Lec 8 Ch.2 Euler-Cauchy / End

Ch.2

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Last time: Euler - Cauchy.

$$x^2 y'' + axy' + by = 0$$

Look for solutions of type $y = x^m$

Char. Eq.

$$m^2 + (a-1)m + b = 0$$

Cas 1 two distinct real roots $m_1 \neq m_2$

$$y = c_1 x^{m_1} + c_2 x^{m_2}$$

Case 2 Double real root. $m_1 = m_2 = m$

$$y = C_1 x^m + C_2 x^m \ln x$$

Cas 3 Two complex conjugate roots: $\alpha + i\beta$; $\alpha - i\beta$

$$y = C_1 x^{\alpha} \cos(\beta \ln x) + C_2 x^{\alpha} \sin(\beta \ln x)$$

Ex 3/ $x^2 y'' - 3xy' + 20y = 0$, $x > 0$, $y(1) = 1$

$y'(1) = 10$

Sol/ Euler-Cauchy with $a=-3,\ b=20$

The Char. eq. is $m^2+(a-1)m+b=0 \iff$

$m^2-4m+20=0$

$$m=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{4\pm\sqrt{(-4)^2-4(1)(20)}}{2}=\frac{4\pm\sqrt{-64}}{2}=\frac{4\pm\sqrt{64i^2}}{2}$$

$m=2\pm4i$: 2 complex conjugate roots

$y_1=x^2\cos(4\ln x)$, $y_2=x^2\sin(4\ln x)$

is a basis of solutions

The General solution is

$$y=C_1x^2\cos(4\ln x)+C_2x^2\sin(4\ln x)$$

$$y'=2C_1x^2\cos(4\ln x)-C_1x^2\sin(4\ln x)\cdot\frac{4}{x}$$
$$+2C_2x\sin(4\ln x)+C_2x^2\cos(4\ln x)\cdot\frac{4}{x}$$

$y(1)=\boxed{1=C_1}$

$y'(1)=10=2C_1+4C_2$

$10=2+4C_2$

$8=4C_2$

$C_2=2$

The solution to the IVP is

$$\boxed{y = x^2\cos(4\ln x) + 2x^2\sin(4\ln x)}$$

Finished Chapter