CARLETON UNIVERSITY

FINAL/DEFERRED EXAMINATION
DECEMBER 2016

DURATION: 3 HOURS

Department and Course Number: Mathematics and Statistics, MATH 1104ABCDE
Course Instructors: Ş. Alaca (c), J. Nilsson, R. Mallick, M. Blenkinshop, M. Sadeghi

AUTHORIZED MEMORANDA:
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Instructions:

1. Please circle your section below.

- Section A (Ş. Alaca)
- Section B (J. Nilsson)
- Section C (R. Mallick)
- Section D (M. Blenkinsop)
- Section E (M. Sadeghi)
- Section F (M. Sadeghi)

2. Please provide your name and student number below.

Last Name ________________________ Given Names ____________________

Student Number ____________________

3. This examination paper contains 13 pages. Please report any missing pages to the proctor.

ANSWER ALL QUESTIONS IN PART I and PART II (pp.3-12)

Question	Maximum Mark	Mark Obtained
Part I: multiple-choice questions	36	
Part II: 1	12	
2	8	
3	8	
4	14	
5	10	
6	12	
Total	100	

Multiple-Choice Answer Sheet

1. (a) (b) (c) (d) (e)
2. (a) (b) (c) (d) (e)
3. (a) (b) (c) (d) (e)
4. (a) (b) (c) (d) (e)
5. (a) (b) (c) (d) (e)
6. (a) (b) (c) (d) (e)
7. (a) (b) (c) (d) (e)
8. (a) (b) (c) (d) (e)
9. (a) (b) (c) (d) (e)
10. (a) (b) (c) (d) (e)
11. (a) (b) (c) (d) (e)
12. (a) (b) (c) (d) (e)

PART I: Multiple Choice Questions. Three marks each. No partial marks. Circle the correct answer on the Multiple-Choice Answer Sheet on page 2. There is only one correct answer for each question.

1. Consider the following augmented matrix of a system of linear equations:

$$\left[\begin{array}{cccc|c} 1 & 2 & 2 & 2 & 3 \\ 0 & 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 5 & 7 \\ -1 & -1 & 0 & 1 & 1 \end{array}\right].$$ The system has

(a) a unique solution

(b) infinitely many solutions with one free variable

(c) infinitely many solutions with two free variables

(d) infinitely many solutions with three free variables

(e) no solutions

2. Let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a linear transformation such that $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$. What is $T\left(\begin{bmatrix} 3 \\ -2 \end{bmatrix}\right)$?

(a) $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ **(b)** $\begin{bmatrix} 9 \\ -8 \end{bmatrix}$ **(c)** $\begin{bmatrix} 9 \\ 8 \end{bmatrix}$ **(d)** $\begin{bmatrix} 3 \\ -4 \end{bmatrix}$ **(e)** $\begin{bmatrix} -3 \\ 4 \end{bmatrix}$

3. Let $A^{-1} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ and $b = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$. If $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is the solution of the matrix equation $Ax = b$, what is x_1?

(a) -2 **(b)** 2 **(c)** 1 **(d)** -1 **(e)** $\dfrac{23}{2}$

4. Let $u = \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}$, $v = \begin{bmatrix} 2 \\ 3 \\ 1 \\ -1 \end{bmatrix}$ and $w = \begin{bmatrix} 3 \\ 1 \\ 5 \\ t \end{bmatrix}$.

   For what value of t is the set $\{u, v, w\}$ linearly **dependent**?

   (a) -3 **(b)** -1 **(c)** 3 **(d)** 2 **(e)** 0

5. Let A, B and C be 3×3 matrices. If $\det A = 2$, $\det B = 4$, and $\det C = 8$, what is $\det(2AB^{-1}C^T)$?

   (a) 2^8 **(b)** 2^6 **(c)** 2^5 **(d)** 2^4 **(e)** 2^3

6. Let A be a 5×8 matrix such that row echelon form has 5 pivot positions (leading entries). Which of the following statements is **FALSE**?

   (a) $\dim \mathrm{Nul} A = 3$.

   (b) $\mathrm{Nul} A = \mathbb{R}^3$.

   (c) $\mathrm{Rank} A = 5$.

