



Université d'Ottawa • University of Ottawa

Faculté des sciences / Faculty of Science
Mathématiques et de statistique / Mathematics and Statistics

Calculus III for Engineers

MAT 2322A - Fall 2014

Final Exam

Professor: Victor G. LeBlanc

Time limit: 3 hours. Closed books.

Name: _____

ID Number: _____

Instructions

- This is a closed book exam. Furthermore, all cell phones, pagers or any other electronic or communication devices are forbidden. **The only calculators which are allowed are Texas Instruments TI-30, TI-34, Casio fx-260 and fx-300, scientific and non programmable.**
- This exam has 15 pages and you have 3 hours to complete it.
- Read each question carefully before answering.
- Questions 1 to 10 are multiple choice questions. These questions are worth 2 points each and no partial marks are possible. **Please write your answers in the corresponding boxes in the grid below entitled “Answers to multiple choice Qs”.**
- Questions 11 to 16 are long answer questions and are worth 5 marks each, so organize your time accordingly. **A correct answer requires a full, clearly-written and detailed solution.** Answer each question in the space provided, using backs of pages or the extra pages at the end if necessary.
- Do not unstaple the test. Good luck!

Answers to multiple choice Qs

1	2	3	4	5	6	7	8	9	10

Grid below is used for grading
(do not write in this grid)

MCQ	11	12	13	14	15	16	Total
/20	/5	/5	/5	/5	/5	/5	/50

1. Consider the function $f(x, y) = x^3y + 2x^2y + x - y$. Which of the following expressions corresponds to the tangent plane to the graph of $z = f(x, y)$ at the point $(x, y, z) = (1, 2, f(1, 2))$?

A. $z = 15x + 2y$

B. $z = (3x^2y + 4xy + 1)(x - 1) + (x^3 + 2x^2 - 1)(y - 2) + 5$

C. $z = 5$

D. $z = 15(x - 1) + 2(y - 2) + 5$

E. $z = 15\vec{i} + 2\vec{j}$

F. This function is not differentiable at the indicated point, so the tangent plane does not exist.

2. Consider the parametrized curve

$$\vec{r}(t) = \frac{1}{2}t^2\vec{i} + \frac{2\sqrt{2}}{3}t^{\frac{3}{2}}\vec{j} + t\vec{k}, \quad 1 \leq t \leq 2.$$

What is the total arclength of this curve?

A. $\frac{5}{2}$

B. $\frac{3}{2}$

C. 0

D. 2

E. $\frac{19}{3}$

F. π

3. For the function $f(x, y) = xe^{xy}$, what is the value of the directional derivative of f at the point $(1, 0)$ in the direction of the vector $\vec{u} = -\frac{\sqrt{2}}{2}\vec{i} + \frac{\sqrt{2}}{2}\vec{j}$?

A. -1

B. 0

C. 1

D. 2

E. 3

F. 4

4. Which of the following vector fields is conservative?

A. $\vec{F}(x, y, z) = x^2\vec{i} + e^y\vec{j} + \cos(z)\vec{k}$

B. $\vec{F}(x, y, z) = (x + y + z)\vec{i}$

C. $\vec{F}(x, y, z) = yz\vec{i} + xz\vec{j} + xyz\vec{k}$

D. $\vec{F}(x, y, z) = -z\vec{i} + x\vec{k}$

E. All of the above

F. None of the above

5. Consider the function $f(x, y) = x^2 - 2y + 6x - y^2 + 2$. Which of the following statements about f is true?

- A. f has a local maximum at the point $(x, y) = (-3, -1)$
- B. f has a saddle point at the point $(x, y) = (-3, -1)$
- C. f has a local minimum at the point $(x, y) = (-3, -1)$ and a saddle point at $(x, y) = (0, 0)$
- D. f has a local maximum at the point $(x, y) = (-3, -1)$ and a saddle point at $(x, y) = (0, 0)$
- E. f has a saddle point at the point $(x, y) = (-3, -1)$ and a local maximum at $(x, y) = (0, 0)$
- F. f has a saddle point at the point $(x, y) = (-3, -1)$ and a local maximum at $(x, y) = (0, 0)$.

6. Which of the following expressions corresponds to the integral $\int_0^1 \int_{1-x^2}^1 f(x, y) dy dx$ with order of integration reversed?

- A. $\int_1^0 \int_1^{1-x^2} f(x, y) dy dx$
- B. $\int_{1-x^2}^1 \int_0^1 f(x, y) dx dy$
- C. $\int_{1-x^2}^1 \int_0^1 f(y, x) dx dy$
- D. $\int_0^1 \int_{1-y^2}^1 f(y, x) dx dy$
- E. $\int_0^1 \int_{\sqrt{1-y}}^1 f(x, y) dx dy$
- F. $\int_0^1 \int_0^1 f(x, y) dx dy$

7. Consider the vector field $\vec{F}(x, y) = (2x + 2y)\vec{i} + (2x + 2y)\vec{j}$, and let C be the circle $x^2 + y^2 = 4$ oriented counter-clockwise. Then the line integral

$$\int_C \vec{F} \cdot d\vec{r}$$

is equal to

- A. -3
- B. -2
- C. -1
- D. 0
- E. 1
- F. 2 .

8. Consider the three-dimensional solid in the **first octant** which is bounded between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$, and the cones $z = \frac{\sqrt{3}}{3}\sqrt{x^2 + y^2}$ and $z = \sqrt{3}\sqrt{x^2 + y^2}$. This solid has a mass-density given by $\delta(x, y, z) = x$. What is the total mass of this solid?

- A. $\frac{5\pi}{16}$
- B. $\frac{3\sqrt{3} - 3}{4}$
- C. $\frac{\pi}{8}$
- D. 0
- E. $\frac{\pi}{12}$
- F. None of the above.

