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COMP 232 (Fall 2018)

# Assignment 1

1) a) $((p \vee r) \wedge (q \vee r) \leftrightarrow ((p \wedge q) \vee r)$

| p | q | r | p∨r | ∧ | q∨r | p∧q | ∨ | R | ((p∨r)∧(q∨r))↔((p∧q)∨r) |
|---|---|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T | | T |
| T | T | F | T | T | T | T | T | | T |
| T | F | T | T | T | T | F | T | | T |
| T | F | F | T | F | F | F | F | | T |
| F | T | T | T | T | T | F | T | | T |
| F | T | F | F | F | T | F | F | | T |
| F | F | T | T | T | T | F | T | | T |
| F | F | F | F | F | F | F | F | | T |

⇒ Tautology.

b) $(p \oplus q) \wedge (p \oplus \neg q)$

| p | q | p⊕q | ¬q | p⊕¬q | p⊕q ∧ p⊕¬q |
|---|---|---|---|---|---|
| T | T | F | F | T | F |
| T | F | T | T | F | F |
| F | T | T | F | F | F |
| F | F | F | T | T | F |

⇒ Contradiction.

☆ c) $(p \rightarrow (q \rightarrow r)) \leftrightarrow (p \rightarrow (q \wedge r))$

| $p$ | $q$ | $r$ | $p$ | $\rightarrow$ | $q \rightarrow r$ | $p$ | $\rightarrow$ | $q \wedge r$ | $(p \rightarrow (q \rightarrow r)) \leftrightarrow (p \rightarrow (q \wedge r$ |
|---|---|---|---|---|---|---|---|---|---|
| T | T | T | | T | T | | T | T | T |
| T | T | F | | F | F | | F | F | T |
| T | F | T | | T | T | | F | F | F |
| T | F | F | | T | T | | F | F | F |
| F | T | T | | T | T | | T | T | T |
| F | T | F | | T | F | | T | F | T |
| F | F | T | | T | T | | T | F | T |
| F | F | F | | T | T | | T | F | T |

$\rightarrow$ Contingency.

d) $(p \wedge (\neg q \rightarrow \neg p)) \rightarrow q$

| $p$ | $q$ | $\neg p$ | $\neg q$ | $p \wedge$ | $(\neg q \rightarrow \neg p)$ | $(p \wedge (\neg p \rightarrow \neg p)) \rightarrow q$ |
|---|---|---|---|---|---|---|
| T | T | F | F | | T | T |
| T | F | F | T | | T | T |
| F | T | T | F | | T | T |
| F | F | T | T | | T | T |

$\Rightarrow$ Tautology.

② a) $(p \to r) \wedge (q \to r) \equiv (p \wedge q) \to r$

| $p$ | $q$ | $r$ | $(p \to r)$ | $\wedge$ | $q \to r$ | $p \wedge q$ | $\to r$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | F | F | F | T | F |
| T | F | T | T | T | T | F | T |
| T | F | F | F | F | T | F | T |
| F | T | T | T | T | T | F | T |
| F | T | F | T | F | F | F | T |
| F | F | T | T | T | T | F | T |
| F | F | F | T | T | T | F | T |

$\to$ Bad line b/c LHS $\not\equiv$ RHS
$\therefore$ invalid where $p = T$ & $r = F$.

b) $(p \to q) \vee (p \to r) \equiv (p \vee q) \to r$

| $p$ | $q$ | $r$ | $(p \to q)$ | $\vee$ | $p \to r$ | $p \vee q$ | $\to r$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | T | F | T | F |
| T | F | T | F | T | T | T | T |
| T | F | F | F | F | F | T | F |
| F | T | T | T | T | T | T | T |
| F | T | F | T | T | T | T | F |
| F | F | T | T | T | T | F | T |
| F | F | F | T | T | T | F | T |

$\to$ Bad line b/c LHS $\not\equiv$ RHS

c) $(((p \vee q) \wedge (p \to r) \wedge (q \to r)) \to r) \equiv T$

LHS: $\neg(((p \vee q) \wedge (p \to r) \wedge (q \to r)) \to r)$

$\equiv \neg((p \vee q) \wedge (p \to r) \wedge (q \to r)) \vee r$ — Conditional

$\equiv \neg(p \vee q) \vee \neg(p \to r) \vee \neg(q \to r) \vee r$ — De Morgan

$\equiv \neg(p \vee q) \vee \neg(p \vee r) \vee \neg(\neg q \vee r) \vee r$ — Conditional 2x

$\equiv (\neg p \vee \neg q) \vee (\neg(\neg p) \wedge \neg r) \vee (\neg(\neg p) \wedge \neg r) \vee r$ — De Morgan 3x

$\equiv (\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee (q \vee \neg r) \vee r$ — Double Negation 2

$\equiv (\neg p \wedge \neg q) \vee ((p \wedge \neg r) \vee (q \wedge \neg r)) \vee r$ — Associative

$\equiv (\neg p \wedge \neg q) \vee (p \wedge q) \vee \neg R) \vee r$ — Distributive

$\equiv (\neg p \wedge \neg q) \vee (p \wedge q) \vee (\neg r \vee r)$ — Associative

$\equiv (\neg p \wedge \neg q) \vee (p \wedge q) \vee T$ — Negation Law

$\equiv T$ — Domination

LHS=RHS

↳ valid.

d) $(((p \to q) \wedge (q \to r)) \to (p \to r)) \equiv T$

$\Rightarrow$ Prove that $\neg(((p \to q) \wedge (q \to r)) \equiv F$ since $\neg A \equiv F$ is equivalent to $A \equiv T$.

