Université d'Ottawa
Faculté de génie

University of Ottawa
Faculty of Engineering

L'Université canadienne
Canada's university

GNG 1105 C
Engineering Mechanics

Mid-Term Exam
Professor A. Skaff

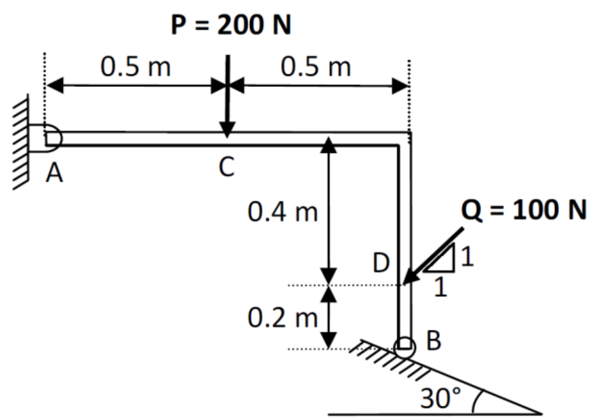
01 November 2018
Time: 80 min.

Closed Book. Non programmable calculators are allowed. Free-body diagrams must be drawn wherever appropriate.

Question 1 (15 marks):

An L-shaped bracket AB is being acted upon by forces **P = 200 N** and **Q = 100 N** as shown. It isbeing held in place by a pin joint at A and a roller at B.

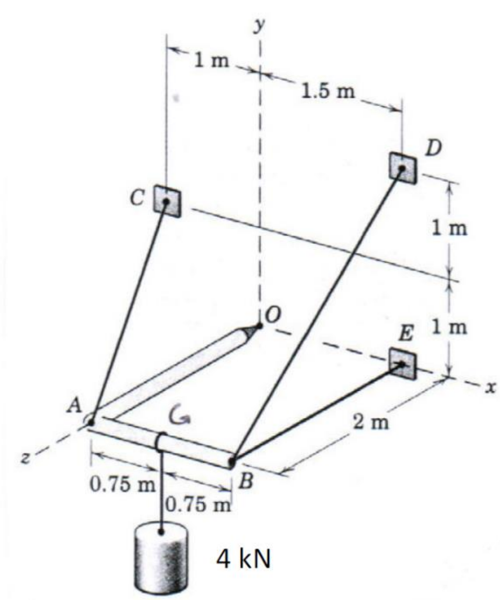
a) Draw the Free-Body-Diagram of this bracket.
b) Reduce the forces P and Q into a force-couple system at A.
c) Calculate the reactions at A and B.



Question 2 (15 marks) :

The right-angle boom supports a **4 kN** force at point G as shown in the diagram. It is being held in equilibrium by a ball and socket joint at O and by three cables AC, BD, and BE. Note that C, D and E lie in the **vertical x-y plane**.

a) Draw the Free-Body-Diagram of this boom.
b) Write the tension in the cable AC, BD and BE in vector form.
c) Find the reaction in each of the cables AC, BD and BE.



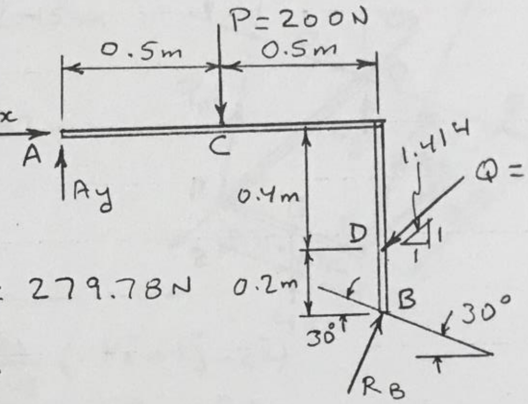
GNG 1105 A&C

Mid-term Examination — Nov. 1, 2018

SOLUTIONS

1.

a) FBD - See Diagram



b) $R_x = -100 \times \frac{1}{1.414} = -70.7N$

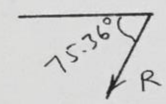
$R_y = -100 \times \frac{1}{1.414} - 200N$

$= -270.7 N$

$R = \sqrt{(-70.7)^2 + (-270.7)^2} = 279.78N$

$\theta = \tan^{-1} \frac{270.7}{70.7} = 75.36°$

$\therefore R = \underline{\underline{279.78 N}}$



ANS.

