21/15

UNIVERSITY OF OTTAWA
Principles of Physics
PHY1321/31 Fall 2018
Dr. A. Czajkowski

STUDENT # [redacted]

NAME: [redacted]

ASSIGNMENT 4:
Maxwell Boltzmann
Distribution Heat Engines,

Released: Oct 5, **Due: Oct 12** **6PM Sharp!**

1 Fill the table below:

Molecule	Degrees of Freedom	AVG Energy of single molecule	Cv	Cp	Gamma
A	1	1/2kT	1/2	3/2	3
B	5	5/2 kT	5/2	7/2 R	7/5
C	3	3/2kT	3/2	5/2	5/3
D	11	11/2 kT	11/2R	13/2	13/11
E	4	4/2kT	4/8	8/2	5/4

3.5

4

2 Given is 1mole of oxygen molecules at atmospheric pressure and temperature of 20°C.

a) Write down the correct expression for N(400,402) , the number of molecules having speeds in the interval (400m/s 402/m/s).

b) Calculate the number of molecules having their speed between 400m/s and 402m/s. (provide the number corresponding to part a (do not show your calculations!)

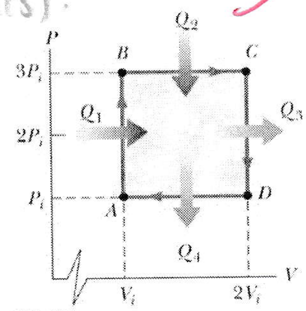
a) $P_v dv = 4\pi\left(\frac{M}{2\pi RT}\right)^{\frac{3}{2}} v^2 e^{-\frac{Mv^2}{2RT}} dv$

$= 4\pi\left(\frac{0.032\,kg/mol}{2\pi(8.3145\,J/mol\cdot K)(298K)}\right)^{\frac{3}{2}}(401\,m/s)^2 e^{-\frac{(0.032\,kg/mol)(401\,m/s)^2}{2(8.3145\,J/mol\cdot K)(298K)}}(2\,m/s)$.

b) 1.41×10^{23} molecule

3

3 A 1.00-mol sample of a monatomic ideal gas is taken through the cycle shown. At point *A*, the pressure, volume, and temperature are P_i, V_i, and T_i, respectively. In terms of *R* and T_i, find (a) the total energy entering the system by heat per cycle, (b) the total energy leaving the system by heat per cycle, (c) the efficiency of an engine operating in this cycle,
(HINT: efficiency of the engine= |Work performed|/|Heat absorbed|



(a) Total energy entering system is $\boxed{\frac{21}{2}T_i}$ (see paper for calculations).

(b) Total energy leaving system is $\boxed{\frac{-17}{2}T_i}$

4

(c) $eff = \frac{|Q_{in} - Q_{out}|}{Q_{in}}$

$= \frac{\left(\frac{21}{2}T_i + \frac{17}{2}T_i\right)}{\frac{21}{2}T_i} \times 100$

$= \frac{4}{21} \times 100$

$\boxed{eff = 19.0\%}$

4 Using Maxwell=Boltzmann Distribution of speeds for Ideal Gas obtain the Boltzmann Distribution of Energies for Ideal Gas. (Follow Lecture Discussions)
/Present your work on the opposite side of this page/

4

4.) Proof:

$P_v dv$ must turn into $P_E dE$.

$$P_v dv = \underbrace{4\pi\left(\frac{m}{2\pi KT}\right)^{\frac{3}{2}}}_{\text{constants}} v^2 e^{\frac{-mv^2}{2KT}}\, dv$$

$\frac{mv^2}{2} = E$, isolate for $v^2 = \frac{2E}{m} = \frac{2mv^2}{2m}$

Change dv to dE.

$$\frac{dE}{dx} = \frac{d\left(\frac{mv^2}{2}\right)}{dx}$$

$$\frac{dE}{dx} = \frac{mv^2}{2}$$

$$\frac{dE}{dx} = \frac{m}{2} \cdot 2v \frac{dv}{dx}$$

$$\frac{dE}{dx} = mv \frac{dv}{dx}$$

$$\frac{dE}{dx} = \sqrt{m}\sqrt{m}\, v \frac{dv}{dx}$$

$$\frac{dE}{dx} = \frac{\sqrt{m}\sqrt{m}\, v\sqrt{2}}{\sqrt{2}} \frac{dv}{dx}$$

* $E = \frac{1}{2}mv^2$

$\sqrt{E} = \sqrt{\frac{1}{2}}\sqrt{m}\, v$

$$\frac{dE}{dx} = \sqrt{m}\sqrt{2} \cdot \frac{\sqrt{m}\, v}{\sqrt{2}} \frac{dv}{dx}$$

$$\frac{dE}{dx} = \sqrt{2m} \cdot \sqrt{E} \frac{dv}{dx}$$

$$dE = \sqrt{2m} \cdot \sqrt{E}\, dv$$

$$dv = \frac{dE}{\sqrt{2m} \cdot \sqrt{E}}$$

Plug into eq.