LHS: $\neg(((p \to q) \wedge (q \to r) \to (p \to r))$

| | |
|---|---|
| $\equiv \neg(\neg((p \to q) \wedge (q \to r)) \vee (p \to r))$ | Conditional |
| $\equiv \neg\neg((p \to q) \wedge (q \to r)) \wedge \neg(p \to r)$ | DeMorgan |
| $\equiv ((p \to q) \wedge (q \to r)) \wedge \neg(p \to r)$ | Double Negation. |
| $\equiv ((\neg p \vee q) \wedge (\neg q \vee r)) \wedge \neg(p \vee r)$ | Conditional 3x |
| $\equiv ((\neg p \vee q) \wedge (\neg q \vee r)) \wedge (\neg(\neg p) \wedge \neg r)$ | Demorgan |
| $\equiv ((\neg p \vee q) \wedge (\neg q \vee r)) \wedge (p \wedge \neg r)$ | Double Negation |
| $\equiv (\neg p \vee q) \wedge ((\neg q \vee r) \wedge p) \wedge \neg r$ | Associative. |
| $\equiv (\neg p \vee q) \wedge (p \wedge (\neg q \vee r)) \wedge \neg r$ | Commutative |
| $\equiv ((\neg p \vee q) \wedge p) \wedge ((\neg q \vee r) \wedge \neg r)$ | Associative 2x |
| $\equiv ((\neg p \wedge p) \vee (q \wedge p)) \wedge ((\neg q \wedge \neg r) \vee (r \wedge \neg r))$ | Distributive 2x |
| $\equiv (F \vee (q \wedge p)) \wedge ((\neg q \wedge \neg r) \vee F)$ | Negation 2x |
| $\equiv (q \wedge p) \wedge (\neg q \wedge \neg r)$ | Identity 2x |
| $\equiv (p \wedge q) \wedge (\neg q \wedge \neg r)$ | Associative |
| $\equiv p \wedge (q \wedge \neg q) \wedge \neg r$ | Commutative. |
| $\equiv p \wedge F \wedge \neg r$ | Negation |
| $\equiv (p \wedge F) \wedge \neg r$ | Associative |
| $\equiv (F \wedge \neg R)$ | Domination |
| $\equiv F$ | Domination |

$\therefore$ LHS $\equiv$ RHS. Since $\neg(((p \to q) \wedge (q \to r)) \to (p \to r) \equiv F$,
then $(((p \to q) \wedge (p \to r) \to (p \to r) \equiv T$

$\therefore$ valid.

Necessary: If n is divisable by 6, then condition
Sufficient: If Condition, then n is divisible by 6.

③ a) Necessary, if n is divisible by 6, then n is divisible by 3.
b) sufficient, if n is divisible by 9, then n is divisible by 6.
c) sufficient; if n is divisible by 12, then n is divisible by 6
d) sufficient, if $n = 24$, then n is divisible by 6.
e) Necessary, if n is divisible by 6, then $n^2$ is divisible by 3.
f) If n is even & divisible by 3, then n is divisible by 6.
↳ Sufficient.

④ L: The file system is locked
Q: The new message will be queued
N: The system is functioning normally
B: The new message will be sent to message buffer.

$2^4 = 16$

$\neg L \to Q$
$\neg L \leftrightarrow N$
$\neg Q \leftrightarrow B$
$\neg L \to B$
$\neg B$

| L | Q | N | B | ¬L | ¬Q | ¬B | ¬L → Q | ¬L ↔ N | ¬Q → B | ¬L → B |
|---|---|---|---|---|---|---|---|---|---|---|
| T | T | T | T | F | F | F | T | F | T | T |
| T | T | T | F | F | F | T | T | F | T | T |
| T | T | F | T | F | F | F | T | T | T | T |
| T | T | F | F | F | F | T | T | T | T | T |
| T | F | T | T | F | T | F | T | F | | |
| T | F | T | F | F | T | T | T | F | | |
| T | F | F | T | F | T | F | T | T | | |
| T | F | F | F | F | T | T | T | T | | |
| F | T | T | T | T | F | F | T | T | | |
| F | T | T | F | T | F | T | T | T | | |
| F | T | F | T | T | F | F | T | F | | |
| F | T | F | F | T | F | T | T | F | | |
| F | F | T | T | T | T | F | F | T | | |
| F | F | T | F | T | T | T | F | T | | |
| F | F | F | T | T | T | F | F | F | | |
| F | F | F | F | T | T | T | F | F | | |

