Tutorial #7 - Questions

Question 1:

Determine the absolute optimum values of the function: $f(x) = x^3 - 12x + 3$ on $[-3,4]$

Solution:

First we derive the function:

$f'(x) = 3x^2 - 12$

We find critical points. Notice that there are no division by 0 in the derivative, this means we only need to find when the derivative is 0.

$0 = 3x^2 - 12$
$12 = 3x^2$
$4 = x^2$
$\pm 2 = x$

Lastly we test all critical points and endpoints to see which is the largest and which is the smallest:

$f(-3) = -27 + 36 + 3 = 12$
$f(-2) = -8 + 24 + 3 = 19$
$f(2) = 8 - 24 + 3 = -13$
$f(4) = 64 - 48 + 3 = 19$

This means the absolute max is 19, and the absolute min is -13.

Question 2:

Determine the intervals of increasing and decreasing: $f(x) = x\ln(x) - 3x$

Solution:

First we derive to find the critical points:

$f'(x) = \ln(x) + \frac{1}{x}x - 3 = \ln(x) - 2$

$0 = \ln(x) - 2$
$2 = \ln(x)$
$x = e^2$

We can see that we don't have any undefined slope that is defined in the domain of the original function (note that the domain of the original function is $D = \{x \in R | x > 0\}$)

We construct an interval table to get:

0 $\qquad$ e^2 $\qquad$ ∞

	1	e^3
$\ln(x) - 2$	$-$	$+$
Total Sign	$-$	$+$

This means that our function is decreasing when $0 < x < e^2$ and is increasing when $x > e^2$

Question 3:

Find all x-values of the critical points and classify them as local max, local min, or neither:

$$f(x) = x^{\frac{1}{3}}e^{3x^2+3x}$$

Solution:

First we derive to find the critical points:

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}e^{3x^2+3x} + x^{\frac{1}{3}}e^{3x^2+3x}(6x+3)$$

$$f'(x) = x^{-\frac{2}{3}}e^{3x^2+3x}\left(\frac{1}{3} + x(6x+3)\right)$$

$$f'(x) = x^{-\frac{2}{3}}e^{3x^2+3x}\left(\frac{1}{3} + 6x^2 + 3x\right)$$

$$f'(x) = \frac{x^{-\frac{2}{3}}e^{3x^2+3x}}{3}(18x^2 + 9x + 1)$$

Here we see that the derivative is undefined when x = 0 due to $x^{-\frac{2}{3}}$, e^{2x^2+5x} is never 0 or undefined, and

$$(18x^2 + 9x + 1) = (6x+1)(3x+1)$$

Which gives $x = -\frac{1}{3}$ or $x = -\frac{1}{6}$

We construct an interval table to get:

$-\infty$ | $-\frac{1}{3}$ | $-\frac{1}{6}$ | 0 | ∞

	-1	$-\frac{1}{4}$	$-\frac{1}{8}$	1
$x^{-\frac{2}{3}}$	$+$	$+$	$+$	$+$
e^{3x^2+3x}	$+$	$+$	$+$	$+$
$(6x+1)$	$-$	$-$	$+$	$+$
$(3x+1)$	$-$	$+$	$+$	$+$
Total Sign	$+$	$-$	$+$	$+$

This means that our critical points are when $x = -\frac{1}{3}, -\frac{1}{6}$ and 0.

$x = -\frac{1}{3}$ is a local max

$x = -\frac{1}{6}$ is a local min

$x = 0$ is not a local extremum

Question 4:
Give an example of any function and interval that does not meet the criteria of the Extreme Value Thoerem, yet has both an absolute max and an absolute minimum.

Solution:

Many answers exist here:

One could simply change the interval to an open interval, but the extreme points are not the endpoints:

Ex: $y = Sin(x)$ on $(0,2\pi)$

Or you could come up with a discontinuous function that has no vertical asymptotes in the interval and absolute extreme points in the interval:

Ex: $y = x + \frac{x}{x}$ on $[-2,2]$ Will be the line $y = x + 1$ with a hole at x=0, but will still reach its absolute max and min at the endpoints.

Question 5:
Determine the inflection point(s) (if any) of the following function:

$$g(x) = \frac{1}{12}x^4 + \frac{2}{3}x^3 + 2x^2 - 5x - 3$$

Solution:

When we derive we get:

$$g'(x) = \frac{1}{3}x^3 + 2x^2 + 4x - 5$$
$$g''(x) = x^2 + 4x + 4$$

Here we see that there are no slopes that are undefined, so we can factor to solve for the zeros:

$$0 = x^2 + 4x + 4$$
$$0 = (x + 2)(x + 2)$$
$$x = -2$$

We could create a table to test, but you can see that our second derivative is $(x + 2)^2$ which is always positive or zero. This means this function has no inflection points.

Question 6:

Show that there is a critical point at x=1 in the following function. Find the second derivative at x=1. What does the second derivative tell you about the critical point?

$$f(x) = x^4 - 4x^3 + 6x^2 - 4x + 1$$

Solution:

$$f'(x) = 4x^3 - 12x^2 + 12x - 4$$
$$f'(1) = 4 - 12 + 12 - 4 = 0$$

Since the first derivative is 0, this means that it is a critical point.

Finding the second derivative gives:

$$f''(x) = 12x^2 - 24x + 12$$
$$f''(1) = 12 - 24 + 12 = 0$$

This tells us nothing about the critical point, we must use another test to find out that this is indeed a minimum (and not an inflection).