$$P_v dv = 4\pi\left(\frac{m}{2\pi KT}\right)^{\frac{3}{2}} v^2 e^{\frac{-mv^2}{2KT}}\, dv$$

$$P(E)dE = 4\pi\left(\frac{m}{2\pi KT}\right)^{\frac{3}{2}}\left(\frac{2}{m} \cdot E\right)\left(e^{\frac{-E}{KT}}\right)\left(\frac{dE}{\sqrt{2m} \cdot \sqrt{E}}\right)$$

$$P(E)dE = \frac{8\pi}{\sqrt{2m} \cdot m}\left(\frac{m}{2\pi KT}\right)^{\frac{3}{2}}\left(E^{1} - E^{-\frac{1}{2}}\right)\left(e^{\frac{-E}{KT}}\right)(dE)$$

$$P(E)dE = \left(\frac{8\pi}{\sqrt{2m} \cdot m}\right)\left(\frac{m}{2\pi KT}\right)^{\frac{3}{2}}\left(\sqrt{E}\right)\left(e^{\frac{-E}{KT}}\right)(dE)$$

$$P(E)dE = \frac{2}{\sqrt{\pi}}\left(\frac{1}{KT}\right)^{\frac{3}{2}} \sqrt{E}\, e^{\frac{-E}{KT}}\, dE$$

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ASSIGNMENT 4: CONT
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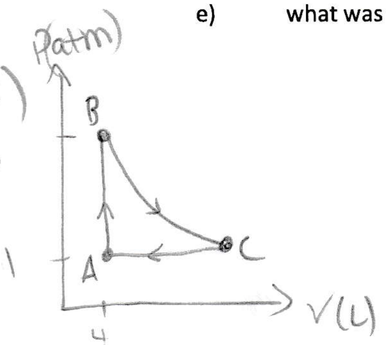
5 A 4 liter sample of a diatomic gas with γ =1.4 confined to a cylinder, is carried through a closed cycle. The gas is initially at 1.00atm. and 300K. First, its pressure is tripled under constant volume. Then it expands isothermally to its original pressure. Finally the gas is compressed isobarically to its original volume.

a) draw pV diagram of this cycle
b) determine the volume of the end of the isothermal expansion
c) find the temperature of the gas at the start of the isothermal expansion
d) find the temperature at the end of the cycle
e) what was the net work done on the gas for this cycle

a.)



b.) $P_B = \frac{1}{V_B}$, $P_C = \frac{1}{V_C}$

$P_B V_B = P_C V_C$

$(3)(4L) = (1)(V_C)$

$V_C = 12L$

c.) $P_A V = nRT_A$, $P_B V = nRT_B$

$\frac{T_A}{P_A} = \frac{T_B}{P_B}$

$\frac{300K}{1atm} = \frac{T_B}{3atm}$

$T_B = 900K$

d.) Temp @ B is 900K,
Temp @ C is 900K (isothermal process)
Temp @ A after 1 cycle?

$P V_C = nRT_C$, $PV_A = nRT_A$

$\frac{T_C}{V_C} = \frac{T_A}{V_A} \Rightarrow \frac{900K}{12L} = \frac{T_A}{4L} \Rightarrow T_A = 300K$

e.) $W_{tot} = W_{AB} + W_{BC} + W_{CA}$

$W_{AB} = 0$ b/c no ΔV.

$W_{BC} = -nRT \ln \frac{V_f}{V_i} = -P_1 V_2 \ln\left(\frac{P_2}{P_1}\right)$

$= -(1atm)(12L)\ln\left(\frac{1atm}{3atm}\right)$

$= +39.6J$ (neg work b/c gas is expanding)

$\therefore = -39.6J$

$W_{CA} = +P(V_f - V_i)$

$W_{CA} = 4(1)(12L - 4L)$

$W_{CA} = +8J$

$W_{tot} = -39.6J + 8J$

$W_{tot} = -31.6J$

3/5

6 A refrigerator has a coefficient of performance of 4.00. The ice tray compartment is at –20.0°C, and the room temperature is 22.0°C. The refrigerator can convert 30.0 g of water at 22.0°C to 30.0 g of ice at -20.0°C each minute. What input power is required? Give your answer in watts.

