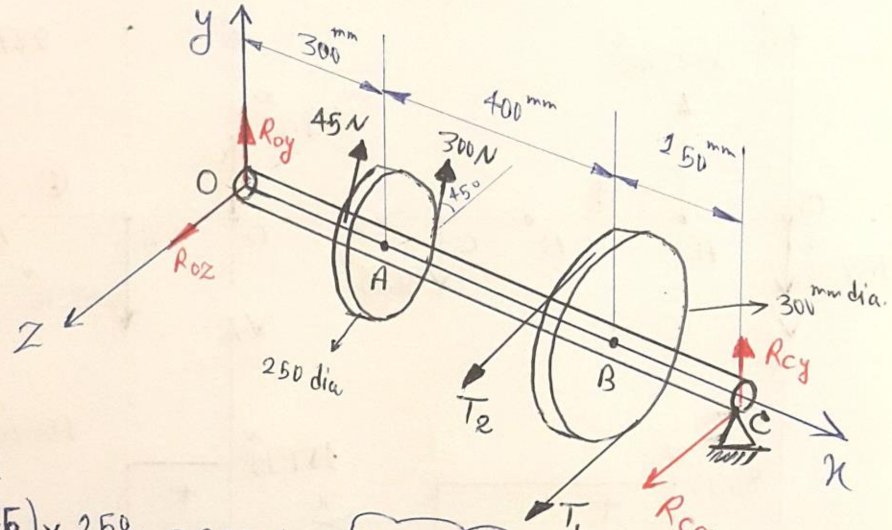
Solution _ Term Test #1 - Mech 344

Problem # 1

(a)



$T_2 = 0.15\ T_1$

$$\underbrace{T_A}_{\text{Torque}} = \underbrace{(300-45)}_{\text{Force}} \times \underbrace{\frac{250}{2}}_{\text{Arm}} = 31875\ N.mm = 31.875 N.m$$

$$T_B = \frac{(T_1 - T_2)\,d_B}{2} = \frac{(T_1 - 0.15T_1)\,300}{2} = 127.5\,T_1\ N.mm$$

Torque Equilibrium: $T_A = T_B \Rightarrow 127.5\,T_1 = 31875 \Rightarrow T_1 = 250\ N$

Thus, $T_2 = 0.15\,T_1 = 0.15 \times 250 = 37.5\ N$

$\sum M_y = 0$ at o $\Rightarrow (300+45)\cos 45 \times 300 - (250+37.5)\times 700 - R_{cz} \times 850 = 0$

