UNIVERSITY OF CALGARY
FACULTY OF ENGINEERING

Monday 30 October 2000
18:30 – 21:00

THE EXAM IS OPEN TEXT

There are 36 questions.
Answer all 36 questions by indicating the letter of the correct answer on the scoring sheet.

Each question answered correctly is awarded 1 mark.
Each question answered incorrectly is awarded 0 marks.

Total possible for the entire exam is 36 marks.

1. If the number of days Microsoft Windows will run without crashing on a certain computer is a random variable that varies according to an exponential distribution with a mean of 30 days, then indicate which of the following is the probability (accurate to 2 significant figures) that Windows will crash on this computer in less than 20 days.

   a) 0.95
   b) 0.66
   c) 0.49
   d) 0.45
   e) none of the above

2. If a destructive avalanche hits a certain ski cabin on average once every 20 years, what is the probability (accurate to 2 significant figures) that three or more destructive avalanches will hit the cabin in a given 15-year period?

   a) 0.041
   b) 0.092
   c) 0.12
   d) 0.28
   e) 0.52

3. An urn contains 10 balls marked $1 and 7 balls marked $2. You take out 3 balls at random without replacement, and you get paid the sum of the $ values on the 3 balls you take out. What is the probability (accurate to 4 significant figures) that you win $5 or more at this game?

   a) 0.3088
   b) 0.3127
   c) 0.3188
   d) 0.3603
   e) 0.3812

4. Suppose that the population of a certain city is 40% male and 60% female. Suppose also that 50% of the males and 30% of the females smoke. Indicate which of the following is the probability (accurate to 2 significant figures) that a randomly selected smoker from the city is male.

a) 0.28
b) 0.53
c) 0.71
d) 0.73
e) 0.77

5. A gambling game consists of three coins being tossed. If the result is all heads or all tails then the player wins $10. If the result is either one or two heads then the player must pay $2. Which of the following is the expected gain for the player?

a) - $ 1.00
b) $ 0.25
c) 0
d) $ 0.625
e) $ 1.00

6. Indicate which of the following is the variance of a random variable X, where X has probability density function:

$$f(x)=\begin{cases}2(x-1) & 1<x<2\\ 0 & \text{elsewhere}\end{cases}$$

a) $\frac{7}{36}$
b) $\frac{1}{18}$
c) $\frac{6}{13}$
d) $\frac{17}{6}$
e) none of the above

7. A manufacturer of automobile tires (not Firestone) reports that among a shipment of 5000 tires sent to a distributor, 1000 are slightly blemished. If a person purchases 10 of these tires at random from the distributor, what is the probability (accurate to 2 significant figures) that this person will get exactly 3 tires that are slightly blemished?

a) 0.1042
b) 0.2013
c) 0.5514
d) 0.6778
e) 0.8791

8. Soil A contains an average of 7 worms per m^2, whereas Soil B contains an average of 14 worms per m^2. A blind bird lands on one of these two types of soil with Soil A being twice as likely as Soil B, and digs out an area of 200 mm by 200 mm. What is the probability (accurate to 3 significant figures) that the bird finds exactly 1 worm in this square?

a) 0.018
b) 0.032
c) 0.106
d) 0.248
e) none of the above

9. A survey reveals that 70% of senior citizens disapprove of pot smoking. If 12 senior citizens are selected at random and asked their opinions, what is the probability (accurate to 4 significant figures) that between 7 and 9 inclusive of this group of 12 disapprove of pot smoking?

a) 0.8650
b) 0.6294
c) 0.4709
d) 0.2528
e) 0.1667

10. The probability that a student excels at social studies is 0.10. The probability that a student excels at physics is 0.14. It is 20% probable that a student excels at social studies given that the student excels at physics. Indicate which of the following is the probability (accurate to 3 significant figures) that a student excels at either social studies or physics or both?

