(out of 4 marks)

1 (a), (c), (d). (Thus (b) is false)

(out of 3)

2 (a) No (b) yes (c) No.

3 B

(out of 3)

4 D

(out of 3)

5 The augmented matrix is

Total = 10 marks



$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 2 & -1 & -2 & 1 \\ 3 & 1 & 2 & -1 \end{array}\right). \quad (2 \text{ marks})$$

We apply elementary row operations to solve it.

$R_2' = -2R_1 + R_2$
$R_3' = -3R_1 + R_3$
$\downarrow$ (2 marks)

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 1 & -4 & -5 \\ 0 & 4 & -1 & -10 \end{array}\right)$$

$R_3' = -4R_2 + R_3$
(2 marks)

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 1 & -4 & -5 \\ 0 & 0 & 15 & 10 \end{array}\right).$$

Thus $x_3 = \frac{10}{15} = 2/3$. (1 mark)

$x_2 = -5 + 4x_3 = -5 + 8/3 = -\frac{7}{3}$ (1 mark)

$x_1 = 3 - x_3 + x_2 = 3 - \frac{2}{3} - \frac{7}{3}$

$= 0$ (1 mark)

$\Rightarrow$ Soln. is $(x_1, x_2, x_3) = \left(0, -\frac{7}{3}, \frac{2}{3}\right)$ (1 mark)

6. (out of 5 marks)

For $1 \leq i \leq 3$, let $C_i$ denote the $i$-th column of $C := AB$. We have $C_i = AB_i$, where $B_i$ is the $i$-th column of $B$. Thus

(1 mark)

$$C_1 = AB_1 = \begin{pmatrix} 1 & 3 & -1 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$ (1 mark)

(1 mark)

$$C_2 = AB_2 = \begin{pmatrix} 1 & 3 & -1 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} -1 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 11 \\ 8 \\ 0 \end{pmatrix}.$$

Similarly $C_3 = \begin{pmatrix} -7 \\ -1 \\ 6 \end{pmatrix}$. (1 mark)

Thus $C = \begin{pmatrix} 2 & 11 & -7 \\ 0 & 8 & -1 \\ 0 & 0 & 6 \end{pmatrix}$. (1 mark)

Rmk: Obviously there are other ways of computing the product, so please use your judgement when marking

7 (total = 10 marks)

(a) (Out of 2) Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $a, b, c, d \in \mathbb{R}$.

Then $\det(A) = ad - bc$. (2 marks)

(b) By (a) and by assumption,

$$(x-5)(x-4) - 8 = -6. \quad \text{(1 mark)}$$

Thus $x^2 - 9x + 18 = 0$ and so

$$(x-6)(x-3) = 0. \quad \text{(1 mark)}$$

Hence either $x = 3$ or $x = 6$. (2 marks)

(c) A is invertible, since $\det(A) = -6 \neq 0$

(1 mark) (1 mark)

Its inverse is

$$\underbrace{\frac{1}{\det(A)} \begin{pmatrix} x-4 & -8 \\ -1 & x-5 \end{pmatrix} = -\frac{1}{6} \begin{pmatrix} x-4 & -8 \\ -1 & x-5 \end{pmatrix}}_{\text{(2 marks)}}$$

8. (out of 7)

We use cofactor expansion (although the students may use, say, row or column operations 1st to make it into a triangular matrix and get its determinant) across the 1st column. We get

$$|M| = 2\begin{vmatrix} 2 & 5 & 4 \\ 0 & -1 & 3 \\ 0 & 2 & 5 \end{vmatrix}. \quad (3 \text{ marks})$$

By a cofactor expansion across the 1st column of this matrix,

$$\begin{vmatrix} 2 & 5 & 4 \\ 0 & -1 & 3 \\ 0 & 2 & 5 \end{vmatrix} = 2\begin{vmatrix} -1 & 3 \\ 2 & 5 \end{vmatrix} = 2(-5-6)$$

$$= -22. \quad (3 \text{ marks})$$

Thus $|M| = 2(-22) = -44$. (1 mark)

Rmk: Please beware the stdts may use different methods to compute $|M|$, such as elementary row operations, etc.