Ex 3/ Solve the I.V.P:

$$y' + \frac{2}{x} y = (\ln x)\sqrt{y} \; ; \; x > 0 \, , \; y(1) = 1$$

Sol/ Bernouilli type with $f(x) = \frac{2}{x}$, $r(x) = \ln x$, $a = \frac{1}{2}$

let $u = y^{1-a} = y^{1-\frac{1}{2}} = y^{\frac{1}{2}}$

$u = y^{1/2} \Rightarrow u' = \frac{1}{2} y^{-\frac{1}{2}} \cdot y' = \frac{1}{2} y^{-\frac{1}{2}} \left(-\frac{2}{x} y + (\ln(x)) y^{1/2}\right)$ (From the ODE

$u' = -\frac{1}{x} y^{1/2} + \frac{1}{2} \ln x \Rightarrow u' + \frac{1}{x} u = \frac{1}{2} \ln x$: linear with $f(x) = \frac{1}{x}$, $r(x) = \frac{1}{2} \ln x$

$$u = \frac{\int e^{\int \frac{1}{x} dx} \frac{1}{2} \ln x \, dx + c}{e^{\int \frac{1}{x} dx}} = \frac{\frac{1}{2} \int x \ln x \, dx + c}{x}$$

$\frac{1}{2} \int x \ln x \, dx$ $\qquad u = \ln x \, , \; v' = x$

$\qquad u' = \frac{1}{x} \qquad v = \frac{x^2}{2}$

$$\frac{1}{2} \int x \ln x \, dx = \frac{1}{2}\left(\frac{x^2}{2} \ln x - \left(\int \frac{1}{2} x \, dx\right)\right)$$

$$= \frac{1}{2}\left(\frac{1}{2} x^2 \ln x - \frac{1}{4} x^2\right)$$

So $u = \dfrac{\frac{1}{4} x^2 \ln x - \frac{1}{8} x^2 + c}{x}$

$x = 1$, $y = 1 \Rightarrow u(1) = (y(1))^{1/2} = (1)^{1/2} = 1$

$u(1) = 1 \Rightarrow -\frac{1}{8} + c = 1$

$C = \frac{7}{8}$

$$u = \frac{\frac{1}{4}x^2 \ln x - \frac{1}{8}x^2 + \frac{9}{8}}{x}$$

$$= \frac{2x^2 \ln x - x^2 + 9}{8x}$$

$$y = u^2 = \left(\frac{2x^2 \ln x - x^2 + 9}{8x}\right)^2$$

Start of Chapter 2