MATH 1104 E

Test 1

Instructor: Aleksandr Tuxanidy

October 1, 2018

10:35–11:25 am (50 minutes)

Tutorial Section:

Last Name:

First Name:

Student Number:

1. (4 marks) For a matrix A and vectors $\mathbf{b}, \mathbf{x}$ of proper dimensions, recall we say that a linear system $A\mathbf{x} = \mathbf{b}$ is *consistent* if it has at least one solution $\mathbf{x}$. Otherwise we say it is *inconsistent*. Moreover $A\mathbf{x} = \mathbf{b}$ is called *homogeneous* if $\mathbf{b}$ is a zero vector. Which of the following are true?

(a) A homogeneous linear system is always consistent.

(b) It is possible that a linear system with coefficients in $\mathbb{R}$ has exactly three solutions.

(c) A homogeneous system of linear equations can have a unique solution.

(d) If a linear system is inconsistent, then it is not homogeneous.

2. (3 marks) For a homogeneous system of 1500 equations in 2018 unknowns, answer each of the following questions with a yes or no.

(a) Can the system be inconsistent?

(b) Can the system have infinitely many solutions?

(c) Can the system have exactly one solution?

3. (3 marks) Determine which of the following matrices A, B, C are in row-echelon form (REF).

$$A = \begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. (3 marks) Determine which of the following matrices D, E, F are in reduced

row-echelon form (RREF).

$$D = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad F = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5. (10 marks) Solve the following system of linear equations.

$$x_1 - x_2 + x_3 = 3$$

$$2x_1 - x_2 - 2x_3 = 1$$

$$3x_1 + x_2 + 2x_3 = -1.$$

6. (5 marks) Let

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 & 2 \\ 0 & 4 & -2 \\ 0 & 0 & 3 \end{bmatrix}.$$

Compute the product AB.

7. (10 marks)

(a) (2 marks) Give the general formula for the determinant of a 2×2 matrix.

(b) (4 marks) For a real number x, let A be the matrix

$$\begin{bmatrix} x-5 & 8 \\ 1 & x-4 \end{bmatrix}.$$

Suppose we are told that $\det(A) = -6$. What is x?

(c) (4 marks) If we assume that $\det(A) = -6$ for A as in (b), is then A invertible? Why? If yes, give its inverse.

8. (7 marks) Evaluate the determinant of the matrix

$$M = \begin{bmatrix} 2 & 1 & -1 & 3 \\ 0 & 2 & 5 & 4 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 2 & 5 \end{bmatrix}.$$

Is M invertible? Why?