Problem 1: Two signals $m_1(t)$ and $m_2(t)$ band-limited to 5000 Hz are to be transmitted simultaneously over a single wireless channel. To transmit these signals, they are frequency multiplexed and then amplitude modulation is used to transmit the multiplexed signal through an antenna. The transmitted signal should have a center frequency of 500000Hz.

Important Note: In this problem, you may not use high-pass or band-pass filters.

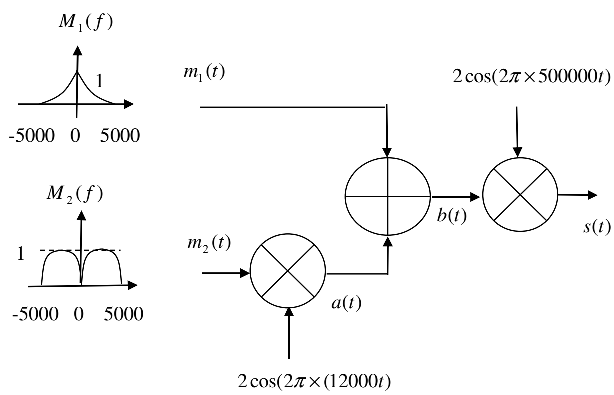
a) Design a transmitter for this system by drawing the structure of the transmitter. Show all the transmitter blocks with exact parameters. Draw the transmitted signal $s(t)$ in frequency domain and determine its bandwidth. Justify all of your answers.

b) Design the coherent receiver to recover the signals $m_1(t)$ and $m_2(t)$. Show all receiver blocks with exact parameters. Justify all of your answers.

Solution 1

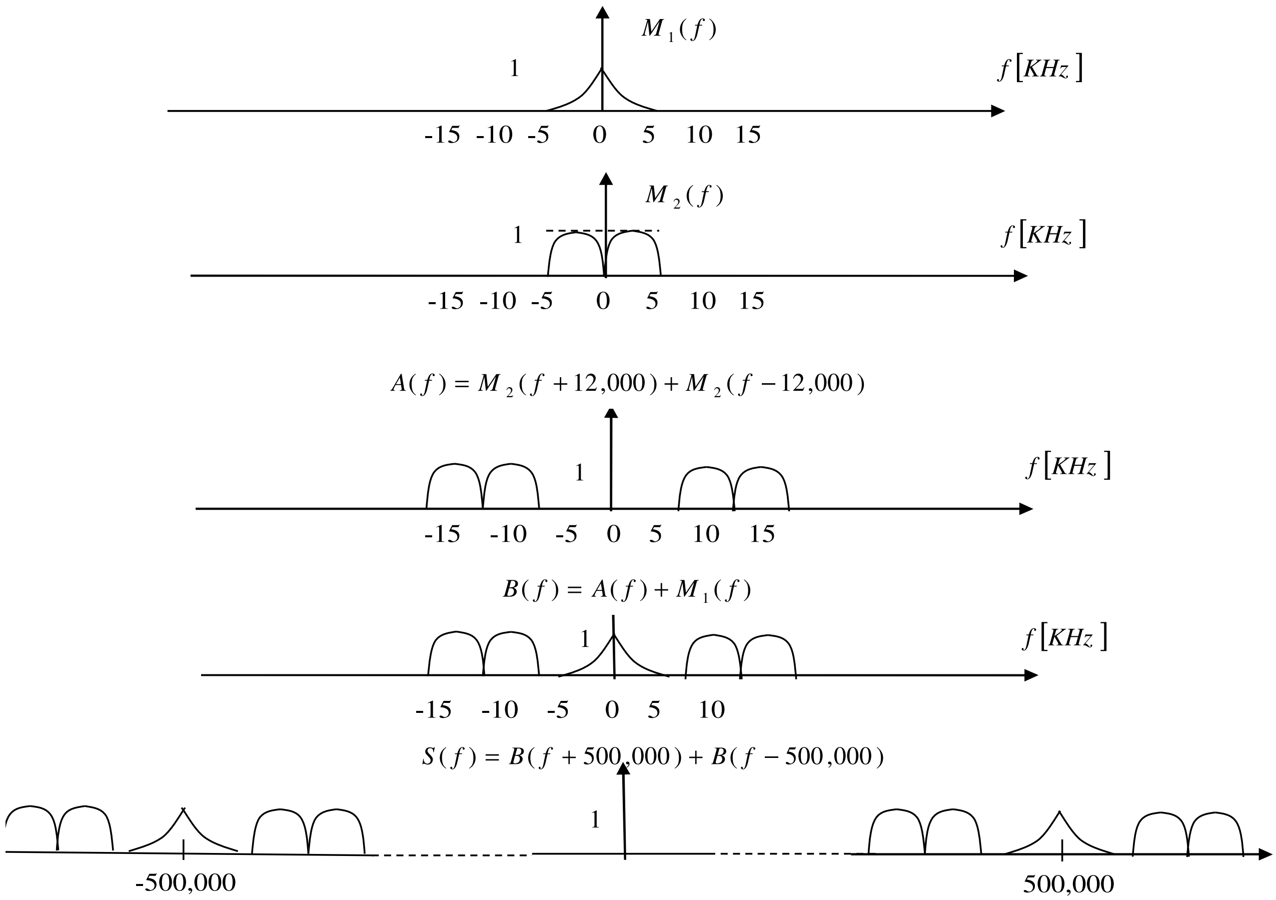
Part a)

