Complete Sol. Attached

CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	209	All
Examination	**Date**	**Pages**
Final	April 22 2017	3
Instructors		**Course Examiner**
ALL		R. Raphael

Special Instructions

▷ Ruled booklets to be used.

▷ Approved calculators allowed.

MARKS

[6] **1.** (a) Find the following limits

(i) $\lim\limits_{x \to 1} \dfrac{2x^5 + 7x - 1}{x^2 + 5x + 3}$

(ii) $\lim\limits_{x \to 2} \dfrac{x^2 + x - 6}{2x^2 - 3x - 2}$

(b) Prove or disprove by giving an example: there exists a function f from the real numbers to the real numbers that is discontinuous at exactly three points.

[4] **2.** Find the derivative $f'(x)$ of the functions $f(x)$: (Do not simplify)

(a) f(x)= $4x^5 - 9x^2 + x - 22$

(b) f(x)= $\dfrac{x^{-7}}{8} + \dfrac{1}{\sqrt[2]{x}}$

[10] **3.** Find $\frac{dy}{dx}$ (do not simplify):

(a) $y = \frac{7 - x^3}{e^{3x}}$

(b) $y = ln(3x^4 + 7)$

(c) $y = (4x - 5)^3(3x^2 + 4)$

(d) $y = (5 + x^3 lnx)^3$

[8] **4.** Let $f(x) = 4x^4 - x^2 - 7$

(a) Find the slope of the tangent line to the curve when $x = 1$

(b) Find the equation of the tangent line to the curve when $x = 1$

[13] **5.** Let $f(x) = x^4 - 2x^3$

Find

(a) the critical and inflection points of $f(x)$

(b) the intervals where $f(x)$ is increasing and where it is decreasing

(c) the intervals on which $f(x)$ is concave up and on which it is concave down

(d) use the above to sketch the graph

[9] **6.** If the cost of a seminar is \$400 per person 1000 people attend. For every \$5 dollar reduction in cost 20 more people will attend the seminar.
How much should be charged for the seminar to maximize revenue?

[6] **7.** Find the absolute extrema of the function $f(x) = x^3 - 12x$ on the interval $[-5, 5]$.

[4] **8.** A country has Lorenz curve $f(x) = x$. Find its Gini index. What can you conclude from the Gini index?

[10] **9.** Find the equation(s) of the tangent line(s)to the graph of $y - xy^2 + x^2 + 1 = 0$ at the point(s) with $x = 1$.

[10] **10.** Compute these antiderivatives:

(a) $\displaystyle\int (5x^7 - 4x^3 - 9)\,dx$

(b) $\displaystyle\int \frac{e^{-2x}}{4 + e^{-2x}}\,dx$

(c) $\displaystyle\int \frac{x^2}{\sqrt{x-5}}\,dx$

[10] **11.** Evaluate the integrals:

(a) $\displaystyle\int_0^1 (x^4 - 5)\,dx$

(b) $\displaystyle\int_6^{10} \frac{2}{x-4}\,dx$

(c) $\displaystyle\int_4^7 \sqrt{x-2}\,dx$

[10] **12.** Find the area bounded by the graphs of $f(x) = 5 - x^2$ and $g(x) = 2 - 2x$.

Math 209 April 2017 Final Exam

1(a) (i) $\lim_{x\to 1} \frac{2x^5+7x-1}{x^2+5x+3} = \frac{2(1)^5+7(1)-1}{(1)^2+5(1)+3} = \frac{8}{9}$

(ii) $\lim_{x\to 2} \frac{x^2+x-6}{2x^2-3x-2} = \frac{2^2+2-6}{2(2)^2-3(2)-2} = \frac{0}{0}$

$\lim_{x\to 2} \frac{(x+3)\cancel{(x-2)}}{(2x+1)\cancel{(x-2)}} = \frac{2+3}{2[2]+1} = \frac{5}{5} = 1$

b) Need function that is undefined at 3 values of x

$f(x) = \frac{1}{(x-1)(x-2)(x-3)}$ is discont. (DNE) when $x=1$, $x=2$, $x=3$

2. a) $f'(x) = 20x^4 - 18x + 1$

b) $f(x) = \frac{1}{8}x^{-7} + x^{-\frac{1}{2}}$

$\Rightarrow f'(x) = \frac{1}{8}(-7)x^{-8} + (-\frac{1}{2})x^{-\frac{3}{2}}$

3. a) $y' = \frac{e^{3x}\frac{d}{dx}(7-x^3) - (7-x^3)\frac{d}{dx}e^{3x}}{(e^{3x})^2}$

