

**MAT 1322 – Fall 2017 Final Exam**

**Multiple Choice:**

1. Evaluate  $\int_1^{\infty} \frac{\ln x}{x} dx$  .

- A) divergent
- B) 0
- C) 1
- D) 1/2
- E)  $\pi$
- F)  $\ln 2$

2. How much work is required to pump all of the water out of a cone with radius 2 m and height 5 m to a height 1 m above the top of the cone?

- A) Insufficient information given.
- B)  $81667\pi$  J
- C)  $169344\pi$  J
- D)  $17280\pi$  J
- E)  $147000\pi$  J
- F)  $15000\pi$  J

3. Determine the volume of the solid formed when the region bounded by  $y = e^{2x}$ ,  $y = 2$  and  $x = 3$  is rotated around  $x = -4$ .

- A) 8025
- B) 3114
- C) -1793
- D) 1793
- E) 7510
- F) 2554

4. Consider the initial value problem  $y' = 2x^2y^2$  with  $y(0) = 3$ . What is  $y(3)$ ?

- A) undefined
- B)  $53/3$
- C)  $-53/3$
- D)  $3/53$
- E)  $-3/53$
- F) a billion!

5. Determine the sum of the series  $\sum_{n=1}^{\infty} 4\left(\frac{5}{9}\right)^{2n+4}$ .

- A)  $-\infty$
- B)  $+\infty$
- C) 9.00
- D) -9.00
- E) 0.170
- F) -0.170

6. What is the error in using the first 7 terms to estimate the sum of the series  $\sum_{n=1}^{\infty} (-1)^n \frac{3^n}{(n^3 + 2)n!}$ ?

- A) Insufficient information given.
- B) The series is not convergent.
- C) 0.000317
- D) 0.000515
- E) 0.00126
- F) 0

7. The equation of the tangent plane to the function  $z = 4x^2y^3$  at the point (3,2) is

A) undefined

B)  $36x + 32y - z = 96$

C)  $z + 288 = 192x + 432y$

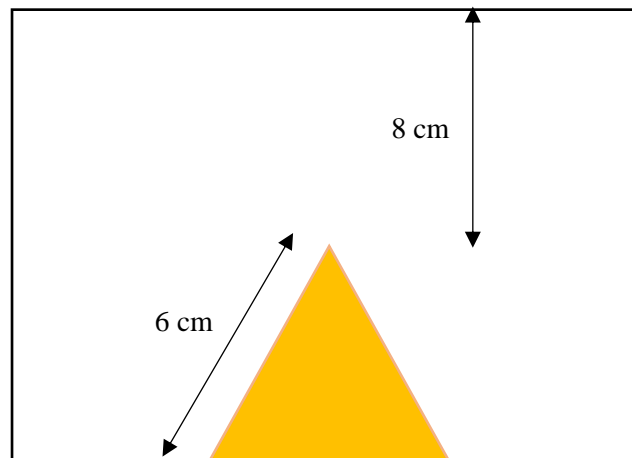
D)  $192x + 432y - z = 1152$

E)  $z = 96 - 36x + 32y$

F)  $-z = 1152 - 192x + 432y$

**Short Answer**

8. Determine the total hydrostatic force on the submerged Doritos chip before it gets soggy. Hurry up! You can assume the chip is an equilateral triangle.



9. Consider the initial value problem  $y' = 2x + y$  with  $y(0) = 2.1$ . Use Euler's Method with  $h = 0.2$  to estimate  $y(0.8)$  to 4 decimal places of accuracy.

10. In an attempt to gain more Instagram followers, you ingest 100 g of polonium-208. #genius Polonium-208 has a half-life of 2.9 years. How long will it take for the level of polonium-208 in your body to reach  $<1$  g?

11. Use the comparison test to determine if the following series converges or diverges:  $\sum_{n=1}^{\infty} \frac{\sin^2 n + 2}{\sqrt{3n^3 + 4}}$

12. What is the interval of convergence of  $\sum_{n=0}^{\infty} \frac{(x-3)^n n^2}{n-2}$ ?

13. Find the first three non-zero terms of the MacLaurin series expansion of  $e^{2x} \sin(4x)$ .

14. Given the following functions, determine  $\frac{\partial z}{\partial s}$  and  $\frac{\partial z}{\partial t}$ .

$$z = x^3 - 2y^2$$

$$x = 2s^4 - t$$

$$y = t^3 + 4s^2$$

**Long Answer:**

15. A 400 L tank is used to mix a salt solution. The tank initially contains pure water. Salt water (40 kg salt/L) enters the tank at a rate of 20 L/min. The well-mixed solution exists at the same rate. Determine the concentration of salt in the tank after 1 h.

16. Do the following series converge or diverge? Use an appropriate test.

a)  $\sum_{n=1}^{\infty} \sin n \cos n$

b)  $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2 + 4}$

c)  $\sum_{n=1}^{\infty} \frac{e^n}{n \ln n}$

d)  $\sum_{n=1}^{\infty} \frac{5^n (n+2)}{n!}$

17. Consider the following multivariable function:  $z = 3x^2 y^3 + 2$ .

a) Find all first- and second-order partial derivatives.

b) Determine the linear approximation to the function at the point (1, 3) and use the linear approximation to estimate the true value of the function at (1.1, 3.1).

c) Find the directional derivative to the function in the direction of the vector  $\langle 3, 4 \rangle$  at the point (1, 1).