   (d) $\dim \mathrm{Col} A = 5$.

   (e) $\mathrm{Col} A = \mathbb{R}^5$.

7. Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 4 & 4 & 4 & 4 & 4 \end{bmatrix}$. What is the dimension of Nul$A$?

(a) 4 **(b)** 1 **(c)** 0 **(d)** 5 **(e)** 3

8. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$. Find the matrix X such that $2X - B = AX + I$.

(a) $\frac{1}{2}\begin{bmatrix} 2 & 1 \\ 2 & 4 \end{bmatrix}$ **(b)** $\frac{1}{3}\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ **(c)** $\frac{1}{3}\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$ **(d)** $\frac{1}{2}\begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix}$ **(e)** $\frac{1}{3}\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$

9. Let $A = \begin{bmatrix} 7 & 4 & 16 \\ 2 & 5 & 8 \\ -2 & -2 & -5 \end{bmatrix}$ and $x = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$.
You are given that x is an eigenvector of A. What is the corresponding eigenvalue?

(a) 1 **(b)** -1 **(c)** -3 **(d)** 2 **(e)** 3

10. If the orthogonal projection of the vector $x = \begin{bmatrix} 6 \\ 0 \\ 9 \end{bmatrix}$ onto the vector $u = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ is $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, what is the value of b?

    (a) -4 **(b)** -1 **(c)** 1 **(d)** 4 **(e)** 0

11. What is the standard form $a + bi$ of the complex number $\dfrac{5+12i}{2-3i}$?

    (a) $-2-3i$ **(b)** $-2+3i$ **(c)** $2+3i$ **(d)** $3-2i$ **(e)** $-3+2i$

12. Let $A = \begin{bmatrix} 1 & -9 \\ 4 & 1 \end{bmatrix}$. What are the eigenvalues of A?

    (a) $1, 6$ **(b)** $2 \pm 4i$ **(c)** $4 \pm 2i$ **(d)** $6 \pm i$ **(e)** $1 \pm 6i$

PART II: Long answer questions. Show all your work.

[12] 1. Find the general solution of the following system of linear equations. Write the solution in vector parametric form.

$$\begin{aligned} -x_1 + 3x_2 - 2x_3 + 4x_4 &= 0 \\ 2x_1 - 6x_2 + x_3 - 2x_4 &= -3 \\ x_1 - 3x_2 + 4x_3 - 8x_4 &= 2 \end{aligned}$$

[**8**] **2.** Let $A = \begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 4 \\ 1 & 1 & 3 \end{bmatrix}$. Find the inverse of the matrix A.

[8] 3. Let $A = \begin{bmatrix} 7 & 0 & 3 & 1 \\ 3 & 6 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 5 & 0 & 1 & 3 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ 7 \\ 4 \\ 9 \end{bmatrix}$ and $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$. You are given that $\det A = 96$.

Use Cramer's Rule to find x_1 (without solving for x_2, x_3 and x_4) in the matrix equation $Ax = b$.

[14] 4. Let $A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 6 \\ 2 & 0 & 6 \end{bmatrix}$.

You are given that the characteristic equation of A is $\lambda(1-\lambda)(\lambda-7)=0$.

(a) Find the eigenvalues of the matrix A.

(b) For each eigenvalue, find a basis for the corresponding eigenspace.

(c) Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

[**10**] **5.** Let $W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 3 \end{bmatrix} \right\}$.

(a) Find a basis for W. What is the dimension of W?

(b) Write $x = \begin{bmatrix} 4 \\ 7 \\ 0 \end{bmatrix}$ as a linear combination of the basis vectors of W, which you found in part (a).

[12] 6. Let $u_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix}$, $u_3 = \begin{bmatrix} -2 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, $W = \text{Span}\{u_1,\ u_2,\ u_3\}$ and $x = \begin{bmatrix} 6 \\ 4 \\ 8 \\ 10 \end{bmatrix}$.

(a) Show that $\{u_1,\ u_2,\ u_3\}$ is an orthogonal set.

(b) Find the orthogonal projection of the vector x onto W.

(c) Write x as the sum of a vector in W and a vector orthogonal to W.

(d) Find the distance from x to W.

This page is left blank for rough work only.