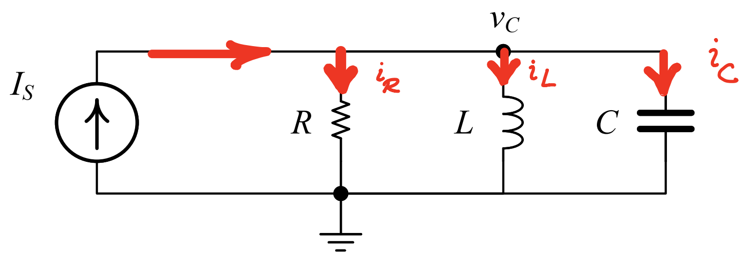

Figure 1: RLC circuit

Consider the circuit shown in Figure 1

1. Write down a differnetial equation for the voltage of v_C.
2. If $I_S = \cos(100t + 30°)$, $R = 100\Omega, L = 0.5$ H and $C = 2 \times 10^{-4}$F, then, using the idea of phasors, write down an expression for the time waveform describing the voltage $v_C(t)$

KCL @ v_C

$$i_R + i_L + i_C = I_S$$

$$i_R = \frac{v_C}{R}, \quad i_C = C\frac{dv_C}{dt}, \quad I_S = \cos(100t + 30°)$$

$$\frac{v_C}{R} + i_L + C\frac{dv_C}{dt} = \cos(100t + 30°)$$

Differentiating both sides with respect to t

$$\frac{1}{R}\frac{dv_C}{dt} + \frac{di_L}{dt} + C\frac{d^2v_C}{dt^2} = -100\sin(100t + 30°$$

But Since

$$v_c(t) = L\frac{di_L}{dt}$$

then $$\frac{di_L}{dt} = \frac{1}{L}v_c(t)$$

Substituting, we get

$$\frac{1}{R}\frac{dv_c}{dt} + \frac{1}{L}v_c(t) + C\frac{d^2v_c}{dt} = 100\cos(100t + 30 + 180 - 90^\circ)$$

This is the differential equation for $v_c(t)$

To use phasors, we represent

$$v_c(t) \Longleftrightarrow V_c\ \angle\theta^\circ$$

$$\frac{dv_c}{dt} \Longleftrightarrow j\omega V_c\angle\theta^\circ = j100\ V_c\angle\theta^\circ$$

$$\frac{d^2v_c}{dt^2} \Longleftrightarrow (j\omega)^2 V_c\angle\theta^\circ = -10^4 V_c\angle\theta^\circ$$

Substituting

$$\frac{1}{100}\ j100\ V_c\angle\theta^\circ + 2\ V_c\angle\theta^\circ + 2\times10^{-4}\times(-10^4)V_c\angle\theta^\circ = 100\ \angle120^\circ$$

$$[j + 2 - 2]V_c\angle\theta^\circ = 100\ \angle120^\circ$$

$$V_c\angle\theta^\circ = \frac{100\ \angle120^\circ}{j}$$

$$= \frac{100 \angle 120^\circ}{1 \angle 90^\circ} = 100 \angle 30^\circ$$

$$= 100 \angle 30^\circ$$

In the time domain

$$v_c(t) = 100 \cos(100t + 30^\circ)$$