The structure of the transmitter is as follows:



Instead of using 10000Hz for down conversion, we are using 12000Hz to be able to use non-ideal filter for recovery of signal in the receiver.

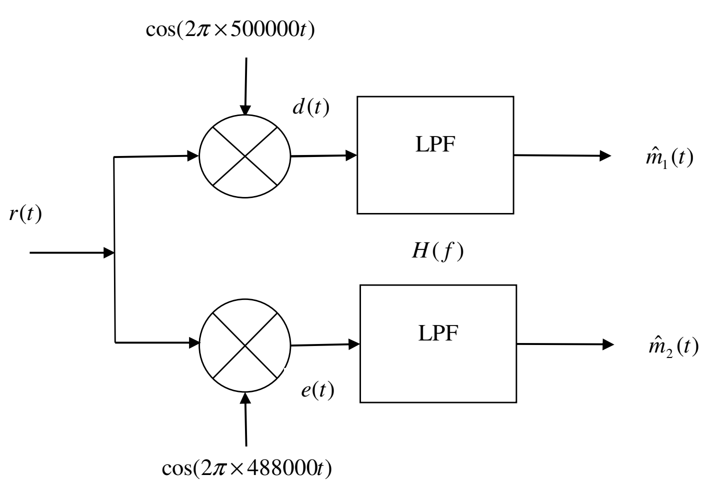
The transmitted signal is centered at 500,000Hz and shown below:



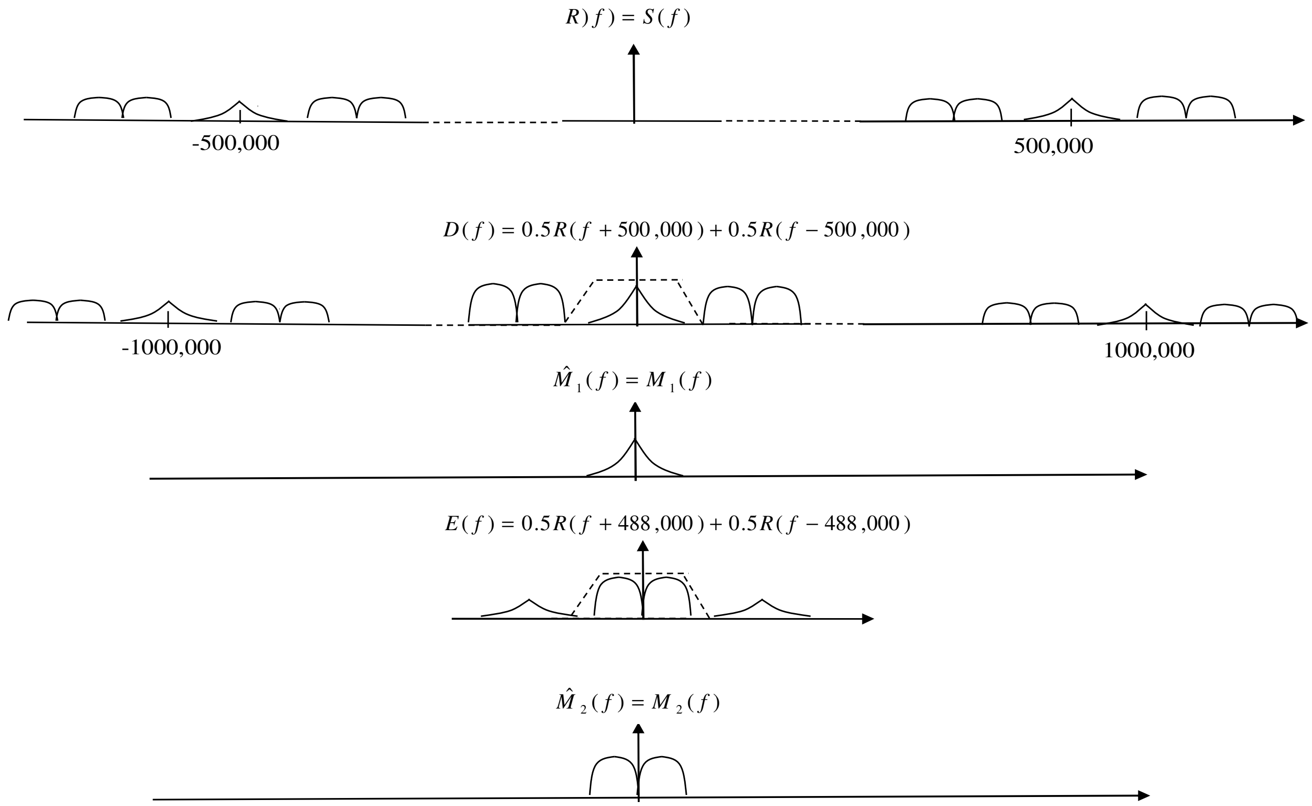
The transmitted signal is centered at 500,000 Hz and the channel is occupied between (500,000-17,000) to 500,000+17,000 Hz, therefore the occupied bandwidth is 34,000 Hz.