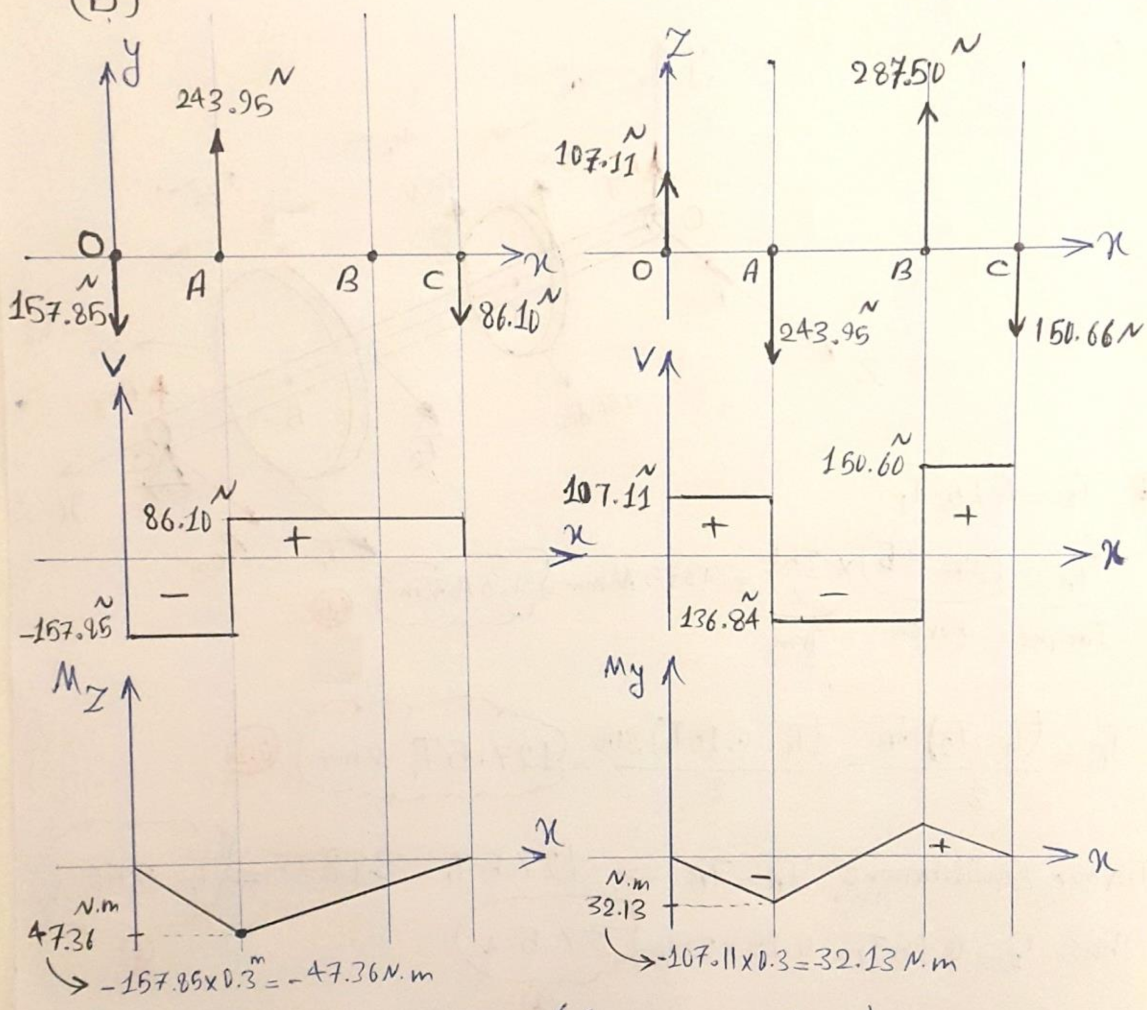
$\Rightarrow R_{cz} = -150.66\ N$

$\sum M_z = 0$ at O $\Rightarrow R_{cy} \times 850 + 345 \sin 45° \times 300 = 0 \Rightarrow R_{cy} = -86.10\ N$

$$\Sigma F_y = 0 \Rightarrow R_{oy} + 345 \sin 45^\circ - 86.10 = 0 \Rightarrow \boxed{R_{oy} = -157.85\ N}$$

$$\Sigma F_z = 0 \Rightarrow R_{oz} - 345 \cos 45^\circ - 150.66 + 287.5 = 0 \Rightarrow \boxed{R_{oz} = 107.11\ N}$$

(b)



As it can be realized Point A (location at pulley A) experiences the Maximum moment of: $M_{z_A} = -47.36$ N.m $\quad M_{y_A} = -32.13$ N.m

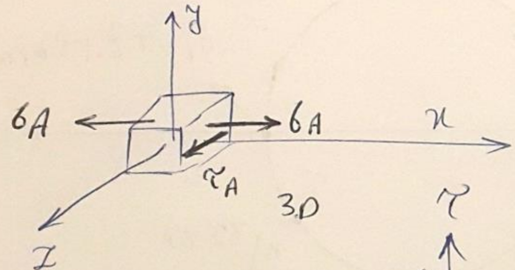
Combined moment: $M_A = \sqrt{(-47.36)^2 + (-32.13)^2} = \boxed{57.23\ N.m}$