$y' = \frac{e^{3x}(-3x^2) - (7-x^3)e^{3x}(3)}{(e^{3x})^2}$

b) $y' = \frac{1}{3x^4+7}\frac{d}{dx}(3x^4+7)$

$y' = \frac{1}{3x^4+7}(12x^3)$

c) $y' = (4x-5)^3\frac{d}{dx}(3x^2+4) + (3x^2+4)\frac{d}{dx}(4x-5)^3$

$= (4x-5)^3(6x) + (3x^2+4)\,3(4x-5)^2\frac{d}{dx}(4x-5)$

$= (4x-5)^3(6x) + (3x^2+4)\,3(4x-5)^2(4)$

d) $y' = 3(5+x^3\ln x)^2\frac{d}{dx}(5+x^3\ln x)$

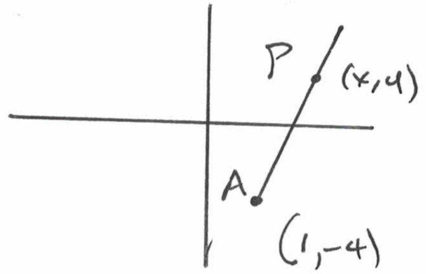
$y' = 3(5+x^3\ln x)^2(x^3\frac{d}{dx}\ln x + \ln x\frac{d}{dx}x^3)$

$y' = 3(5+x^3\ln x)^2(x^3\cdot\frac{1}{x} + \ln x\cdot 3x^2)$

4. a) $f'(x) = 4(4)x^3 - 2x$

$f'(1) = 16(1)^3 - 2(1) = 14$

b)



Step 1 when $x=1$, $f(1) = 4(1)^4 - (1)^2 - 7 =$

$\Rightarrow$ pt is $(1,-4)$

Step 2 Let $P(x,y)$ be any pt on line

$m_{AP} = \frac{y-(-4)}{x-1}$

$14 = \frac{y+4}{x-1} \Rightarrow y = 14x-18$

5. Step 1 $f'(x) = 4x^3 - 6x^2$

$f'(x) = 0$	$f'(x) = \frac{1}{0}$
$2x^2(2x-3) = 0$	No x
$2x^2 = 0$ / $2x - 3 = 0$	
$x = 0$ $x = 3/2$	

$f''(x) = 12x^2 - 12x$

$f''(x) = 0$	$f''(x) = \frac{1}{0}$
$12x(x-1)$	No x
$12x = 0$ / $x - 1 = 0$	
$x = 0$ $x = 1$	

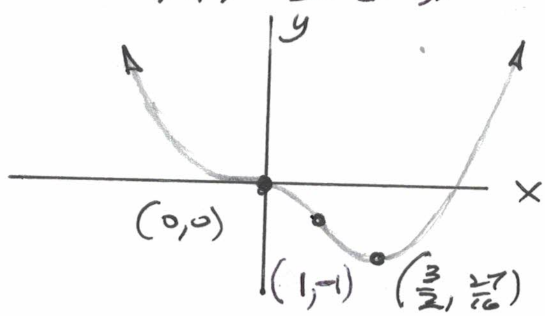
Step 2

x	$-\infty < x < 0$	$x = 0$	$0 < x < 1$	$x = 1$	$1 < x < 3/2$	$3/2$	$\frac{3}{2} < x <$
$f'(x)$	$f'(-1) = -$	0	$f'(1/2) = -$	$-$	$-$	0	$f'(2) =$
$f''(x)$	$f''(-1) = +$	0	$f''(1/2) = -$	0	$+$	$+$	$f''(2) = +$
$f(x)$	Dec, CU.	Infl. pt $(0,0)$	Dec CD.	Dec Infl. pt $(1,-1)$	Dec CU	CU Loc Min $(3/2, \frac{27}{16})$	Inc CU

Step 3 Signs: use

$f(x) = \cancel{2}x^2(x-3)$

$f''(x) = \cancel{4x(3x-4)}$
$= 12x(x-1)$



6. Step 1 $R = x * p$

Rev. (No. persons)(price/person)

Step 2 too many variable

$R = (1000 + 20r)(400 - 5r)$

No. person	Cost/person
1000	400\$
$1000 + 1(20)$	$400 - 1(5)$
$1000 + 2(20)$	$400 - 2(5)$
$1000 + 3(20)$	$400 - 3(5)$
$\vdots$	
$1000 + r(20)$	$400 - r(5)$

where r is No. of 5\$ dec.