Part b)

The structure of the receiver is as follows:



Following spectrum analysis shows that the transmitted signals can be recovered using the above receiver.



The low pass filter in the receiver is non-ideal with passband of 5000 Hz and transition band of 2000Hz.

Problem 2: Two signals $m_1(t)$ and $m_2(t)$ band-limited to 5000 Hz are to be transmitted simultaneously over a single wireless channel. The power of each of these signals is equal to 20 mWatts. To transmit these analog signals, each signal is converted to digital using an Analog-to-Digital Convertor with dynamic range of -1 volt to 1 volt. Each Analog-to-Digital Convertor includes a sampler at twice the Nyquist rate, a linear quantizer with SQNR of 48 dB and a PCM encoder. Then, the output of PCM encoder for signal $m_1(t)$ is time multiplexed with the output of PCM encoder for signal $m_2(t)$. The output of time multiplexer is modulated by using a Quadrature Phase Shift Keying (QPSK) scheme which has a roll-off factor of 0.25. The transmitted signal should have a center frequency of 500000Hz.

a) Design a transmitter for this system by drawing the structure of the transmitter. Show all the transmitter blocks with exact parameters. Draw the transmitted signal $s(t)$ in frequency domain and determine its bandwidth. Justify all of your answers.
b) Design the coherent receiver to recover the signals $m_1(t)$ and $m_2(t)$. Show all receiver blocks with exact parameters. Justify all of your answers.

Solution 2

Part a)

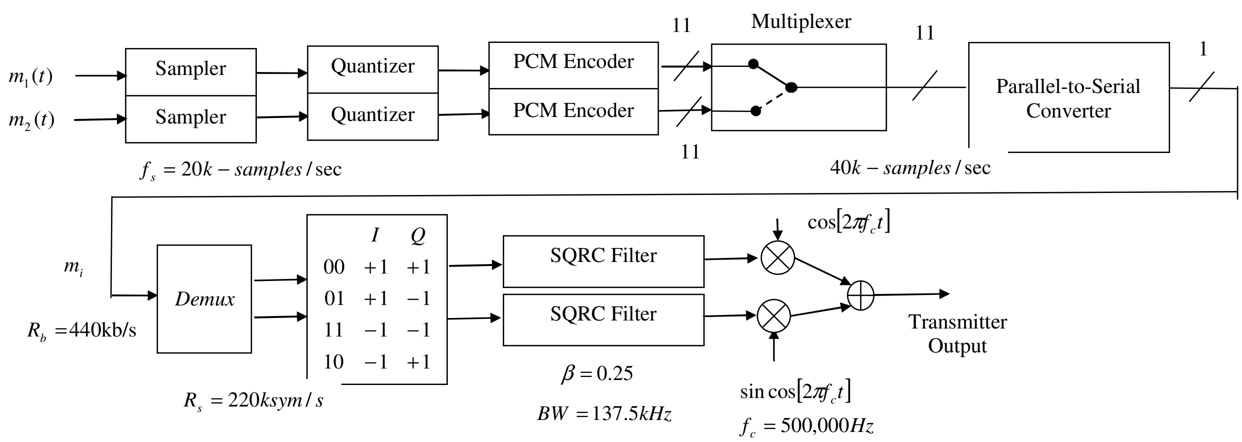
Nyquist sampling frequency is double the maximum frequency of $M_1(f)$ and $M_2(f)$ which is $2\times 5 = 10kHz$. We sample twice the Nyquist rate at $f_s = 20000Hz$.