9. Consider the two-dimensional region D drawn below, whose boundary is the oriented curve C , also drawn. Let $\vec{F}(x, y) = P(x, y)\vec{i} + Q(x, y)\vec{j}$ be a vector field with continuous partial derivatives. Then which of the following equations corresponds to Green's theorem?

A. $\int_C P dx + Q dy = \iint_D \left(\frac{\partial Q}{\partial y} - \frac{\partial P}{\partial x} \right) dx dy$

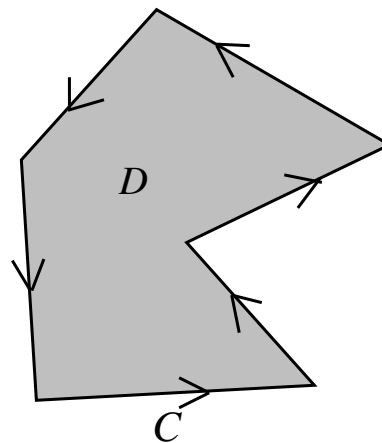
B. $\int_C P dx + Q dy = \iint_D \left(\frac{\partial P}{\partial x} - \frac{\partial Q}{\partial y} \right) dx dy$

C. $\int_C P dx + Q dy = \iint_D \left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x} \right) dx dy$

D. $\int_C P dx + Q dy = \iint_D \left(\frac{\partial Q}{\partial x} + \frac{\partial P}{\partial y} \right) dx dy$

E. $\int_C P dx + Q dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy$

F. $\int_C Q dx + P dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy$



10. Consider the parametrized surface S described by

$$\vec{r}(\theta, z) = 2 \cos \theta \vec{i} + 2 \sin \theta \vec{j} + z \vec{k}, \quad \frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}, \quad 0 \leq z \leq 3,$$

and the scalar function $f(x, y, z) = yz$. What is the value of the surface integral $\iint_S f dS$?

A. $7\sqrt{2}$

B. $8\sqrt{2}$

C. $9\sqrt{2}$

D. $10\sqrt{2}$

E. $11\sqrt{2}$

F. $12\sqrt{2}$

11. Consider the solid defined by the inequalities

$$0 \leq z \leq 8 - x^2 - y^2,$$

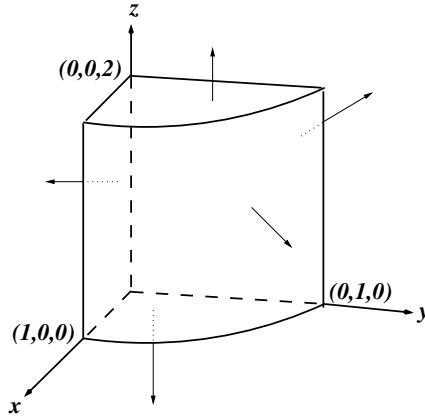
$$0 \leq x \leq 2,$$

$$0 \leq y \leq 1.$$

This solid has a mass density given by $\delta(x, y, z) = x + 2y$. Find the total mass of this solid.

12. Find the global extrema for the function $f(x, y) = 2x^2 + 3y^2 - 4x - 5$ on the disk of radius 4, centered at the origin, $0 \leq x^2 + y^2 \leq 16$.

13. For the vector field $\vec{F}(x, y, z) = (x^2y + y \sin z)\vec{i} + (xy^2 + ze^x)\vec{j} + (xy + y^2)\vec{k}$, compute the divergence of \vec{F} , i.e. compute $\vec{\nabla} \cdot \vec{F}$. Then, **using Gauss' divergence theorem**, compute the surface (flux) integral $\int \int_S \vec{F} \cdot d\vec{S}$, where S is the outward-oriented surface illustrated below. Note that S is the boundary of the solid region D which is the portion of the cylinder $x^2 + y^2 \leq 1$, $0 \leq z \leq 2$ which lies in the first octant.



14. Consider the integral $\int_{y=0}^{y=1} \int_{x=0}^{x=\sqrt{1-y^2}} e^{x^2+y^2} dx dy$. Convert it to polar coordinates, and then evaluate the integral.

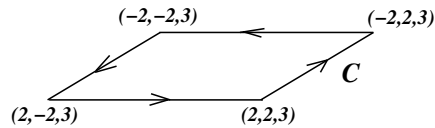
15. Consider the vector field $\vec{F}(x, y) = (2x + y \sin y) \vec{i} + (x \sin y + xy \cos y) \vec{j}$.

(a) Show that the vector field \vec{F} is conservative.

(b) Find a potential for \vec{F} , i.e. find a scalar function $f(x, y)$ such that $\vec{F}(x, y) = \vec{\nabla} f(x, y)$

(c) Compute the line integral $\int_C \vec{F} \cdot d\vec{r}$, where C is the oriented straight line segment that starts at $(x, y) = (0, 0)$ and ends at $(x, y) = (1, \pi/2)$.

16. Consider the vector field $\vec{F}(x, y, z) = (-yz + xe^{x^2})\vec{i} + (xz + y \sin(y^2))\vec{j} + (e^z + z \cos(z))\vec{k}$, and let C be the oriented curve illustrated below consisting of the four straight line segments which run from $(2, -2, 3)$ to $(2, 2, 3)$ to $(-2, 2, 3)$ to $(-2, -2, 3)$ and then back to $(2, -2, 3)$. Compute the line integral $\int_C \vec{F} \cdot d\vec{r}$ using either a direct computation, or using Stokes' theorem. Note that one of these two methods will yield a much simpler computation than the other one, so choose the method carefully.



(Extra page)

(Extra page)

(Extra page)