot m}$$

**7. Radical operations**

$$\sqrt{a} = \sqrt[2]{a}; \quad \sqrt[n]{a^n} = a; \quad \sqrt[n]{a} = a^{\frac{1}{n}}; \quad \sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}; \quad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}; \quad \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m = a^{\frac{n}{m}}$$

**8. Rationalization**
If the denomimator has only 1 term, then multiply the top and bottom by this term and simplify.
If the denominator has 2 terms $(\boldsymbol{a \pm b})$, then multiply the top and bottom by $(\boldsymbol{a \mp b})$ and simplify.

**9. Linear equation/functions**: $y = mx + b$

where m is the slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and b is the y-intercept.

The 2 lines $L_1 : y = m_1x + b_1$ and $L_2 : y = m_2x + b_2$ are parallel if $m_1 = m_2$

The 2 lines $L_1 : y = m_1x + b_1$ and $L_2 : y = m_2x + b_2$ are perpendicular if $m_1 \cdot m_2 = -1$

**10. Quadratic equation**: $ax^2 + bx + c = 0$

$$\text{If } x^2 = a \text{ then } x = \pm\sqrt{a}; \qquad x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**11. Inequalities**

$$\text{If } a < b \text{ then if } c < 0 \ ac > bc; \qquad \text{If } |x| = a \text{ then } x = \pm a;$$

$$\text{If } |x| < a \text{ then } -a < x < a; \qquad \text{If } |x| > a \text{ then } x > a \text{ or } x < -a$$

**12. Distance and midpoint**: given 2 points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$

$$Distance(P_1, P_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \qquad Midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

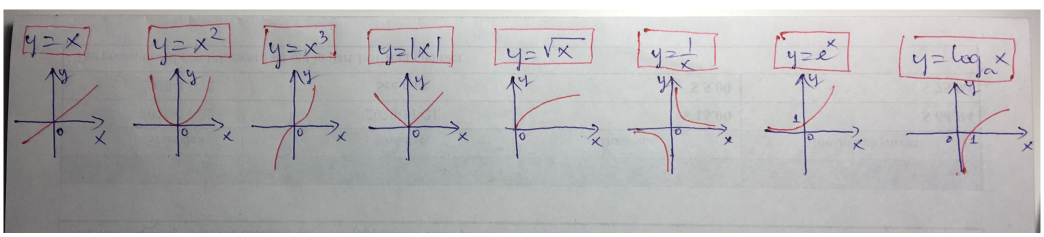
**13. Circles**: given the center $(h, k)$ and the radius r, the equation of a circle is

$$(x - h)^2 + (y - k)^2 = r^2$$

**14. Intercepts**: to find y-intercepts, let $x = 0$ then solve for x; to find x-intercepts, let $y = 0$ then solve for y.

**15. Even and odd functions:**
Test for even, replace $x = -x$ and simplify, if the resulting function is the same as the original one, then it is even.
Test for odd, replace $x = -x$ and $y = -y$, if the result function is the same as the original one then it is odd.

**16. Graph of basic functions**




**17. Graph transformation techniques:** given a graph of function $f(x)$.
Vertically transformations:
The graph of function $f(x) + a$ is obtained by shift the graph of $f(x)$ vertically by a units.
The graph of function $af(x)$ is obtained by compress $(0 < a < 1)$ or stretch $(a > 1)$ $f(x)$ vertically by factor a.
Horizontal transformations:
The graph of function $f(x + a)$ is obtained by shift the graph of $f(x)$ to the left by a units.
The graph of function $f(x - a)$ is obtained by shift the graph of $f(x)$ to the right by a units.
The graph of function $f(ax)$ is obtained by stretch $(0 < a < 1)$ or compress $(a > 1)$ $f(x)$ horizontally by factor a.
Reflection
The graph of function $-f(x)$ is obtained by flip the graph of $f(x)$ over the x-axis.
The graph of function $f(-x)$ is obtained by flip the graph of $f(x)$ over the y-axis.

**18. Quadratic functions:** $f(x) = ax^2 + bx + c$
If $a > 0$ then the graph concave up and it has the minimum at the vertex=$(\frac{-b}{2a}, f(\frac{-b}{2a}))$.
If $a < 0$ then the graph concave down and it has the maximum at the vertex=$(\frac{-b}{2a}, f(\frac{-b}{2a}))$.

**19. Domain of a function**
If $f(x) =$ polynomial then the domain is all real number $\mathbb{R}$.

If $f(x) = \frac{A}{B}$ then we need to put a condition $B \neq 0$.
If $f(x) = \sqrt{A}$ then we need to put a condition $A \geq 0$.
If $f(x) = \frac{A}{\sqrt{B}}$ then we need to put a condition $B > 0$.
If $f(x) = \log_a B$ then we need to put a condition $B > 0$.

**20. Vertical Asymptotes (VA)**
If $f(x) = \frac{A(x)}{B(x)}$, then the VA are the solutions of $B(x) = 0$.
If $f(x) = \log_a b(x)$, then the VA are the solutions of $B(x) = 0$.

**21. Horizontal Asymptotes (HA)**. For $f(x) = \frac{a_n x^n + \ldots}{b_m x^m}$
If $n < m$ (i.e. the highest degree of the top is smaller than the degree of the bottom), then HA: $y = 0$.
If $n = m$ (i.e. the highest degree of the top is equal to the degree of the bottom), then HA: $y = \frac{a_n}{b_m}$.
If $n > m$ (i.e. the highest degree of the top is bigger than to the degree of the bottom), then no HA.

**22. Composition functions:** $fog(x) = f(g(x))$
To find the domain of $fog$, first we need to find the domain of $g(x)$ (the one on the right of little "o"), and the domain of the resulting $fog(x)$, then we combine the two domains into one.

**23. Inverse functions:** $f^{-1}(x)$
To find the inverse of one-to-one function, we interchange x and y, then we solve for y to get the inverse $y = f^{-1}(x)$.
f and g are inverse of each other if and only if $fog(x) = gof(x) = x$.
If f and g are inverse of each other then their graphs are symmetric with respect to the line $y = x$.
Domain of $f(x)$ = range of $f^{-1}(x)$. And range of $f(x)$ = domain of $f^{-1}(x)$.

**24. Exponential functions.** $f(x) = a^x$
If $a > 1$, then $f(x)$ is increasing, going through (0,1), and has HA: $y = 0$.
If $0 < a < 1$, then $f(x)$ is decreasing, going through (0,1), and has HA: $y = 0$.
If $a^u = a^v$, then $u = v$.

**25. Logarithmic functions**. $f(x) = y = \log_a x$ if and only if $a^y = x$
$\log_a x$ and $a^x$ are inverse of each other.

$$\log_a 1 = 0 \qquad \log_a a = 1 \qquad a^{\log_a x} = x \qquad \log_a b^n = n \log_a b \qquad \log_a x + \log_a y = \log_a (x \cdot y)$$

$$\log_a x - \log_a y = \log \frac{x}{y} \qquad \log_a b = \frac{\log_c b}{\log_c a} \text{ for } c > 0$$

If $\log_a x = \log_a y$ then $x = y$.

**26. Compound interest** $A = P(1 + \frac{r}{n})^{nt}$
where A is the future value, P is the principle, r is the interest rate (in decimal), n is the number of compounds (annually n=1, semi-annual n=2, quarterly n=4, monthly n=12, daily n=365, weekly n=52), t is number of years.
Continuous compound interest: $A = Pe^r t$.
Effective rate: $r_e = (1 + \frac{r}{n})^n - 1$.