a) 0.121
b) 0.124
c) 0.212
d) 0.316
e) none of the above

11. X is a random variable with probability density function:

$$f(x) = \begin{cases} c(x^3 + 3x^2) & 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

Indicate which of the following is correct concerning the allowable value or values for c:

a) $0 < c \le 1$
b) $c = 1.25$
c) $c = 1.00$
d) $c = 0.80$
e) none of the above

12. How many distinctly different ways can 3 red, 4 yellow, and 2 blue bulbs be arranged in a string of Christmas tree lights with 9 sockets?

a) 1260
b) 860
c) 630
d) 240
e) none of the above

13. Two random variables X and Y have a joint density function given by

$f(x,y) = (1/50)(x^2 + y^2)$ for $0 < x < 2$, $1 < y < 4$
$= 0$ elsewhere

Indicate which of the following is the most appropriate response to the question: Are X and Y statistically independent?:

a) yes, they are independent
b) no, they are dependent
c) maybe
d) it depends
e) none of the above

14. Eight thousand tickets are to be sold at $1.00 each in a lottery. There is one prize of $3000.00 that is to be awarded by randomly selecting one ticket out of the eight thousand sold. Indicate the expected gain for a person who purchases exactly two tickets?

a) $ 0.75
b) $ 0.38
c) - $ 0.63
d) - $ 1.25
e) - $ 2.75

15. Suppose that there are three chests, each having two drawers. The first chest has a gold coin in each drawer, the second chest has a gold coin in one drawer and a silver coin in the other drawer, and the third chest has a silver coin in each drawer. A chest is chosen at random, one of the two drawers of this chest is opened and it is found that the drawer contains a gold coin. What is the probability that the other drawer in this chest also contains a gold coin?

a) 1/2
b) 2/3
c) 3/4
d) 1/4
e) none of the above

16. Suppose that the time, in years, for the failure of an electronic thingy is a random variable that follows a gamma distribution with parameters $\alpha = 2$ and $\beta = ½$. What is the probability that the thingy will fail before 2 years have passed?

a) $\int_0^2 8x^2 e^{-x/2}\,dx$

b) $\int_0^2 4xe^{-x/2}\,dx$

c) $\int_0^2 \frac{x}{8} e^{-2x}\,dx$

d) $\int_0^2 \frac{x}{4} e^{-2x}\,dx$

e) none of the above

17. Determine the expected value of $Z = 2X^2 + 4XY - 3$ given that $\mu_x = 2$, $\sigma_x^2 = 4$, $\mu_y = 1$, $\sigma_y^2 = 2$, and $\sigma_{xy} = 1$

a) 13
b) 17
c) 21
d) 25
e) none of the above

18. Out of 12 people, what is the probability that exactly 5 are born on the same day of the week (i.e. on Monday, ..., or Sunday)?

a) $7(1/7)^5(6/7)^7$

b) $(1/7)^5(6/7)^7$

c) $\left(\frac{12!}{7!5!}\right)\left(\frac{6^7}{7^{12}}\right)$

d) $\left(\frac{12!}{6!5!}\right)\left(\frac{6^7}{7^{12}}\right)$

e) none of the above

19. An electrical firm manufactures light bulbs that have a length of life that is normally distributed with a mean of 600 hours and a standard deviation of 25 hours. Indicate which of the following is the probability (accurate to 3 significant figures) that a randomly selected bulb from the firm has a life between 560 and 638 hours.

a) 0.00952
b) 0.0539
c) 0.878
d) 0.881
e) 0.935

20. X is a random variable with probability density function

$$f(x)\begin{cases}\frac{1}{10}(2x^3+x) & 0\le x\le 2\\ 0 & \text{elsewhere}\end{cases}$$