Step 3 (1) $R' = (1000 + 20r)(-5)$
$+ (400 - 5r)(20)$

$R' = -5000 - 100r + 8000 - 100r$

$R' = -200r + 3000$

(2)

$R' = 0$	$R' = \frac{1}{0}$
$-200r + 3000 = 0$	No r
$r = \frac{-3000}{-200}$	
$r = 15$	

(3) test $R'' = - \Rightarrow R$ is CD for all r
$\Rightarrow r = 15$ produces Local MAX

Step 4 Cost/person $400 - 5(15) = 325\$$

Note: No. person $1000 + 20(15) = 1300$ persons

7. Step1 $f'(x) = 3x^2 - 12$

$f'(x) = 0$	$f'(x) = \frac{c}{0}$
Step2 $3x^2 - 12 = 0$	No x
$3x^2 = 12$	
$x^2 = 4$	
$x = \pm 2$	

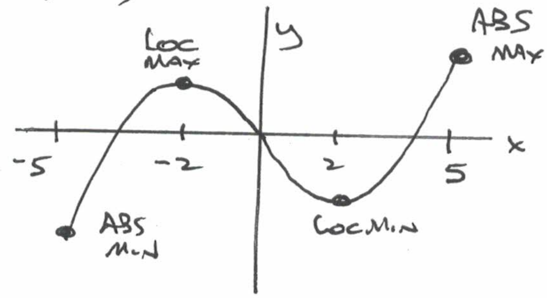
Step3 test

$f(2) = -16$

$f(-2) = 16$

$f(-5) = -65$ MIN

$f(5) = 65$ MAX



8 Gini Index $= 2\int_0^1 [x - f(x)]\,dx$

$= 2\int_0^1 [x - x]\,dx$

$= 0$

This is perfect Income distribution (Equality)

ie 10% of pop earn 10% Inc

20% .. 20% "

⋮

x% .. x% ..

9. $\frac{d}{dx}y - \frac{d}{dx}xy^2 + \frac{d}{dx}x^2 + \frac{d}{dx}1 = \frac{d}{dx}0$

$\frac{dy}{dx} - \left(x\frac{d}{dx}y^2 + y^2\frac{d}{dx}x\right) + 2x\frac{d}{dx}x + 0 = 0$

$\frac{dy}{dx} - x(2y)\frac{d}{dx}y - y^2 + 2x = 0$

$\frac{dy}{dx}[1 - 2xy] = +y^2 - 2x$

$\frac{dy}{dx} = \frac{y^2 - 2x}{1 - 2xy}$

when $x = 1$

$\Rightarrow y - 1y^2 + 1^2 + 1 = 0$

$-y^2 + y + 2 = 0$

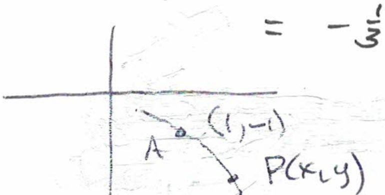
$y^2 - y - 2 = 0$

$(y+1)(y-2)$

$y + 1 = 0$	$y - 2 = 0$
$y = -1$	$y = 2$
pt $(1,-1)$	pt $(1,2)$

$\left.\frac{dy}{dx}\right|_{x=1,\ y=-1} = \frac{(-1)^2 - 2(1)}{1 - 2(1)(-1)}$

$= -\frac{1}{3}$



$(1,-1)$

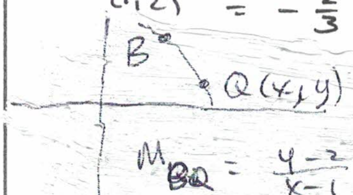
$m_{AP} = \frac{y+1}{x-1}$

$-\frac{1}{3} = \frac{y+1}{x-1}$

$3(y+1) = -(x-1) \Rightarrow y = -\frac{1}{3}x - \frac{2}{3}$

$\left.\frac{dy}{dx}\right|_{x=1,\ y=2} = \frac{(2)^2 - 2(1)}{1 - 2(1)(2)}$

$(1,2)$ $= -\frac{2}{3}$



$m_{BQ} = \frac{y-2}{x-1}$

$-\frac{2}{3} = \frac{y-2}{x-1}$

$3(y-2) = -2(x-1)$

$y = -\frac{2}{3}x + \frac{8}{3}$

10. a) $5\int x^7 dx - 4\int x^3 dx - 9\int dx$

$\frac{5x^8}{8} - \frac{4x^4}{4} - 9x + c$

b) $\int \frac{e^{-2x}}{4+e^{-2x}} dx$

Aim AT $\int \frac{1}{x} dx$ #2

Le $u = 4 + e^{-2x}$

$\frac{du}{dx} = e^{-2x}(-2)$

$du = e^{-2x}(-2)dx$

$-\frac{1}{2} du = e^{-2x} dx$

$= \int \frac{-\frac{1}{2} du}{u}$

$= -\frac{1}{2}\int \frac{1}{u} du$

$= -\frac{1}{2} \ln|u| + c$

$= -\frac{1}{2} \ln|4 + e^{-2x}| + c$

c) $\int \frac{x^2}{\sqrt{x-5}} dx$

Get an VAR. in DEN then we can use Div tech.