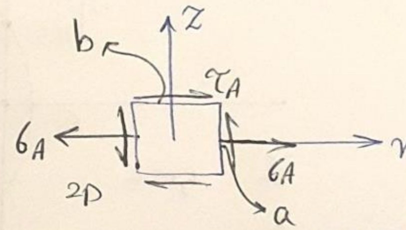
(C) point A [on the surface at top] experiences both normal stress and shear stress due to bending and torsion, respectively; thus,

$$\sigma_A = \frac{M_A c}{I} \; ; \; c = d/2 \; ; \; I = \frac{\pi d^4}{64} \; ; \; d = 20^{mm}$$

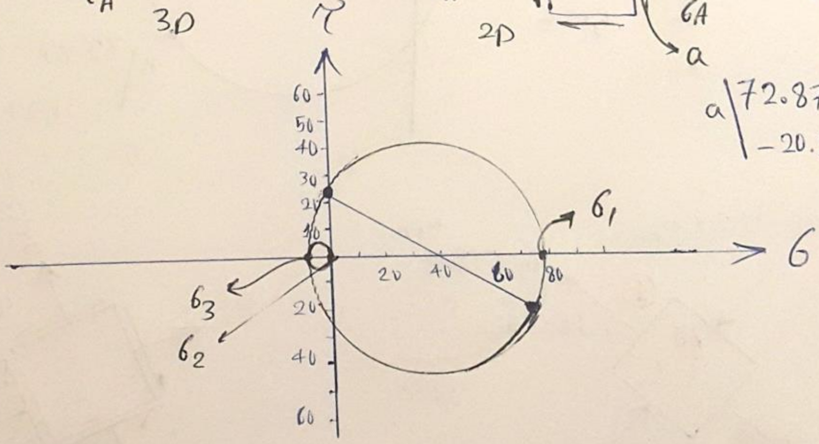
$$\Rightarrow \sigma_A = \frac{32 M_A}{\pi d^3} = \frac{32 \times 57.23}{\pi (20 \times 10^{-3})^3} = 72.87\ MPa = 72867499.15\ Pa$$

$$\tau_A = \frac{T_A c}{J} , \; J = \frac{\pi d^4}{32} \Rightarrow \tau_A = \frac{16 T_A}{\pi d^3} = \frac{16 \times 31.875}{\pi \times (20 \times 10^{-3})^3} = 20.29\ MPa$$





a | 72.87 MPa, −20.29 MPa
b | 0, +20.29 MPa



principal stresses:

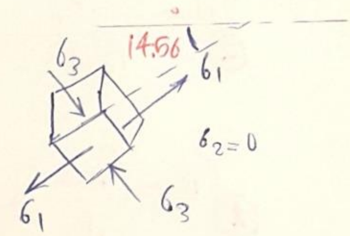
$$\sigma = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\tau_{xz}^2 + \left(\frac{\sigma_x - \sigma_z}{2}\right)^2} = \frac{\sigma_A}{2} \pm \sqrt{\tau_A^2 + \left(\frac{\sigma_A}{2}\right)^2}$$

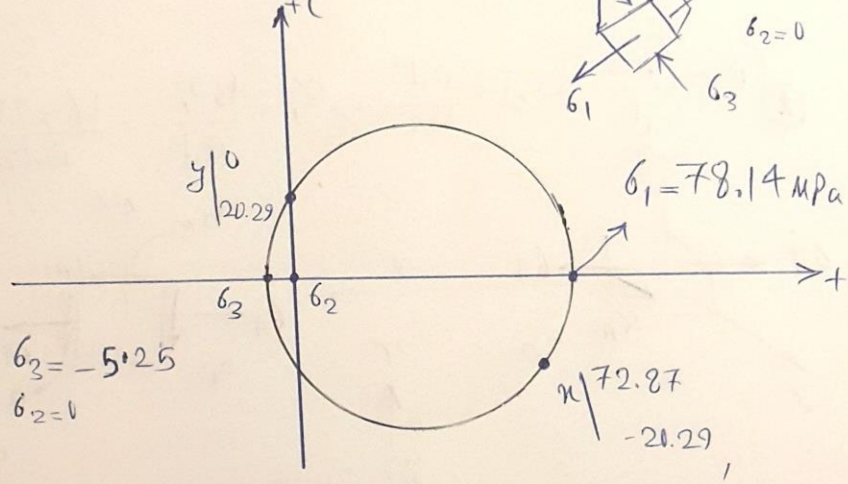
$$= \frac{72.87}{2} \pm \sqrt{(20.29)^2 + \left(\frac{72.87}{2}\right)^2} \Rightarrow \sigma_1 = 78.14\ MPa , \; \sigma_2 = 0 , \; \sigma_3 = -5.27\ MPa$$

