MATH1004E - Test 3: 13:35–14:25, Nov 19

Surname _____ **First Name** _____ **Student #** _____

Total: 30 points. Closed book! Calculators are allowed!

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1. (8 points, 2 points each) Multiple choices, no partial marks.

(i) $e^{\ln a+\ln b} =$

(a) $a+\ln b$ (b) $\ln a+b$ (c) $a+b$ (d) ab (e) $a-b$

Solution: (d).

(ii) Let $f(x)=\ln(x^2+1)$, then $f'(1) =$

(a) 0 (b) 1 (c) 2 (d) 1/2 (e) Does not exist.

Solution: (b). $f'(x)=\frac{2x}{x^2+1}$.

(iii) $\lim\limits_{t\to 0}\dfrac{e^t-t-1}{t^2} =$

(a) 1 (b) 0.5 (c) 2 (d) 1.5 (e) does not exist.

Solution: (b). $\lim\limits_{t\to 0}\frac{e^t-t-1}{t^2}=\lim\limits_{t\to 0}\frac{e^t-1}{2t}=\lim\limits_{t\to 0}\frac{e^t}{2}=\frac{1}{2}$.

(iv) The vertical and horizontal asymptotes of $y=\dfrac{x+3x^2-1}{x^2-x}$ are:

(a) $x=0, x=1, y=0$ (b) $x=0, y=1$ (c) $x=1, y=3$ (d) $x=1, y=0$
(e) $x=0, x=1, y=3$

Solution: (e)

2. (4 points) Given $y=x^{2x+1}$. Find $\frac{dy}{dx}$.

Solution: By logarithmic differentiation,

$$\ln y=(2x+1)\ln x,$$

$$\frac{d}{dx}\ln y=\frac{d}{dx}((2x+1)\ln x),\Rightarrow$$

$$\frac{y'}{y} = 2\ln x + (2x+1)\left(\frac{1}{x}\right) = 2\ln x + \frac{2x+1}{x}, \Rightarrow$$
$$y' = y\left(2\ln x + \frac{2x+1}{x}\right) \quad \text{or} \quad y' = x^{2x+1}\left(2\ln x + \frac{2x+1}{x}\right).$$

3. (4 points) Let $f(x) = \dfrac{x^2-1}{x^2+1}$. Find the intervals of increase and decrease.

Solution: $f'(x) = \frac{4x}{(x^2+1)^2}$.
$f'(x) = 0$ implies that $x = 0$. Thus only one critical point is $x = 0$.
$f'(x) > 0$ when $x > 0$, which is the interval of increase;
$f'(x) < 0$ when $x < 0$, which is the interval of decrease;

4. (5 points) Let $f(x) = xe^{2x}$. Study the concavity and find points of inflection.

Solution: By the product rule, $f'(x) = (1+2x)e^{2x}$. $f''(x) = 4(1+x)e^{2x}$.
$f''(x) = 0 \Rightarrow x = -1$.
when $x < -1$, $f''(x) < 0$, $f(x)$ is concave down;
when $x > -1$, $f''(x) > 0$, $f(x)$ is concave up.
Point of inflection is at $x = -1$, or $(-1, -e^{-2})$.

5. (5 points) Find the function $f(x)$ such that $f'(x) = \dfrac{3x^7+x^3}{x^5}$ and $f(1) = 2$.

Solution:

$$f(x) = \int f'(x)\,dx = \int \frac{3x^7+x^3}{x^5}\,dx = \int (3x^2 + x^{-2})\,dx = x^3 - \frac{1}{x} + C.$$
$$f(1) = 2 \Rightarrow C = 2, \Rightarrow$$
$$f(x) = x^3 - \frac{1}{x} + 2.$$

6. (4 points) Let $g(x) = \displaystyle\int_0^x \sin(t^2+1)dt$. Find $g'(2)$ and $g''(2)$.

Solution: By the fundamental theorem of calculus,
$g'(x) = \sin(x^2+1)$, $g'(2) = \sin 5$.
$g''(x) = 2x\cos(x^2+1)$, $g''(2) = 4\cos 5$.