∴ L = T
Q = T
N = F
B = F

(5) a) $P(1,3) \vee P(2,3) \vee P(3,3)$

b) $\neg P(2,1) \wedge \neg P(2,2) \wedge P(2,3)$

c) $(P(1,1) \vee P(1,2) \vee P(1,3)) \wedge (P(2,1) \vee P(2,2) \vee P(2,3)) \wedge$
$(P(3,1) \vee P(3,2) \vee P(3,3))$

d) $(\neg p(1,1) \wedge \neg p(2,1) \wedge \neg p(3,1)) \vee (\neg p(2,1) \wedge \neg p(2,2) \wedge \neg p(2,3)) \vee$
$(\neg p(3,1) \wedge \neg p(3,2) \wedge \neg p(3,3))$

(6) a) $P(\text{Carlos, Bulgaria})$

b) $P(x, \text{United States})$

c) $\forall x \exists y P(x,y)$

d) $\exists x \forall y \neg P(x,y)$

e) $\exists x \exists y (x \neq y \wedge \forall z (Q(x,z) \vee Q(v,z)))$

f) $\forall x \forall y (P(x,y) \rightarrow Q(x,y))$

(7) a) $\neg p(\text{carlos, Bulgaria})$

b) $\neg p(x, \text{united states})$

c) $\neg(\forall x \exists y P(x,y) \equiv \exists x \neg(\exists y\, P(x,y) \equiv \exists x \forall y \neg P(x,y)$

d) $\neg(\exists x \forall y \neg P(x,y)) \equiv \forall x \neg(\forall y \neg P(x,y) \equiv \forall x \exists y \neg(\neg P(x,y))$
$\equiv \forall x \exists y P(x,y)$

e) $\neg[\exists x \exists y (x \neq y \wedge \forall z (Q(x,y) \vee Q(y,z)))]$
$\equiv \forall x \neg[\exists y (x \neq y \wedge \forall z\, Q(x,z) \vee Q(y,z)))]$
$\equiv \forall x \forall y \neg[x \neq y \vee \forall z (Q(x,z) \vee Q(y,z))]$
$\equiv \forall x \forall y (x = y \vee \neg(\forall z (Q(x,z) \vee Q(y,z))))$
$\equiv \forall x \forall y (x = y \vee \exists z \neg(Q(x,z) \vee Q(y,z)))$
$\equiv \forall x \forall y (x = y \vee \exists z (\neg Q(x,z) \wedge \neg Q(y,z)))$

f) $\neg[\forall x \forall y (P(x,y) \rightarrow Q(x,y))]$
$\equiv \exists x \neg[\forall y (P(x,y) \rightarrow Q(x,y))]$
$\equiv \exists x \exists y \neg(P(x,y) \rightarrow Q(x,y))$
$\equiv \exists x \exists y \neg(\neg P(x,y) \vee Q(x,y))$
$\equiv \exists x \exists y (\neg(\neg P(x,y)) \wedge \neg Q(x,y))$
$\equiv \exists x \exists y (P(x,v) \wedge \neg Q(x,y))$

8b) $\neg[\forall x \forall y (Q(x,y) \leftrightarrow Q(y,x))]$

$\equiv \exists x \neg[\forall y (Q(x,y) \leftrightarrow Q(y,x))]$

$\equiv \exists x \exists y \neg[Q(x,y) \leftrightarrow Q(y,x)]$

$\equiv \exists x \exists y \neg[(Q(x,y) \land Q(y,x)) \lor (\neg Q(x,y) \land \neg Q(y,x))]$

$\equiv \exists x \exists y [\neg(Q(x,y) \land Q(y,x)) \land \neg(\neg Q(x,y) \land \neg Q(y,x))]$

$\equiv \exists x \exists y [(\neg Q(x,y) \lor \neg Q(y,x)) \land (Q(x,y) \lor Q(y,x))]$

$\equiv \exists x \exists y (Q(x,y) \oplus Q(y,x))$

8c) $\neg(\forall y \exists x \exists z (T(x,y,z) \land Q(x,y)))$

$\equiv \exists y \neg[\exists x \exists z (T(x,y,z) \land Q(x,y))]$

$\equiv \exists y \forall x \neg[\exists z (T(x,y,z) \land Q(x,y))]$

$\equiv \exists y \forall x \forall z \neg[T(x,y,z) \land Q(x,y)]$

$\equiv \exists y \forall x \forall z (\neg T(x,y,z) \lor \neg Q(x,y)]$

8a) $\neg[(\exists x \exists y P(x,y)) \lor (\forall x \forall y Q(x,y))]$

$\equiv \neg(\exists x \exists y P(x,y)) \land \neg(\forall x \forall y Q(x,y))$

$\equiv (\forall x \neg(\exists y P(x,y))) \land (\exists x \neg(\forall y Q(x,y)))$

$\equiv (\forall x \forall y \neg P(x,y)) \land (\exists x \exists y \neg Q(x,y))$