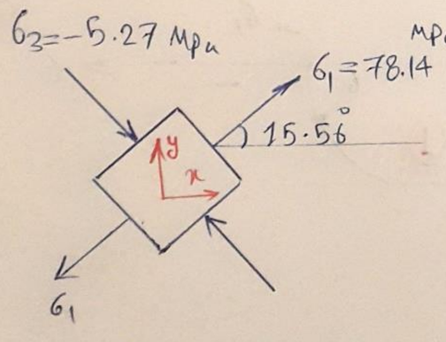
$$2\phi = \tan^{-1}\frac{2\tau_{xy}}{\sigma_x-\sigma_y} = \tan^{-1}\frac{2\tau_A}{\sigma_A} = \tan^{-1}\frac{2\times 20.29}{72.87} \Rightarrow \boxed{\phi = 14.56^\circ}$$

$$\tau_{max} = \pm\sqrt{\tau_{xy}^2+\left(\frac{\sigma_x-\sigma_y}{2}\right)^2} = \pm\sqrt{\tau_A^2+\left(\frac{\sigma_A}{2}\right)^2} = \pm\sqrt{(20.29)^2+\left(\frac{72.87}{2}\right)^2}$$

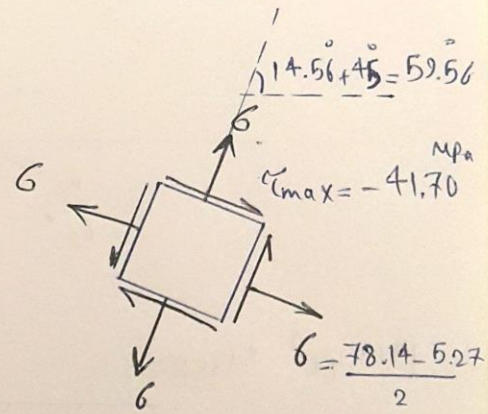
$$\Rightarrow \boxed{\tau_{max} = \pm 41.70\ \mathrm{MPa}}$$







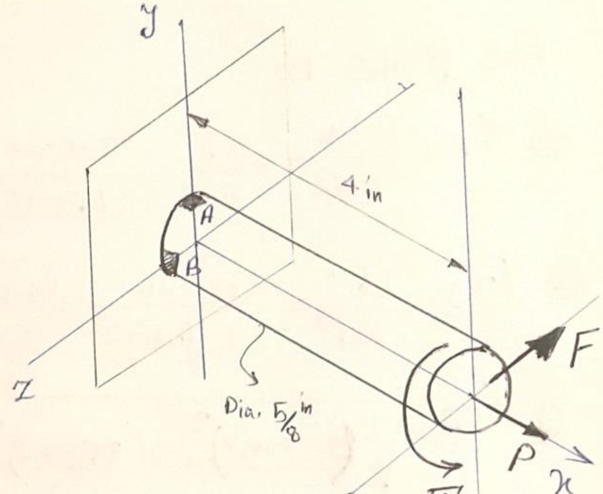
principal element



$$\sigma = \frac{78.14-5.27}{2}$$

$\sigma = 36.4$ MPa

Problem # 2



For point A

$$\sigma_x = \frac{P}{\left(\frac{\pi d^2}{4}\right)} = \frac{4 \times 900}{\pi (0.625)^2} = 2933.5 \text{ psi} = 2.9 \text{ KSi}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} - \frac{4}{3}\frac{V}{A} = \frac{16 \times 220}{\pi (0.625)^3} - \frac{4 \times 125 \times 4}{3\pi (0.625)^2} = 4046.12 \text{ psi} \simeq 4 \text{ ksi}$$