Therefore, we can use the following formula:

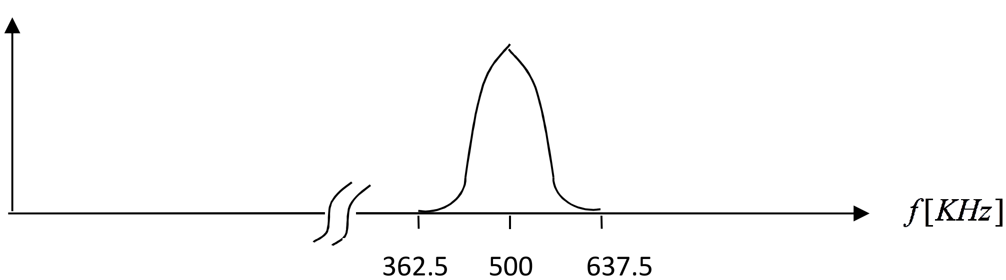
$$\frac{S_o}{N_o} = 3L^2 \frac{\overline{m^2(t)}}{m_p^2} \Rightarrow 10^{4.8} = 3L^2 \frac{20\times 10^{-3}}{1^2} \Rightarrow L \geq 1025.47$$

L should be power of 2 and larger than 1025. Therefore, $2^n = 2048$ and we need at least $n = 11$ bits for the quantizer. Note that for each signal $n = 11$ bits.

The sampling frequency is $f_s = 20000Hz$ and each sample is $n = 11$ bits per signal, therefore the bit rate for bit-by-bit serial transmission is $R_b = 2\times f_s \times n = 2\times 20000\times 11 = 440 \quad kbits/\sec$.

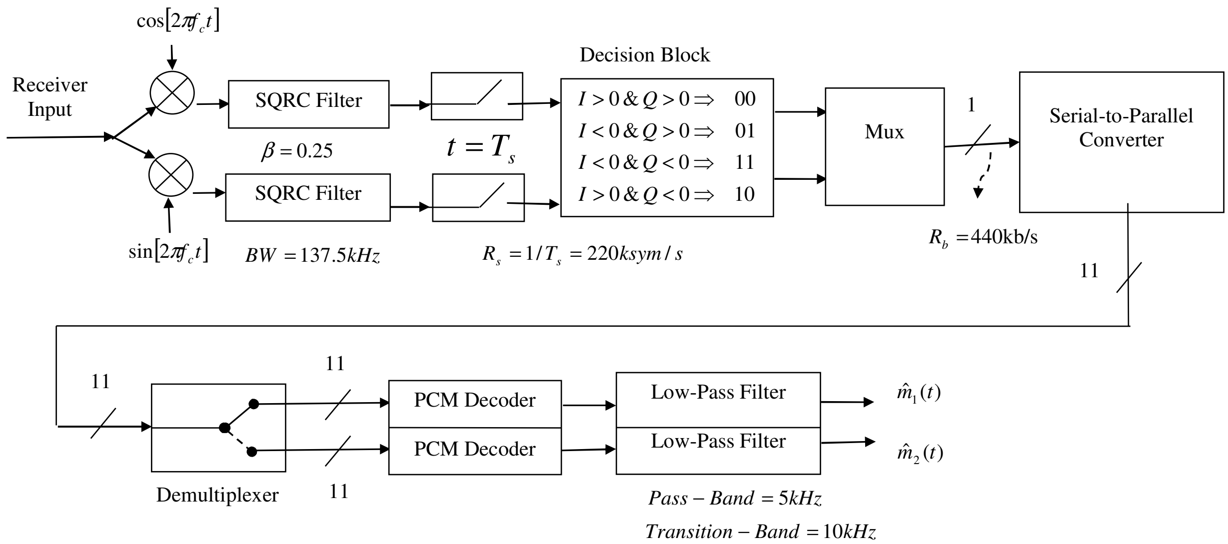


The bandwidth of QPSK modulator is $BW = (1+\alpha)R_b/2 = (1+0.25)\times 220 = 275 \quad kHz$



Part b:

The block diagram of the receiver is shown below:



The received signal is a QPSK modulated signal with bandwidth of 275 kHz and carrier frequency of $f_c = 500kHz$.

At the output of the mixers, there will be the baseband signals and a high frequency signal centered at 1MHz. The high frequency signal will be removed by the SQRC low pass filter.

The baseband signal will go to the samplers which will sample the signal at every $T_s = \frac{1}{220000}[\sec]$. Decision will be made at the output of the sampler and bits with bit rate of $R_b = 440{,}000 bits/\sec$ will be produced.

