Chapter 0 Introduction and basic concepts

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Newton's law of motion :

$$F = m\frac{dv}{dt}$$

This is a first order differential equation

Definition:

Let y(x1,x2,x3,...xn) be an unknown function of the independent varable (x1,x2,x3,...xn.
A Differential Equation (DE for short) is an equation relating the function y, some of its derivatives,
Maybe the independent variables (x1,x2,x3,...xn and some other known function of (x1,x2,x3,...xn.

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The order: of DE is the longest order of the derivative in the equation.If y=f(x) is a function of one variable, we say that we have an ordinary DE (OED for short), otherwise , the DE is called partial DE (PDE for short).

ex1/ $x^3 y''' - 3e^x \sin x\, y'' = 3y \sin y$

Function of one Var because

ODE of order 3

If it had more Var. there would be $\frac{\partial y}{\partial x}$

ex2/ $x^2 t^3 \frac{\partial^2 z}{\partial x \partial t} - 3\frac{\partial z}{\partial t} = e^{xt}$

PDE of order 2

Function of more than 1 Var because of the partial derivative

In this course, we only study ODE'S

Definition: We say that a function Y is a solution to a ODE in the interval I if:
(1) y is defined in I
(2) y satisfies the ODE

ex3/ Consider the ODE :

$$y'' - 5y' + 4y = 0$$

Verify that the function $y = Ae^x + Be^{4x}$ is a solution for the ODE on $]-\infty, +\infty[$ for any values of the constants A and B

Solution

(1) yes it is defined on the interval.

(2) $y = Ae^x + B^{4x} \Rightarrow y' = Ae^x + 4B^{4x}$

$y'' = Ae^x + 16B^{4x}$

Back into the ODE

$$y'' - 5y' + 4y = Ae^x + 16Be^{4x} - 5(Ae^x + 4Be^{4x}) + 4(Ae^x + Be^{4x})$$

$$= \cancel{Ae^x} + \cancel{16Be^{4x}} - \cancel{5Ae^x} - \cancel{20Be^{4x}} + \cancel{4Ae^x} + \cancel{4Be^{4x}} = 0$$

is a solution to the ODE.

Remark The above example shows in particular that an ODE has infinetly many solutions. The solution $y = Ae^x + Be^{4x}$

Is called the general solution.

To specify the values for A and B one needs 2 conditions called initial conditions.

ex/ find the particular solution of the ODE : $y'' - 5y' + 4y = 0$ that satisfies the conditions

$y(0) = 1, \quad y'(0) = -2.$

Solution We know that the general sol. is $y = Ae^{x} + Be^{4x}$

$$y' = Ae^{x} + 4Be^{4x}$$

$$\left.\begin{array}{l} y(0) = 1 \Rightarrow A + B = 1 \quad (1) \\ y'(0) = -2 \Rightarrow A + 4B = -2 \quad (2) \end{array}\right\}$$

$-(1) + (2) = 3B = -3$

$B = -1$

$(1) \Rightarrow A = 2$

This particular solution is $\boxed{y = 2e^{x} - e^{4x}}$

Definition: An initial value problem (IVP) consists of an ODE of order n and a set of n initial conditions:

$$y(x_0) = v_0, \; y'(x_0) = v_1, \ldots, \; y^{(n-1)}(x_0) = v_{n-1}$$

Under certain conditions stated by a theorem known as the "theorem of existence and uniqueness" every IVP will have a unique solution. We assume these conditions satisfied in this course!