Let $u = x-5$

$du = dx$

Not $u+5 = x$

$(u+5)^2 = x^2$

$\int \frac{(u+5)^2}{\sqrt{u}} du$

$\int \frac{u^2+10u+25}{\sqrt{u}} du$

$\int (u^{3/2} + 10u^{\frac{1}{2}} + 25u^{-\frac{1}{2}}) du$

$\frac{2}{5}u^{5/2} + 10\left(\frac{2}{3}\right)u^{3/2} + 25(2)u^{\frac{1}{2}} + c$

$\frac{2}{5}(x-5)^{5/2} + \frac{20}{3}(x-5)^{3/2} + 50(x-5)^{\frac{1}{2}} + c$

11. a) $\int x^4 dx - 5\int dx$

$\left[\frac{x^5}{5} - 5x\right]\Big|_0^1 = \left[\frac{1^5}{5} - 5(1)\right] - \left[\frac{0^5}{5} - 5(0)\right] = -\frac{24}{5}$

b) $\int \frac{2}{x-4} dx$

Aim at $\int \frac{1}{x} dx$ #2

Let $u = x-4$

$du = dx$

$= 2\int \frac{du}{u}$

$= 2\ln|u|$

$= \left[2\ln|x-4|\right]_6^{10} =$

$= \left[2\ln|10-4|\right] - \left[2\ln|6-4|\right]$

$= 2\ln 6 - 2\ln 2$

$= 2(\ln 6 - \ln 2)$

$= 2\ln\frac{6}{2}$

11.c) $\int_4^7 \sqrt{x-2}\,dx$

Get one var under Root
let $u = x-2$
$du = dx$

Note $u+2 = x$

$\int \sqrt{u}\,du$

$\int u^{\frac{1}{2}}\,du$

$\frac{2}{3}u^{\frac{3}{2}}$

$\frac{2}{3}(x-2)^{\frac{3}{2}}\Big|_4^7 = \frac{2}{3}(7-2)^{3/2} - \frac{2}{3}(4-2)^{3/2}$

12. Step 1 Int. pts

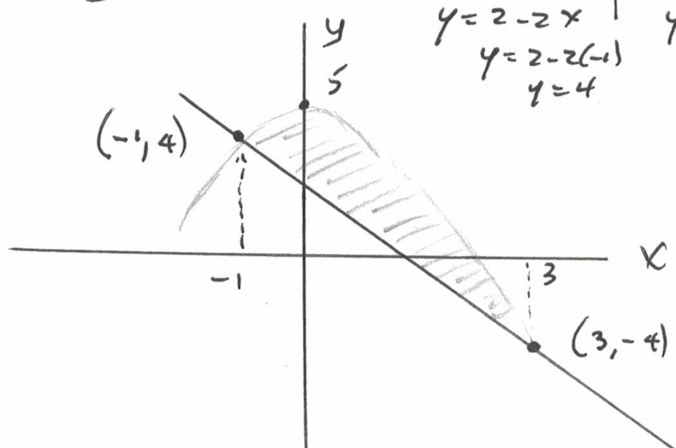
$2-2x = 5-x^2$

$x^2-2x-3=0$

$(x+1)(x-3)=0$

$x=-1$	$x=3$
$y=2-2x$	$y=2-2x$
$y=2-2(-1)$	$y=2-2(3)$
$y=4$	$y=-4$

Step 2 Rough Sketch



Extrapt on parabola
if $x=0$
$y=5$

$$\text{Area} = \int_{-1}^{3} \left|(5-x^2)-(2-2x)\right|\,dx$$

$$= \int_{-1}^{3} \left|-x^2+2x+3\right|\,dx$$

$$= \left|-\frac{x^3}{3}+\frac{2x^2}{2}+3x\right|\Bigg|_{-1}^{3}$$

$$= \left|\left(-\frac{3^3}{3}+3^2+3(3)\right)-\left(-\frac{[-1]^3}{3}+[-1]^2+3[-1]\right)\right|$$

$$= \left|\cancel{-9}+\cancel{9}+9-\frac{1}{3}-1+3\right|$$

$$= 11-\frac{1}{3}$$

$$= \frac{33-1}{3}$$

$$= \frac{32}{3}$$