Then serial to parallel converter will group the bits 11-bit by 11-bit which is one sample at sampling frequency of 20kHz. Then, the demultiplexer produces two streams of 11-bit samples each with sampling frequency of 20 kHz. These samples are passed through PCM decoder.

Output of PCM Decoders are filtered by a low-pass filter to remove ripples which has a pass-band of 5kHz to pass the signal and transition band of 10 kHz which will remove high frequency signals.. The estimates of original signals will be produced at the output of these filters.

Problem 3: A stream of digital data with data rate of $R_b = 1/T_b$ is to be transmitted using an 8-ary modulation scheme. Each group of 3-bit data is mapped to one of the following signals which are defined as,

$$s_1(t) = +p(t)\cos(2\pi f_c t),\quad s_2(t) = +p(t)\cos(2\pi f_c t) + p(t)\sin(2\pi f_c t)$$
$$s_3(t) = +p(t)\sin(2\pi f_c t),\quad s_4(t) = -p(t)\cos(2\pi f_c t) + p(t)\sin(2\pi f_c t)$$
$$s_5(t) = -p(t)\cos(2\pi f_c t),\quad s_6(t) = -p(t)\cos(2\pi f_c t) - p(t)\sin(2\pi f_c t)$$
$$s_7(t) = -p(t)\sin(2\pi f_c t),\quad s_8(t) = +p(t)\cos(2\pi f_c t) - p(t)\sin(2\pi f_c t)$$

where $p(t) = \begin{cases} 1 & 0 < t < 3T_b \\ 0 & otherwise \end{cases}$ and $f_c >> \dfrac{1}{T_b}$ is the carrier frequency.

a) Using the above signal set, draw the structure of the transmitter and identify the function of the mapper. Then, conclude the constellation diagram of the receiver and determine maximum likelihood decision regions for a coherent receiver. On the constellation diagram, assign 3-bit values to signal points such that Gray coding is formed.
b) Evaluate the bit error rate of the system with coherent receiver if the received power is P_r and the noise power spectral density is N_0.

Solution 3:

Part a)

Signals can be written as linear combination of carriers $\sqrt{2}\cos(2\pi f_c t)$ and $\sqrt{2}\sin(2\pi f_c t)$ as , $s_i(t) = [I_i]p(t)\sqrt{2}\cos(2\pi f_c t) + [Q_i]p(t)\sqrt{2}\sin(2\pi f_c t)$ for $i = 1,2,3,4,5,6,7,8$.

Then, we can find, coordinates of the constellation points in transmitter as $s_i\left|\begin{matrix} I_i \\ Q_i \end{matrix}\right.$ for $i = 1,2,3,4,5,6,7,8$

$$s_1(t) = [C]p(t)\sqrt{2}\cos(2\pi f_c t) + [0]p(t)\sqrt{2}\sin(2\pi f_c t) \Rightarrow \qquad s_1\left|\begin{matrix} I_1 = C \\ Q_1 = 0 \end{matrix}\right.$$

$$s_2(t) = [C]p(t)\sqrt{2}\cos(2\pi f_c t) + [C]p(t)\sqrt{2}\sin(2\pi f_c t) \Rightarrow \qquad s_2\left|\begin{matrix} I_2 = C \\ Q_2 = C \end{matrix}\right.$$

$$s_3(t) = [0]p(t)\sqrt{2}\cos(2\pi f_c t) + [C]p(t)\sqrt{2}\sin(2\pi f_c t) \Rightarrow \qquad s_3\left|\begin{matrix} I_3 = 0 \\ Q_3 = C \end{matrix}\right.$$

$$s_4(t) = [-C]p(t)\sqrt{2}\cos(2\pi f_c t) + [C]p(t)\sqrt{2}\sin(2\pi f_c t) \Rightarrow \qquad s_4\left|\begin{matrix} I_4 = -C \\ Q_4 = +C \end{matrix}\right.$$

