11 Evaluate the integral $\int \sin^{-1}(x)\,dx$

(a) $x\sin^{-1}(x) - \frac{\sqrt{1-x^2}}{2} + C$ (b) $x\sin^{-1}(x) + \ln(\sqrt{1-x^2}) + C$ (c) $x\sin^{-1}(x) + \sqrt{1-x^2} + C$
(d) $\ln|\sin x| + C$ (e) None of these

12 Evaluate the integral $\int \frac{dx}{x^2+2x}$

(a) $\ln(x^2+2x)+C$ (b) $\frac{1}{2}\{\ln|x| - \ln|x+2|\}+C$ (c) $\tan^{-1}(x+1)+C$ (d) $\tan^{-1}(x+2\sqrt{x})+C$
(e) None of these

13 Find that antiderivative of the function $\frac{1}{x^2+2x+2}$ whose value at $x=0$ is 0.

(a) $\ln(x^2+2x+2) - \ln 2$ (b) $-\frac{1}{x} + 2\ln|x| + \frac{x}{2} + 1$ (c) $\tan^{-1}(x+1) - \frac{\pi}{4}$ (d) $\frac{1}{2}\tan^{-1}(\frac{x}{2})$

14 Interpret the following integral geometrically $\int_0^3 \sqrt{9-x^2}\,dx$

(a) It is the area of a semicircle of radius 3 (b) It is the area of a quarter circle of radius 3 in the first quadrant (c) It is the area of the quarter circle of radius 9 in the first quadrant (d) It is the volume of the quarter circle of radius 3 in the first quadrant when rotated about the x-axis

15 Evaluate the integral $\int \cos^5 x\,dx$

(a) $\frac{1}{6}\sin^6 x + C$ (b) $\sin x - \frac{1}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C$ (c) $\sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C$
(d) $1 - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C$ (e) None of these

16 Evaluate the integral $\int \tan^3 x\,dx$

(a) $\frac{1}{4}\tan^4 x + C$ (b) $\frac{1}{2}\tan^2 x - \sec^2 x + C$ (c) $\frac{1}{2}\tan^2 x + \ln|\cos x| + C$
(d) $\frac{1}{2}\tan^2 x + \ln|\sin x| + C$

17 Evaluate the integral $\int_1^\infty \frac{dx}{x}$

(a) 0 (b) 1 (c) -1 (d) ∞ (e) None of these

18 Compute the area of the region enclosed by the curves $y = x^2$ and $y = 2x$.

(a) 2 (b) 4 (c) $\frac{4}{3}$ (d) $\frac{20}{3}$

19 Compute the volume of the solid of revolution obtained by rotating the region bounded by $|y| = 3x$, $x = 1$, about the x-axis.

(a) 2π (b) 3π (c) 4π (d) 6π

20 Evaluate $\int_1^e \frac{3(\ln x)^2 + x}{x}\,dx$.

(a) $1 + \ln 3$ (b) $\ln 4$ (c) $3e + 1$ (d) e (e) $2 + \ln 3$

21 Let $f(x) = |x|x^2 + 2$. Calculate $\lim_{h\to 0} \frac{f(-3+h) - f(-3)}{h}$.

(a) -18 (b) -9 (c) -27 (d) does not exist. (e) -14

22 Let $f(x) = \arctan(x^2)$. Calculate $f''(-1)$.

(a) 1 (b) 9 (c) -9 (d) -1 (e) -14

23 Find an expression for the volume V of the solid of revolution obtained by rotating the region bounded by the graphs of $y = e^x + 5x$, $y = e^x + 2x$, $x = 0$ and $x = 1$ about the y-axis.

(a) $\int_0^1 \pi(e^x + 3x)dx$ (b) $\int_0^1 2\pi x e^x dx$ (c) $\int_0^1 2\pi x dx$ (d) $\int_0^1 6\pi x^2 dx$ (e) $\int_0^1 \pi x e^x dx$

24 Evaluate the improper integral $\int_0^1 16x^3 \ln x\,dx$.

(a) divergent (b) -1 (c) -8 (d) -14 (e) -16

25 Evaluate $\int_0^1 xe^x\,dx$.

(a) e (b) -1 (c) 1 (d) 0 (e) $e - 1$

[Total: 100 marks]

END OF THE EXAMINATION.