$$\sigma_{eA} = \sqrt{\sigma_x^2 + 3\tau_{xz}^2} = \sqrt{(2933.5)^2 + 3(4046.12)^2} = 7597.3 \text{ psi} = 7.6 \text{ ksi}$$

$$\tau_{max\,A} = \sqrt{\left(\frac{2933.5}{2}\right)^2 + (4046.12)^2} = 4303.8 \text{ psi} = 4.3 \text{ ksi}$$

Max. shear stress theory: $SF_{A_1} = \frac{0.5 S_y}{\tau_{max_A}} = \frac{45000 \times 0.5}{4303.8} = 5.2$

more Conservative

Max. distortion energy theory: $SF_{A_2} = \frac{S_y}{\sigma_{eA}} = \frac{45000}{7597.3} = 5.9$

For point B

$$\sigma_x = \frac{32M}{\pi d^3} + \frac{4P}{\pi d^2} = \frac{32 \times 125 \times 4}{\pi (0.625)^3} + \frac{4 \times 900}{\pi (0.625)^2} = 23794.3 \text{ psi} = 23.8 \text{ Ksi}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16 \times 220}{\pi (0.625)^3} = 4589.4 \text{ psi} = 4.6 \text{ Ksi}$$

$$\sigma_{eB} = \sqrt{(23794.3)^2 + 3(4589.4)^2} = 25087.0 \text{ psi} = 25 \text{ Ksi}$$

$$\tau_{max\,B} = \sqrt{\left(\frac{23794.3}{2}\right)^2 + (4589.4)^2} = 12751.7 \text{ psi} = 12.8 \text{ Ksi}$$

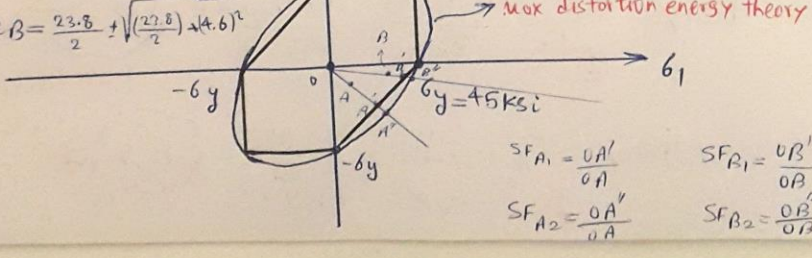
Max shear stress theory: $SF_{B_1} = \dfrac{0.5 S_y}{\tau_{max\,B}} = \dfrac{0.5 \times 45000}{12751.7} = 1.76$

Max distortion energy theory: $SF_{B_2} = \dfrac{S_y}{\sigma_{eB}} = \dfrac{45000}{25087} = 1.79$ (Almost the same)

Max shear-stress theory, generally, gives smaller (SF) than max. distortion energy theory; In other words, the max shear-stress theory is more conservative than max. distortion energy theory as shown

$$\sigma_{1,2A} = \frac{2.9}{2} \pm \sqrt{\left(\frac{2.9}{2}\right)^2 + 4^2}$$

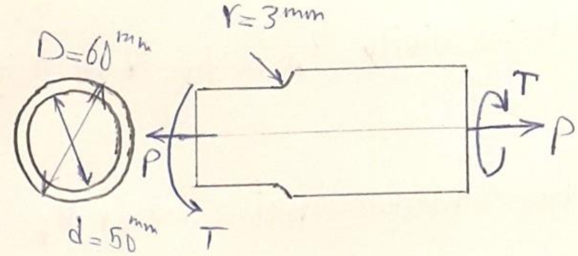
$$\sigma_{1,2B} = \frac{23.8}{2} \pm \sqrt{\left(\frac{23.8}{2}\right)^2 + (4.6)^2}$$