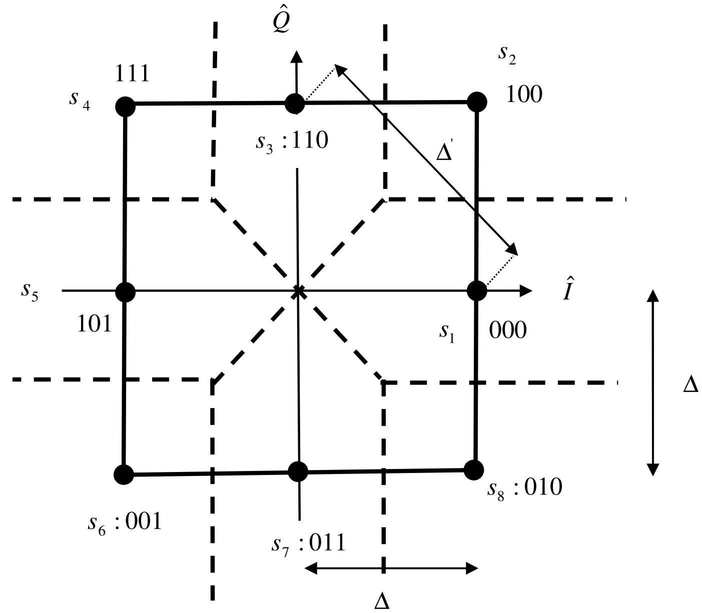
$$s_5(t) = [-C]p(t)\sqrt{2}\cos(2\pi f_c t) + [0]p(t)\sqrt{2}\sin(2\pi f_c t) \Rightarrow \qquad s_5\left|\begin{matrix} I_5 = -C \\ Q_5 = 0 \end{matrix}\right.$$

$$s_6(t) = [-C]p(t)\sqrt{2}\cos(2\pi f_c t) + [-C]p(t)\sqrt{2}\sin(2\pi f_c t) \Rightarrow \qquad s_6\left|\begin{matrix} I_6 = -C \\ Q_6 = C \end{matrix}\right.$$

$$s_7(t) = [0]p(t)\sqrt{2}\cos(2\pi f_c t) + [-C]p(t)\sqrt{2}\sin(2\pi f_c t) \Rightarrow \quad s_7 \left| \begin{array}{l} I_7 = 0 \\ Q_7 = -C \end{array} \right.$$

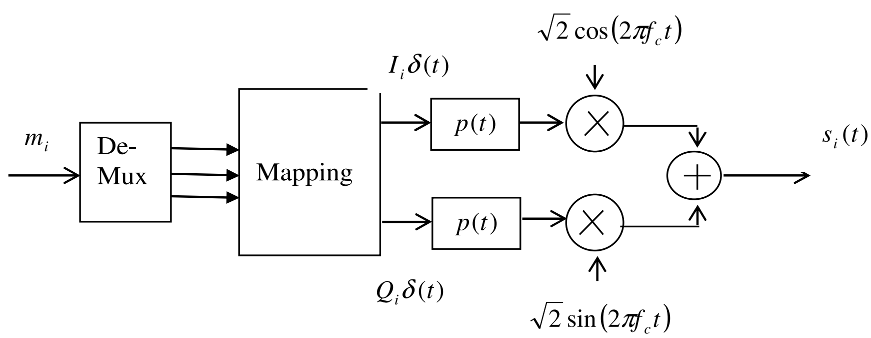
$$s_8(t) = [C]p(t)\sqrt{2}\cos(2\pi f_c t) + [-C]p(t)\sqrt{2}\sin(2\pi f_c t) \Rightarrow \quad s_8 \left| \begin{array}{l} I_8 = C \\ Q_8 = -C \end{array} \right.$$

The value of constant in above equations is $C = 1/\sqrt{2}$. Based on these constellation points we can draw the constellation diagram in the transmitter. The constellation diagram in the receiver has the same shape as the one in the transmitter as shown below (based on coordinates of above signal points where C in the transmitter is replaced by Δ in the receiver):



In the above figure, the boundaries between maximum likelihood decision regions are shown by dashed lines. Gray coding is also assigned in the diagram as well by assigning 3-bit values to signal points such that adjacent points (vertically and horizontally) with distance of Δ differ only in one bit.

Transmitter structure is:



The mapper detail is as follows:

	000	100	110	111	101	001	011	010
	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8

I	C	C	0	-C	-C	-C	0	C
Q	0	C	C	C	0	-C	-C	-C

Part b)

$$P_E = P(symbol - error) = \frac{1}{8}\sum_{i=1}^{8} P(error|s_i)$$

Because of symmetry, there are two kinds of points which are s_1, s_3, s_5, s_7 and s_2, s_4, s_6, s_8. Therefore,

$$P_E = \frac{1}{8}\{4P(error|s_1) + 4P(error|s_2)\}$$

Based on Union bound, we have:

$P(error|s_1) = 2p + 2p^{'}$ and $P(error|s_2) = 2p$

where $p = Q\left(\frac{\Delta/2}{\sigma_0}\right)$ and $p^{'} = Q\left(\frac{\Delta^{'}/2}{\sigma_0}\right)$. Since $\Delta^{'} = \sqrt{2}\Delta > \Delta$ then $p^{'} << p$ and

$$P_E = \frac{1}{8}\{4\times 2p + 4\times 2p\} = 2p = 2Q\left(\frac{\Delta}{2\sigma_0}\right)$$

Considering $\sigma_0 = \sqrt{\frac{N_0}{2}}$, the average symbol error rate is $P(symbol - error) = P_E = 2Q\left(\frac{\Delta}{\sqrt{2N_0}}\right)$.

