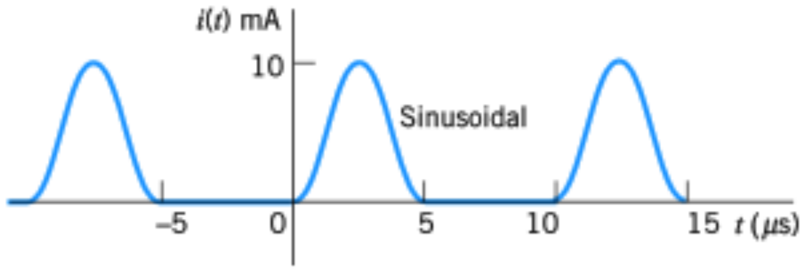


Problem (1)

The period of the sinusoid is half the period of $i(t)$. Let T be the period of $i(t)$ and $T/2$ be the period of the sinusoid. Then

$$i(t) = \begin{cases} 10\sin\frac{4\pi}{T}t & 0 < t < \frac{T}{4} \\ -10\sin\frac{4\pi}{T}t & \frac{T}{2} < t < \frac{3T}{4} \\ 0 & otherwise \end{cases}$$

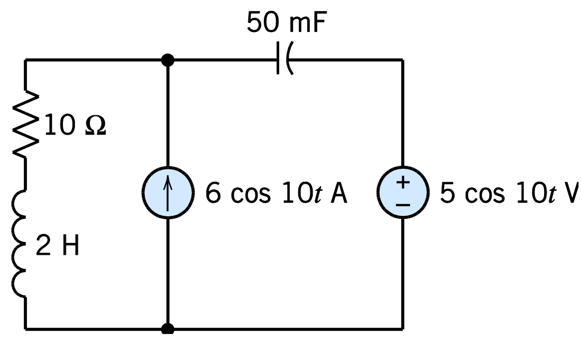
$$I_{RMS} = \frac{100}{T}\left[\int_0^{T/4} \sin^2\frac{4\pi}{T}t\,dt + \int_{T/4}^{3T/4} \sin^2\frac{4\pi}{T}t\,dt\right]$$

$$= \frac{100}{T}\left[\left(\frac{t}{2} - \frac{\sin\frac{8\pi}{T}t}{\frac{18\pi}{T}}\right)_0^{T/4} + \left(\frac{t}{2} - \frac{\sin\frac{8\pi}{T}t}{\frac{18\pi}{T}}\right)_{T/2}^{2T/4}\right] = 25$$

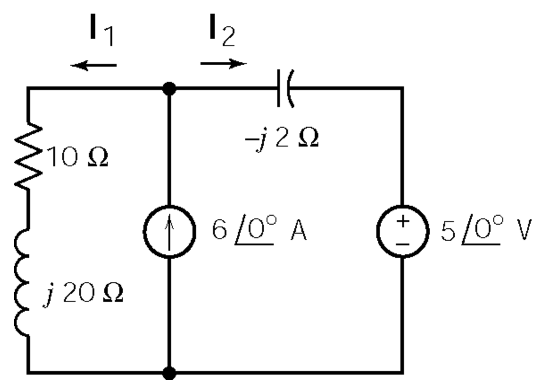
$$I_{\text{rms}} = \sqrt{25} \quad \Rightarrow \quad I_{\text{rms}} = 5 \text{ mA}$$

Problem (2)

For the circuit shown, determine the complex power of the *R, L*, and *C* elements and show that the complex power delivered by the sources is equal to the complex power absorbed by the *R, L*, and *C* elements.



Solution:



KVL:

$$(10+j20)\mathbf{I}_1 = 5\angle 0^\circ - j2\mathbf{I}_2$$

$$\Rightarrow \quad (10+j20)\mathbf{I}_1 + j2\mathbf{I}_2 = 5\angle 0^\circ$$

KCL:

$$\mathbf{I}_1 + \mathbf{I}_2 = 6\angle 0^\circ$$

Solving these equations using Cramer's rule:

$$\Delta = \begin{vmatrix} 10+j20 & j2 \\ 1 & 1 \end{vmatrix} = 10+j18$$

$$\mathbf{I}_1 = \frac{1}{\Delta}\begin{vmatrix} 5 & j2 \\ 6 & 1 \end{vmatrix} = \frac{5-j12}{10+j18} = 0.63\angle 232^\circ \text{ A} = -0.39 - j0.5 \text{ A}$$

$$\mathbf{I}_2 = 6 - \mathbf{I}_1 = 6 + 0.39 + j.5 = 6.39 + j.5 = 6.41\angle 4.47^\circ \text{ A}$$

Now we are ready to calculate the powers. First, the powers delivered:

$$\mathbf{S}_{5\angle 0^\circ} = \frac{1}{2}\left(5\angle 0^\circ\right)\left(-\mathbf{I}_2^*\right) = 2.5\left(6.41\angle(180-4.47)\right) = -16.0 + j1.25 \text{ VA}$$

$$\mathbf{S}_{6\angle 0^\circ} = \frac{1}{2}\left[5 - j2\mathbf{I}_2\right]\left(6\angle 0^\circ\right) = \left[5 - j2\left(6.39 + j.5\right)\right]3 = 18.0 - j38.3 \text{ VA}$$

$$\mathbf{S}_{\substack{\text{Total}\\ \text{delivered}}} = \mathbf{S}_{5\angle 0^\circ} + \mathbf{S}_{6\angle 0^\circ} = \underline{2.0 - j37.2} \text{ VA}$$

Next, the powers absorbed:

$$\mathbf{S}_{10\Omega} = \frac{1}{2}10\left|\mathbf{I}_1\right|^2 = \frac{10}{2}\left(.63\right)^2 = 2.0 \text{ VA}$$

$$\mathbf{S}_{j20\Omega} = \frac{j20}{2}\left|\mathbf{I}_1\right|^2 = j4.0 \text{ VA}$$

$$\mathbf{S}_{-j2\Omega} = \frac{1}{2}\left(-j2\right)\left|\mathbf{I}_2\right|^2 = -j\ \left(6.41\right)^2 = -j41.1 \text{ VA}$$

$$\mathbf{S}_{\substack{\text{Total}\\ \text{absorbed}}} = \underline{2.0 - j37.1 \text{ VA}}$$

To our numerical accuracy, the total complex power delivered is equal to the total complex power absorbed.