CARLETON UNIVERSITY

FINAL EXAMINATION
MATH 1004
2018

DURATION: 3 HOURS

Department Name and Course Number: School of Mathematics and Statistics, MATH 1004 A, B, C, D, E, F
Course Instructor(s): Mr. Blenkinsop, Dr. Cova, Dr. Hua.

PAGES-2

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CALCULATORS PERMITTED, NO WIRELESS DEVICES

In addition to the EXAMINATION PAPER students will require an ADDITIONAL ANSWER BOOK, and a SCANTRON SHEET.

1. Please verify that you are in possession of a Scantron FORM.
2. Please **fill in your COURSE CODE** (e.g., MATH 1004) and **COURSE SECTION** (e.g., A, B, C, D, E, F), **YOUR NAME** and **YOUR STUDENT NUMBER** where required on the Scantron form.
3. **The examination consists of one sheet of legal size paper with text on BOTH sides.** It is out of a total of 100 and consists of 25 multiple choice questions each worth 4 marks. **Please fill in only one answer on your Scantron sheets with a pencil** as there is only one answer to any given question. Circling two or more answers to any question invalidates that question (*i.e.*, you get 0 marks for that question).
4. **This exam may be released to the Library and may be taken away by the student.**

Return the SCANTRON form ONLY. .

1 Find the equation of the tangent line to the curve $y = \ln(\ln x)$ at the point where $x = e$.

(a) $y = \frac{1}{e}x - 1$ (b) $y = \frac{1}{e}x - e$ (c) $x - ey + e = 0$ (d) $y = ex$ (e) None of these

2 Let

$$f(x) = \begin{cases} x^2 - \alpha^2, & \text{if } x < 4, \\ \alpha x + 20, & \text{if } x \geq 4. \end{cases}$$

What value of α will make f continuous on $(-\infty, \infty)$?

(a) $\alpha = 12$ (b) $\alpha = 4$ (c) $\alpha = -2$ (d) $\alpha = 2$ (e) None of these

3 Given that $\lim_{x\to 4}[f(x) + g(x)] = 2$, and $\lim_{x\to 4}[f(x) - g(x)] = 1$, compute $L = \lim_{x\to 4}[f(x)g(x)]$.

(a) $L = \frac{3}{2}$ (b) $L = \frac{3}{4}$ (c) $L = 4$ (d) $L = 2$ (e) None of these

4 Let If $h(2) = 4$, $h'(2) = 3$, then $\dfrac{d}{dx}\left(\dfrac{h(x)}{x}\right)$ evaluated at $x = 2$ is equal to:

(a) 12 (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{2}$ (e) None of these

5 Let $r = \sin f(t)$ where $f(0) = \pi/3$ and $f'(0) = 4$. Evaluate $\dfrac{dr}{dt}$ at $t = 0$.

(a) $\frac{1}{2}$ (b) 4 (c) $-\frac{1}{2}$ (d) -1 (e) 2

6 Find the derivative of the function $y = x^x$

(a) $x\,x^{x-1}$ (b) $x^x \ln x$ (c) $x^x(\ln x + 1)$ (d) $e^{x \ln x}$ (e) $x\,x^{x-1} \ln x$

7. Compute $\lim\limits_{\alpha\to 0} \dfrac{\tan\alpha}{\alpha}$.

(a) 1 (b) 0 (c) $\frac{\pi}{4}$ (d) ∞ (e) $-\frac{\pi}{2}$

8. Evaluate $\lim\limits_{x\to-\infty} \dfrac{\sqrt{1+x^2}}{5x-7}$.

(a) 0 (b) $-\frac{1}{5}$ (c) $\frac{1}{7}$ (e) $\frac{1}{5}$ (e) $-\frac{7}{5}$

9 Evaluate the integral $\int 6x^2 e^{x^3+1}\,dx$

(a) $3e^{x^3+1} + C$ (b) $2e^{x^3+1} + C$ (c) $x^3 e^{x^3+1} + C$ (d) $2x^3 e^{x^3+1} + C$ (e) None of these

10 Evaluate the integral $\int (x^2 + x)e^{2x}\,dx$

(a) $\frac{1}{2}x^2 e^{2x} + C$ (b) $\frac{1}{2}(x^2+2x+1)e^{2x} + C$ (c) $\frac{1}{2}(x^2 - x - 1)e^{2x} + C$ (d) $(\frac{x^3}{6} + \frac{x^2}{4})e^{2x} + C$