Lec 17 Ch.6(Laplace Transform)

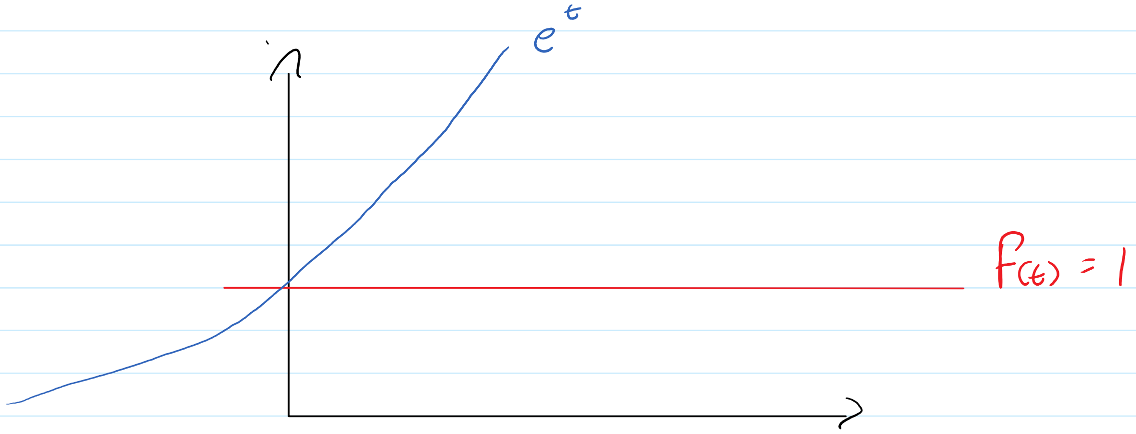
November 6, 2018 9:24 AM

Definition Let $\alpha \in \mathbb{R}$, $f(t)$ be a Function of one Variable t $(t \geq 0)$ we say that the function $f(t)$ is of exponential order α if there exists positive constant t_0 and K such that

$$|f(t)| \leq K e^{\alpha t} \qquad \forall\, t \geq t_0$$

for all

Ex/ $f(t) = 1$ (constant function)



$$|f(t)| \leq e^t$$
$$\forall t \geq 0$$

So $f(t) = 1$ is of exponential order $\alpha = 1$

Ex/ $f(t) = e^{\alpha t}$

$$|f(t)| = |e^{\alpha t}| = e^{\alpha t}$$

$$|f(t)| = |e^{\alpha t}| = e^{\alpha t}$$

$e^{\alpha t}$ is of exponential order α

Ex/ $f(t) = \cos t$

$$|\cos(t)| \leq 1$$

$$= 1\, e^{0t}$$

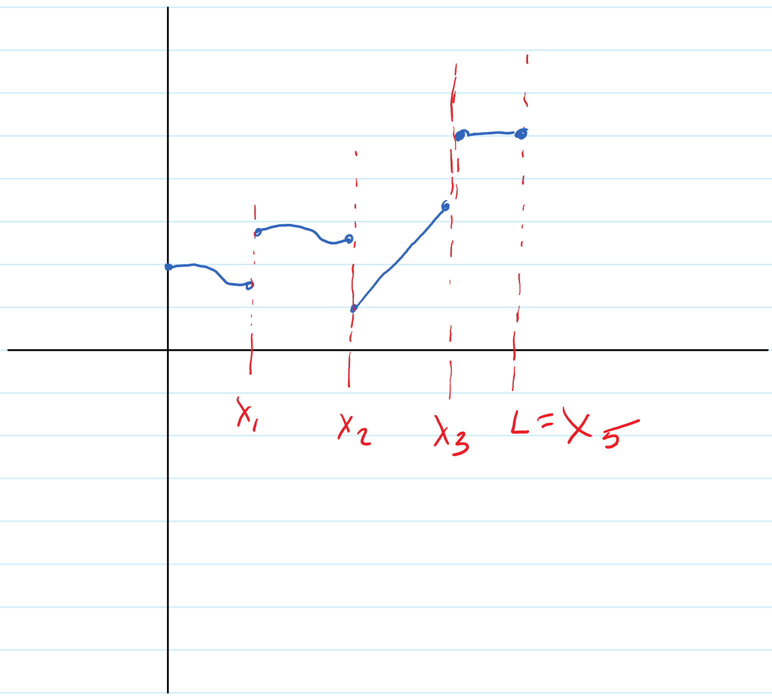
$\cos(t)$ is of exponential order $\emptyset$ same for $\sin(t)$

Definition: We say that the function $f(t)$ is <u>Piecewise continuous</u> on $[0, L]$ if we have a subdivision $x_0 = 0, x_1, x_2, \ldots, x_n = L$ of $[0, L]$ such that:

1) f is continuous on every subinterval $]x_i, x_{i+1}[$

2) For every i

$\lim\limits_{t \to x_i^-} f(t)$ and $\lim\limits_{t \to x_i^+} f(t)$ are finite



<u>Recall</u> we say that the improper integral

$\int_a^{\infty} f(t)\,dt$ is <u>convergent</u> if

$$\lim_{L \to \infty} \int_a^{L} f(t)\,dt \quad \text{exists (a real number)}$$

We say that $\int_a^{+\infty} f(x)$ is <u>divergent</u> if

the limit does not exists (or ∞)

Ex/ $\displaystyle\int_0^{+\infty} \frac{1}{1+x}\,dx$

Sol $\displaystyle\int_0^{+\infty} \frac{1}{1+x}\,dx = \lim_{L\to+\infty} \int_0^{L} \frac{1}{1+x}\,dx$

$$= \lim_{L\to+\infty} \Big[\ln(1+x)\Big]_0^L = \lim_{L\to+\infty} \ln(L+1) - \ln(1)$$

$$= \lim_{L\to+\infty} \big(\ln(1+L)\big) = +\infty$$