$\overset{+}{\circlearrowleft} \Sigma M_A = -100 \times \frac{1}{1.414} \times 0.4m$

$- 100 \times \frac{1}{1.414} \times 1.0m - 200 \times 0.5m = \underline{\underline{-198.98 N.m}}$. ANS

$\therefore$ The force-Couple System at A is: $R = \underline{\underline{279.78 N}}$ 75.36° R AN.

$M = \underline{\underline{198.98 N.m}}$

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c) $\overset{+}{\circlearrowleft} \Sigma M_A = -100 \times \frac{1}{1.414} \times 0.4m - 100 \times \frac{1}{1.414} \times 1.0m - 200 \times 0.5m$

$+ R_B \sin 30° \times 0.6m + R_B \cos 30° \times 1.0m = 0$

$1.166 R_B = 198.98$; $\therefore R_B = \underline{\underline{170.65N}}$ ANS

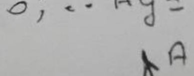
$\overset{+}{\rightarrow} \Sigma F_x = 0$

$A_x - 100 \times \frac{1}{1.414} + 170.65 \sin 30° = 0$, $\therefore A_x = \underline{\underline{-14.6N}}$

$+\uparrow \Sigma F_y = 0$

$A_y - 200N - 100 \times \frac{1}{1.414} + 170.65 \cos 30° = 0$; $\therefore A_y = 122.9N$

$A = \sqrt{(-14.6)^2 + (122.9)^2} = \underline{\underline{123.76N}}$



ANS.

$\theta = \tan^{-1} \frac{122.9}{-14.6} = \underline{\underline{83.2°}}$

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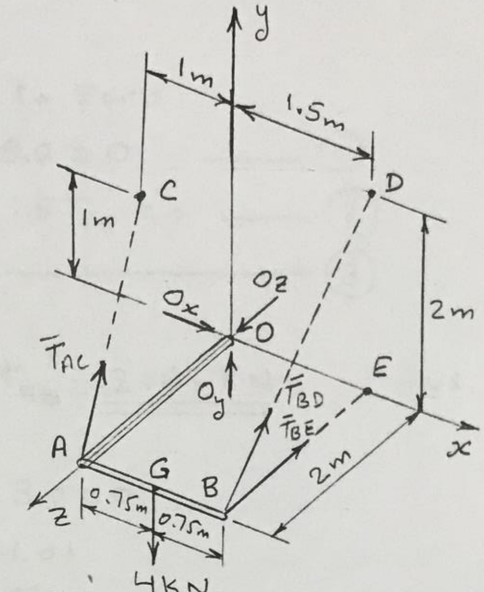

3 2. a) FBD - See diagram

b) $\overline{AC} = -1m\,\bar{i} + 1m\,\bar{j} - 2m\,\bar{k}$

$AC = \sqrt{(-1)^2+(1)^2+(-2)^2} = 2.45\,m$

$\overline{BD} = +2m\,\bar{j} - 2m\,\bar{k}$

$BD = \sqrt{(2)^2+(-2)^2} = 2.83\,m$

$\overline{BE} = -2m\,\bar{k}$

$BE = 2.00\,m$

$\bar{T}_{AC} = T_{AC}\,\bar{\lambda}_{AC} = T_{AC}\dfrac{\overline{AC}}{AC} = \dfrac{T_{AC}}{2.45}(-1\bar{i}+1\bar{j}-2\bar{k})$

