Université d'Ottawa
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Canada's university

University of Ottawa
Faculty of Engineering

Department of
Civil Engineering

CVG 4148
Theory of Structures II

FINAL EXAMINATION

Length of Examination: 3 hours **December 7th, 2012, 14:00**

Professor: Dr. B. Martín-Pérez

Family Name: _____

First Name: _____

Student Number: _____

Number of booklets submitted: _____

Signature: _____

OPEN BOOK EXAM.

- If you do not understand a question, clearly state an assumption and proceed.
- Programmable calculators or other electronic devices are not allowed.
- Questions can be done in any order.
- You are required to provide all details of your calculations.
- Clearly label your answers and provide units, if applicable.

RETURN THE EXAMINATION SHEETS AT THE END OF THE EXAM.

1. (25 marks)

The steel rod (element 2) is used to stiffen the cantilever beam (element 1) subjected to a uniform distributed load of 200 kN/m. Assemble the structure stiffness matrix [S] and the load vector $\{F\}$ for the assembly shown in Fig. 1. For the steel rod, E = 210 GPa and A = 1×10^{-3} m^2. For the cantilever beam, E = 210 GPa, A = 2×10^{-3} m^2, and I = 5×10^{-5} m^4. The angle between the beam and the steel rod is 45°. Clearly indicate the numbering of degrees of freedom on your analytical model.

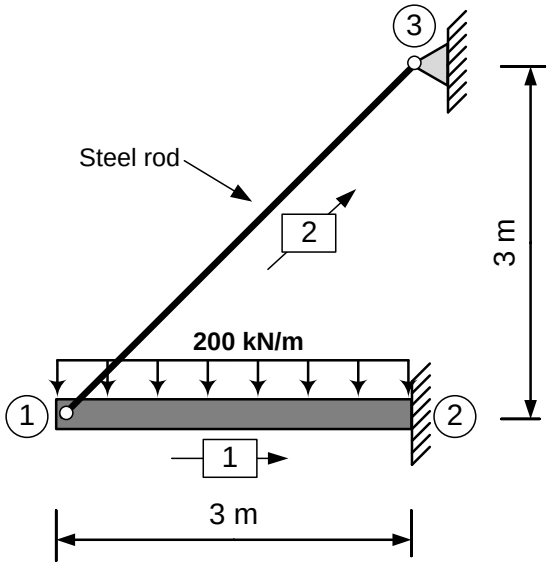

Fig. 1

2. (25 marks)

If the global end displacements for member 1 in Fig. 1 are given by $\{v\}^{(1)}$, calculate the member end forces for members 1 and 2. Are these members in equilibrium under these forces?

$$\{v\}^{(1)} = \begin{Bmatrix} 0.00152 \text{ m} \\ -0.01014 \text{ m} \\ -0.00565 \text{ rad} \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

3. (25 marks)

Determine the member end forces and support reactions for the beam shown in Fig. 3, due to the combined effect of the loading shown and a settlement of 30 mm of the right support. Note that E = 200 GPa and $I = 145\times10^6$ mm^4. Clearly indicate the numbering of degrees of freedom on your analytical model.

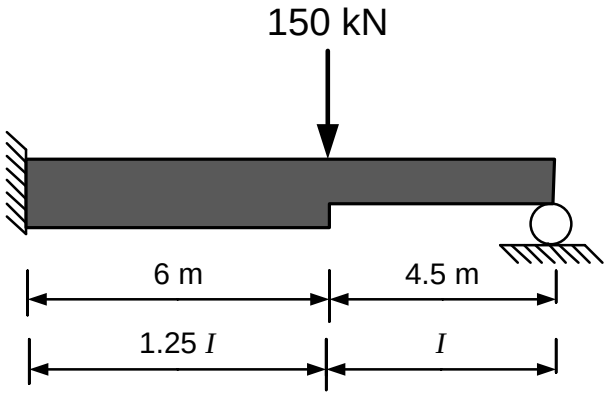

Fig. 3

4. (25 marks)

A frame consists of three uniform members, as shown in Fig. 4. Member 1 experiences an increase of temperature with 10°C at the top surface and 70°C at the bottom surface. The temperature increase varies linearly over the depth d = 600 mm of the member cross section. Members 2 and 3 remain at ambient temperature. Determine the resultant reactions at nodes 1, 2, 3 and 4 due to the temperature increase in member 1. Note that L = 2 m, E = 200 GPa, A = 5,000 mm^2, $I = 127\times10^6$ mm^4, and $\alpha = 12\times10^{-6}$/°C. Clearly indicate the numbering of degrees of freedom on your analytical model.

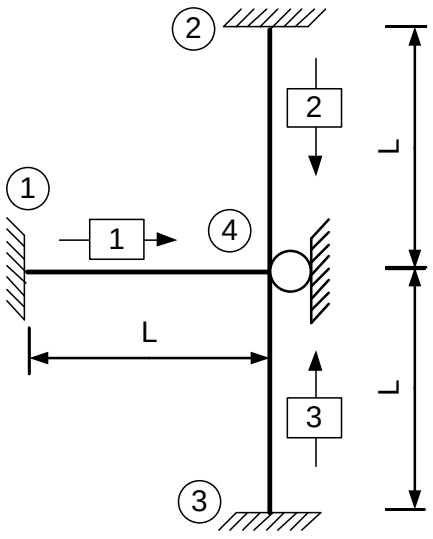

Fig. 4

The inverse of a 2×2 matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$A^{-1} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

The inverse of a 3×3 matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} a_{33}a_{22} - a_{32}a_{23} & -(a_{33}a_{12} - a_{32}a_{13}) & a_{23}a_{12} - a_{22}a_{13} \\ -(a_{33}a_{21} - a_{31}a_{23}) & a_{33}a_{11} - a_{31}a_{13} & -(a_{23}a_{11} - a_{21}a_{13}) \\ a_{32}a_{21} - a_{31}a_{22} & -(a_{32}a_{11} - a_{31}a_{12}) & a_{22}a_{11} - a_{21}a_{12} \end{bmatrix}$$

$$|A| = a_{11}(a_{33}a_{22} - a_{32}a_{23}) - a_{21}(a_{33}a_{12} - a_{32}a_{13}) + a_{31}(a_{23}a_{12} - a_{22}a_{13})$$