$COP = \frac{|Q_c|}{W}$

$4 = \frac{|Q_c|}{W}$

$|Q_c| = ?$

$= (mc_{water}\Delta T + mL_f + mc_{ice}\Delta T)$ / minute

$= ((0.030kg)(4186 \frac{J}{kg°C})(0-22°C) + (0.030kg)(3.33\times10^5 J/kg) + (0.030kg)(2090 J/kg°C)(-20-0°C))$ / min

$= -2,762.76 + 9,990J - 1,254J$ (1)

(2) $|Q_c| = 5,973.24$ J/min
$= 99.55$ J/sec

$\Rightarrow 4 = \frac{|Q_c|}{W}$

$W = \frac{99.55 J/s}{4}$

Input Power $= 24.89W$

3/5

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Ass #4 PHY: 1331 A.

2.) $P_v dv = 4\pi \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{\frac{mv^2}{2kT}} dv$

use 401 m/s as interval is small enough to get accurate velocity probability.

a) $P_v dv = \frac{N_A}{N_A} = 4\pi \left(\frac{MM_{O_2}}{2\pi RT}\right)^{\frac{3}{2}} v^2 e^{\frac{-MM_{O_2} v^2}{2RT}} dv$

b) (1) $= 4\pi \left(\frac{0.032 \text{ kg/mol}}{2\pi (8.3145 \frac{J}{mol K})(298K)}\right)(401 \text{ m/s})^2 e^{\frac{-0.032 kg (401 m/s)^2}{2(8.3145 \frac{J}{mol K})(298K)}} (2 \text{ m/s})$

(2) $P = (0.331)(0.354)(2)$

$P = 0.234$

(3) $N = N_A \cdot P$

$= 6.022 \times 10^{23} \frac{\text{molecules}}{\text{mole}} \times 1 \text{mol} \times 0.234$

(4) $N = 1.41 \times 10^{23}$ molecules.

Find all temp relations for each step first.

3) Q_1 for isovolumetric, $Q = nC_v \Delta T$, we know for monoatomic gases, γ is $\frac{5}{3}$ $\therefore C_v = \frac{3}{2}$ $C_p = \frac{5}{2}$

(A→B)

$PV = nRT$

$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$\frac{P_1 V_1}{T_i} = \frac{3P_1 V_1}{T_B}$

$T_B = 3T_i$

(B→C)

$PV = nRT$

$\frac{3P_1 V_1}{3T_1} = \frac{3P_1 2V_1}{T_C}$

$T_C = 6T_1$

(C→D)

$PV = nRT$

$\frac{3P_1 2V_1}{6T_1} = \frac{P_1 2V_1}{T_D}$

$T_D = 2T_1$

(D→A)

$PV = nRT$

$\frac{P_1 2V_1}{2T_1} = \frac{P_1 V_1}{T_1}$

$T_1 = T_1$

$Q_1 = A \to B$

$Q_1 = nC_v \Delta T$

$= (1)\left(\frac{3}{2}\right)(3T_1 - T_1)$

$= \frac{3}{2}(2T_1)$

$= 3T_1$

Q_2 B→C

$Q_2 = nC_p \Delta T$

$= (1)\left(\frac{5}{2}\right)(6T_1 - 3T_1)$

$= \frac{5}{2}(3T_1)$

$= \frac{15}{2} T_1$

Q_3 C→D

$Q_3 = nC_v \Delta T$

$= (1)\left(\frac{3}{2}\right)(2T_1 - 6T_1)$

$= \frac{3}{2}(-4T_1)$

$= -6T_1$

Q_4 D→A.

$Q_4 = nC_p \Delta T$

$= (1)\left(\frac{5}{2}\right)(T_1 - 2T_1)$

$= \frac{5}{2}(-T_1)$

$= -\frac{5}{2} T_i$

tot energy entering systems $3T_1 + \frac{15}{2}T_1 = \frac{21}{2}T_1$

" " leaving system is $-6T_1 + \frac{-5}{2}T_1 = \frac{-17}{2}T_1$