We should find average energy of signals $E_s = \frac{1}{8}\sum_{i=1}^{i=8} E_{s_i}$.

Because of symmetry, there are two kinds of points which are s_1, s_3, s_5, s_7 and s_2, s_4, s_6, s_8.

$$E_s = \frac{1}{8}\left(4E_{s_1} + 4E_{s_2}\right)$$

$E_{s_1} = \Delta^2$ and $E_{s_2} = \Delta^2 + \Delta^2$

$$E_s = \frac{1}{8}\left(4E_{s_1} + 4E_{s_2}\right) = \frac{1}{8}\left(4\Delta^2 + 4(\Delta^2 + \Delta^2)\right) = \frac{3}{2}\Delta^2 \Rightarrow \qquad \Delta = \sqrt{\frac{2E_s}{3}}$$

Therefore, $P(symbol-error) = P_E = 2Q\left(\frac{\Delta}{\sqrt{2N_0}}\right) = 2Q\left(\sqrt{\frac{E_s}{3N_0}}\right) = 2Q\left(\sqrt{\frac{3P_r}{3N_0R_b}}\right)$

$$P_b = \frac{P_E}{\log_2 8} = \frac{P_E}{3} = \frac{2}{3}Q\left(\sqrt{\frac{3P_r}{3N_0R_b}}\right) \Rightarrow \qquad P_b = \frac{2}{3}Q\left(\sqrt{\frac{3P_r}{3N_0R_b}}\right)$$

Problem 4:

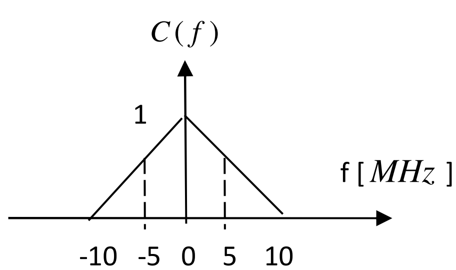
A stream of digital data with bit rate of R_b is to be transmitted. This data is mapped to one of 4 signals as follows: $s_i(t) = I_i\, p(t)\sin(2\pi f_c t) + Q_i\, p(t)\cos(2\pi f_c t)$ where I_i and Q_i are given in the table below:

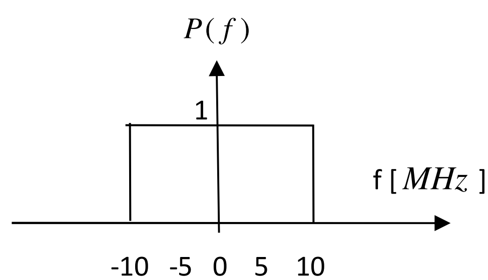
i	1	2	3	4
I_i	**+1**	**+1**	**-1**	**-1**
Q_i	**+1**	**-1**	**+1**	**-1**

In these signals, $f_c >> R_b$ is the carrier frequency and the pulse shaping filter is represented by $p(t)$.

a) Draw the block diagram of the transmitter and coherent receiver. Prove that the above pass band system is equivalent to two one-dimensional baseband systems from inter-symbol interference point of view. Draw the block diagram of the baseband transmitters and receivers.

b) Assume that information bit rate is R_b = 10 $Mbits/\sec$ and also assume that frequency responses of the channel $C(f)$ and the pulse shaping filters $P(f)$ are as given in the figure. If the receiver low pass filters $M(f)$ are matched to $P(f)$, prove that the system has no Intersymbol Interference.





Solution 4:

See the solution of problem 1 of assignment 5

Problem 5:

A stream of digital data with data rate of 1 Mb/s is to be transmitted using a modulation scheme. The communication system is required to achieve a bit error rate of at most 10^{-5}. The center frequency of signal in the channel after up-conversion is 500 MHz and the receiver is coherent.

a) If the modulation scheme is 8PSK with roll-off factor of 0.25, determine P_r/N_0 in dB-Hz to achieve the required performance (P_r is the received power and N_0 is the single sided noise spectral density). Draw the frequency spectrum of the transmitted signal and determine the bandwidth occupied by the transmitted signal.

b) Determine the best modulation scheme if the channel bandwidth is 4.5 MHz and P_r/N_0 is 68 dB-Hz. Draw the frequency spectrum of the transmitted signal.