$\bar{T}_{BD} = T_{BD}\,\bar{\lambda}_{BD} = T_{BD}\dfrac{\overline{BD}}{BD} = \dfrac{T_{BD}}{2.83}(2\bar{j}-2\bar{k})$

6 $\bar{T}_{BE} = T_{BE}\,\bar{\lambda}_{BE} = T_{BE}\dfrac{\overline{BE}}{BE} = \dfrac{T_{BE}}{2.00}(-2\bar{k})$

c) $\Sigma\bar{M}_o = 0$

$\Sigma\bar{M}_o = \bar{r}_{A/o}\,\bar{T}_{AC} + \bar{r}_{B/o}\,\bar{T}_{BD} + \bar{r}_{B/o}\,T_{BE} + \bar{r}_{G/o}\,(-4KN)\bar{j} = 0$

where,

$\bar{r}_{A/o} = (2.0m)\bar{k}$; $\bar{r}_{B/o} = (1.5m)\bar{i} + (2.0m)\bar{k}$; $\bar{r}_{G/o} = (0.75m)\bar{i} + (2.0m)$

$\therefore\ \Sigma\bar{M}_o = 2\bar{k} \times \dfrac{T_{AC}}{2.45}(-1\bar{i}+1\bar{j}-2\bar{k})$

$+ (1.5\bar{i} + 2.0\bar{k}) \times \dfrac{T_{BD}}{2.83}(2\bar{j} - 2\bar{k})$

$+ (1.5\bar{i} + 2.0\bar{k}) \times \dfrac{T_{BE}}{2.00}(-2\bar{k})$

$+ (0.75\bar{i} + 2.0\bar{k}) \times (-4\bar{j}) = 0$

$\bar{k}$ $\bar{j}$ $\bar{i}$

$\Sigma M_o = -0.82\,T_{AC}\,\bar{j} - 0.82\,T_{AC}\,\bar{i}$

$+ 1.06\,T_{BD}\,\bar{k} + 1.06\,T_{BD}\,\bar{j} - 1.41\,T_{BD}\,\bar{i}$

$+ 1.5\,T_{BE}\,\bar{j}$

$- 3.0\bar{k} + 8.0\bar{i} = 0$

Equate Coefficients of $\bar{i}$, $\bar{j}$ and $\bar{k}$ to Zero

$\bar{i}$: $-0.82\,T_{AC} - 1.41\,T_{BD} + 8.0 = 0$ ——— 1

$\bar{j}$: $-0.82\,T_{AC} + 1.06\,T_{BD} + 1.5\,T_{BE} = 0$ ——— 2

$\bar{k}$: $1.06\,T_{BD} - 3.0 = 0$ ——— 3

From Eq. 3 : $T_{BD} = \underline{\underline{2.83\,KN}}$ ANS.

Insert value of T_{BD} in eq. 1 :

$$-0.82\,T_{AC} - 1.41 \times 2.83 + 8.0 = 0$$

$$0.82\,T_{AC} = 4.01$$

$$\therefore\ T_{AC} = \underline{\underline{4.89\,KN}}$$ ANS.

Insert value of T_{AC} & T_{BD} in eq. 2 :

$$-0.82 \times 4.89 + 1.06 \times 2.83 + 1.5\,T_{BE} = 0$$

$$1.5\,T_{BE} = 1.01$$

$$\therefore\ T_{BE} = \underline{\underline{0.67\,KN}}$$ ANS.

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Determinant Form:

$$\Sigma M_O = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -1 & 1 & -2 \\ 0 & 0 & 2 \end{vmatrix} \times \frac{T_{AC}}{2.45} + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 2 & -2 \\ 1.5 & 0 & 2 \end{vmatrix} \times \frac{T_{BD}}{2.83}$$

$$+ \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 0 & -2 \\ 1.5 & 0 & 2 \end{vmatrix} \times \frac{T_{BE}}{2.00} + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & -4 & 0 \\ 0.75 & 0 & 2 \end{vmatrix} = 0$$