$SF_{A_1} = \dfrac{OA'}{OA}$ $\quad SF_{B_1} = \dfrac{OB'}{OB}$

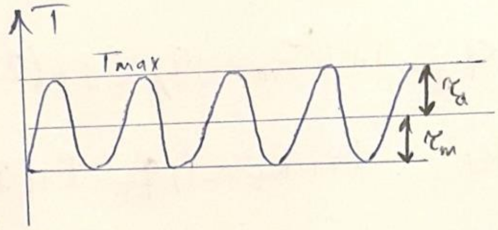
$SF_{A_2} = \dfrac{OA''}{OA}$ $\quad SF_{B_2} = \dfrac{OB''}{OB}$

Problem # 3



$S_y = 855\ MPa = 124\ Ksi$

$S_{ut} = 965\ MPa = 140\ Ksi$



$$T_a = T_m = \frac{T_{max}}{2} = \frac{2000\ kN}{2} = 1000\ kN$$

Endurance limit

$$S_n = S_n' C_\ell C_G C_s C_T C_R$$

$$S_n' = 0.5\ S_{ut} = 0.5 \times 965 = 482.5\ MPa$$

$C_l = 1.0$ $C_G = 0.9$ $C_s = 0.9$ (Fig. 8.13 - ground surface finish)

$C_R = 0.865$ for 95% reliability $C_T = 1$ room Temp.

$$\Rightarrow S_n = 482.5 \times 1 \times 0.9 \times 0.9 \times 1 \times 0.865 \simeq 339\ MPa$$

$r/d = \frac{3}{50} = 0.06$ $\frac{D}{d} = \frac{60}{50} = 1.2$

A: Axial
B: Torsion

K_t from Fig. 4.35 Theoretical stress concentration factor

$K_{t_A} = 2$ and $K_{t_T} = 1.5$

Notch Sensitivity q_A & q_T from Fig. 8.24 $\begin{cases} q_A = 0.9 \\ q_T = 0.92 \end{cases}$

Fatigue stress concentration factor K_f

$$\Rightarrow K_{f_A} = 1 + (K_{t_A} - 1) q_A = 1 + (2-1) \times 0.9 = 1.9$$

$$K_{f_T} = 1 + (K_{t_T} - 1) q_T = 1 + (1.5 - 1) \times 0.92 = 1.46$$

$$\Rightarrow \sigma_m = K_{f_A} \frac{P}{A} = K_{f_A} \frac{4P}{\pi d^2} = 1.9 \times \frac{4 \times 22500}{\pi (50)^2} = 21.77 \text{ MPa}$$

$\sigma_a = 0$ since Axial force is constant.

$$\tau_a = \tau_m = K_f \frac{TC}{J} = K_f \frac{16 T_a}{\pi d^3} = 1.46 \times \frac{16 \times 1000 \times 10^3}{\pi (50)^3} = 59.49 \text{ MPa}$$