Indicate which of the following is the value for F(1):

a) $1/10$
b) $3/10$
c) $1/2$
d) $6/10$
e) none of the above

21. The factory of the Gourmand Cookie Company has three cookie machines: machine M produces milk chocolate chip cookies, machine W produces white chocolate chip cookies, and machine D produces dark chocolate chip cookies. On a normal day, machines M, W and D produce 45%, 35% and 20% of the total cookies at the factory, respectively. If a cookie has no chips, it is considered defective. The percentage of defective cookies made by machines M, W and D are 2%, 6% and 7%, respectively. Suppose that a cookie from the factory is selected at random on a given day and is found to have no chips, what is the probability (accurate to 2 significant figures) that machine M produced the cookie – given the additional information that machine D was not working on that day and therefore could not have produced the defective cookie?

a) 0.04
b) 0.20
c) 0.30
d) 0.43
e) 0.70

22. A coin is tossed 4 times. What is the probability that at least one head occurs given that at least one tail occurs?

a) $14/15$
b) $15/16$
c) $(15/16)^2$
d) $1-(1/16)^2$
e) $1-(15/16)(1/16)^2$

23. A child is making an ice cream sundae. He must choose one flavour: chocolate, vanilla, strawberry or butterscotch. He must choose to have the ice cream in a cone or in a bowl. Finally, he must choose to add one of either chocolate sprinkles, nuts, or nothing for a topping. How many different combinations are possible for the sundae?

a) 9
b) 20
c) 24
d) 24!
e) none of the above

24. A mixture of nuts contains 9 walnuts, 12 almonds and 14 peanuts. If a person randomly selects five nuts from this mixture, indicate which of the following is the probability (accurate to 6 significant figures) of getting at least one almond or peanut.

a) 0.000388132
b) 0.226218
c) 0.742857
d) 0.999612
e) none of the above

25. The random variables X and Y have the joint probability density function:

$f(x,y) = 2$ for $0 < x < y$, $0 < y < 1$
$= 0$ elsewhere

Indicate which of the following is the value of σ_{xy}

a) $1/36$
b) $1/32$
c) $1/8$
d) $1/4$
e) $1/2$

26. The average number of oil tankers arriving each day at a certain port in a city across the Pacific Ocean is known to be 10. The facilities at the port can handle at most 15 tankers per day. What is the probability (accurate to 3 significant figures) that on a given day tankers will have to be sent away?

a) 0.0230
b) 0.0352
c) 0.0487
d) 0.0563
e) none of the above

27. Indicate which of the following is correct concerning the difference between a binomial random variable and a hypergeometric random variable:

a) The binomial concerns trials with two possible outcomes whereas the hypergeometric concerns trials with multiple outcomes
b) The binomial requires independence among the trials whereas the hypergeometric does not require independence among the trials.
c) The binomial concerns the number of successes in n trials whereas the hypergeometric concerns the number of trials to get k successes.
d) The binomial is for trials without replacement and the hypergeometric is for trials with replacement
e) none of the above, that is, all the above are NOT correct.

28. The radius of a circle of random size is represented by the random variable R. What is the variance of the area A of this circle if it is known that $E(R) = 2$, $E(R^2) = 6$, and $E(R^4) = 38$?

a) $36\pi^2$
b) $32\pi^2$
c) $4\pi^2$
d) $2\pi^2$
e) π^2

29. A study has been undertaken to determine the relationship between the sand content of soil (X) and soil density (Y). An analysis of observations of X and Y provide a value of 1642.3% kg/m^3 for the covariance between X and Y. Indicate which of the following interpretations concerning the relationship between the sand content of soil and soil density is most appropriate on the basis of this covariance.

a) there is no relationship between soil density and the sand content of soil
b) there is a strong linear relationship where soil density decreases as the sand content of soil increases
c) there is a strong linear relationship where soil density increases as the sand content of soil increases
d) there is a relationship where soil density increases as the sand content of soil increases
e) none of the above.