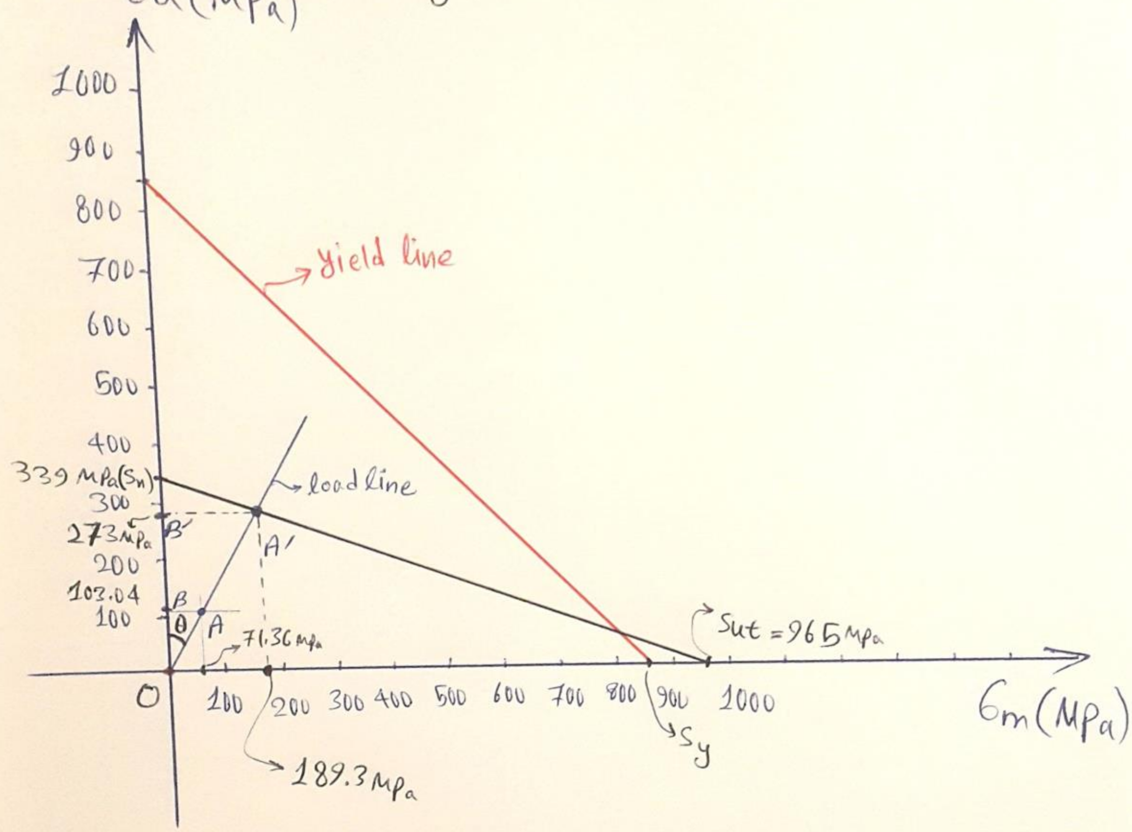
$\Rightarrow$ Equivalent stresses can be obtained as

$$\sigma_{ea} = \sqrt{\underbrace{\cancel{\sigma_a^2}}_{0} + 3\tau_a^2} = \sqrt{3}\,\tau_a = \sqrt{3} \times 59.49 \cong 103.04 \text{ MPa}$$

$$\sigma_{em} = \frac{\sigma_m}{2} + \sqrt{\tau_m^2 + \left(\frac{\sigma_m}{2}\right)^2} = \frac{21.77}{2} + \sqrt{(59.49)^2 + \left(\frac{21.77}{2}\right)^2}$$

$$\Rightarrow \sigma_{em} = 71.36 \text{ MPa}$$

$S_y = 855\,MPa$



Analytically if the load line intersects the Goodman line, thus

$$\frac{1}{SF} = \frac{\sigma_{em}}{S_{ut}} + \frac{\sigma_{ea}}{S_n} = \frac{71.36}{965} + \frac{103.04}{339} \Rightarrow \boxed{SF \cong 2.65}$$

Graphically, the safety factor can be found as:

$$SF = \frac{OA'}{OA} = \frac{OB'}{OB} = \frac{273}{103.04} \cong \boxed{2.65}$$

$Eq_{load\ line} = Eq_{Goodman} \Rightarrow A' \begin{cases} 189.3\ MPa \\ 273\ MPa \end{cases}$; $Eq_{load\ line}: \sigma_a = 1.44\,\sigma_m$; $Eq_{Goodman}: \sigma_a = -\frac{339}{965}\sigma_m + 339$