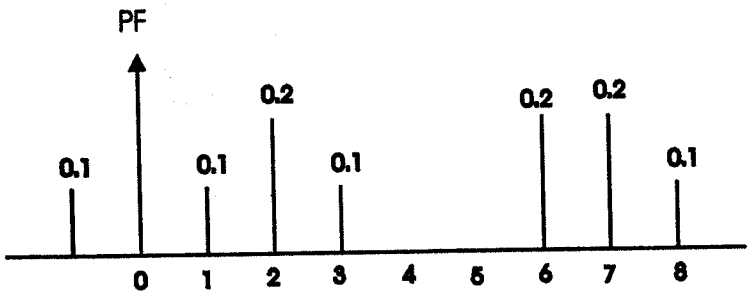
30. The probability of you winning a unicycle race in any weather conditions is 10%. However, when it is raining this probability doubles because you are more skilled than the other racers on slippery roads. The probability that it will rain on race day is one third. What is the probability of the union of the two events 'you win the race' and 'it rains'?

a) 3/30
b) 4/30
c) 11/30
d) 12/30
e) none of the above

31. For the two events A and B, P(A) = 0.60, P(B) = 0.35 and P(AUB) = 0.74. Indicate which of the following is the most appropriate response to the question: Are A and B independent?

a) yes, they are independent
b) no, they are dependent
c) only if A and B are mutually independent
d) only if A and B are mutually exclusive
e) none of the above

32. X is a discrete random variable with the following probability mass function:



What is the value of the cumulative distribution function for x = 6.3 ?

a) 0.20
b) 0.63
c) 0.70
d) undefined
e) none of the above

33. The joint probability function for two discrete random variables X and Y is

$$f(x,y) = \begin{cases} \dfrac{1+x(y+1)}{21} & \text{for}(x,y) = (2,-1),(2,0),(2,1),(3,-1),(3,0),(3,1) \\ 0 & \text{elsewhere} \end{cases}$$

Indicate which of the following is the marginal probability function for X.

a) $\dfrac{1+3x}{21}$ for x = 2, 3 and 0 elsewhere

b) $\dfrac{3x+1}{21}$ for x = 2, 3 and 0 elsewhere

c) $\dfrac{x+3}{11}$ for x = 2, 3 and 0 elsewhere

d) $\dfrac{2+2(x+2)}{24}$ for x = 2, 3 and 0 elsewhere

e) $\dfrac{x+1}{7}$ for x = 2, 3 and 0 elsewhere

34. The probability that it will rain tomorrow is 0.24, the probability that it will be windy tomorrow is 0.61 and the probability that it will both rain and be windy tomorrow is 0.35. Indicate the probability (accurate to 3 significant figures) that it will be windy tomorrow given that it does not rain tomorrow.

a) 0.500
b) 0.461
c) 0.257
d) 0.146
e) none of the above

35. The diameter of a metal screw is a normally distributed random variable with mean of 16 mm and a variance of 0.25 mm^2. What is the probability (accurate to 3 significant figures) that a randomly selected screw will have a diameter greater than 16.25 mm?

a) 0.0794
b) 0.159
c) 0.309
d) 0.841
e) none of the above

36. Solve either Problem (i) or Problem (ii) and mark your response as the answer for Question 36:

(i) A local company manufactures telephone wire. The average length of the wire is 52 inches, with a standard deviation of 6.5 inches. At most, what percentage (accurate to 3 significant figures) of the telephone wire from this company exceeds 71.5 inches? Assume that the distribution is symmetric about the mean.

a) 88.9
b) 11.1
c) 33.3
d) 5.56
e) none of the above

(ii) The average rate of occurrence of through-cracks in a 100 km segment of highway pavement is 0.4 per km. If X is the distance between km 0 and the 4^{th} through-crack, then the distribution for this random variable is:

a) negative binomial (n=40, r=4)
b) negative binomial (n=40, r=3)
c) gamma (α=3, β=2.5 km)
d) gamma (α=4, β=2.5 km)
e) none of the above