Important Note: In this problem, you may use formula sheet to calculate bit error rates of modulation schemes and bandwidth of the transmitted signals.

Solution 5:

Part a)

From the formula sheet the symbol error rate of 8PSK is found as

$P_E(8PSK) = 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{8}\right)$. Since Gray coding is possible for MPSK system then

$$BER(8PSK) = \frac{P_E(8PSK)}{\log_2 8} = \frac{2}{3}Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{8}\right).$$

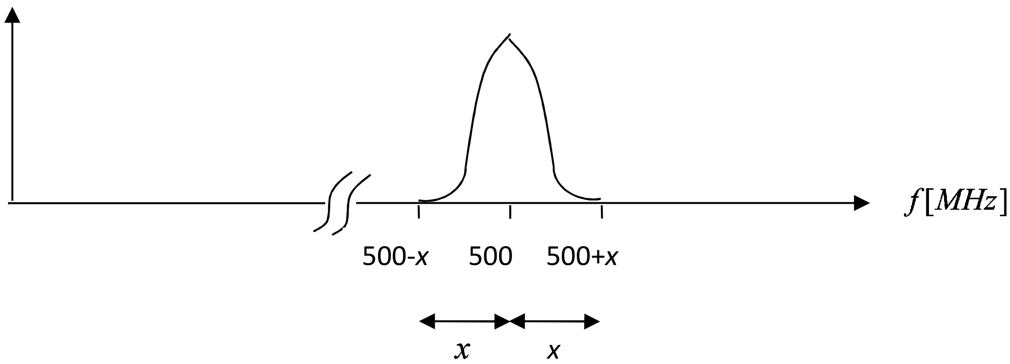
$E_s = (\log_2 8)E_b = 3E_b$ and $E_s = 3E_b = \frac{3P_r}{R_b}$, therefore $BER(8PSK) = \frac{2}{3}Q\left(\sqrt{\frac{6P_r}{N_0 R_b}}\sin\frac{\pi}{8}\right)$.

$$10^{-5} = \frac{2}{3}Q\left(\sqrt{\frac{6P_r}{N_0 R_b}}\sin\frac{\pi}{8}\right) \Rightarrow 1.5\times 10^{-5} = Q\left(\sqrt{\frac{6P_r}{N_0 R_b}}\sin\frac{\pi}{8}\right) = Q(x) \Rightarrow$$

$$x = 4.185 = \sqrt{\frac{6P_r}{N_0 R_b}}\sin\frac{\pi}{8} = \sqrt{\frac{6P_r}{N_0\times 10^6}}0.3826 \quad \Rightarrow \frac{P_r}{N_0} = 19.94\times 10^6 \quad \Rightarrow \left.\frac{P_r}{N_0}\right|_{dB} = 73dB - Hz$$

$$BW(8PSK) = (1+\beta)R_s = (1+0.25)\frac{R_b}{\log_2 8} = (1+0.25)\frac{10^6}{3} = 416.67 \quad KHz$$

Carrier frequency is the center frequency as shown in the figure and $x = 208.33$ KHz.



Part b)

Since the available bandwidth of 4.5 MHz is very wide considering the data rate of 1 Mb/s, we try to use MFSK which is a power limited modulation scheme.

$BW(MFSK) = \frac{MR_s}{2} = \frac{MR_b}{2\log_2 M} = \frac{10^6 M}{2\log_2 M} \le 4.5\times 10^6$ and therefore $M = 2, \ 4, \ 8, \ 16, \ 32$.

We would like to choose "M" value such that we get a bit error rate of 10^{-5}. Obviously, we would like to have the lowest possible "M" for having low complexity.

For MFSK, we cannot have Gray coding and therefore:

$$BER(MFSK) = \frac{M/2}{M-1}P_E = \frac{M/2}{M-1}(M-1)Q\left(\sqrt{\frac{E_s}{N_0}}\right) = \frac{M}{2}Q\left(\sqrt{\frac{E_b Log_2 M}{N_0}}\right) = \frac{M}{2}Q\left(\sqrt{\frac{P_r Log_2 M}{R_b N_0}}\right)$$

Since $\left.\frac{P_r}{N_0}\right|_{dB} = 68 \quad dB-Hz$, we have $68 = 10\log\frac{P_r}{N_0} \Rightarrow \frac{P_r}{N_0} = 10^{(68/10)} \Rightarrow \frac{P_r}{N_0} = 6309573 \quad Hz$

Now, we find the BER considering $\frac{P_r}{N_0} = 6309573 Hz$ and $R_b = 10^6 b/\sec$.

$$BER(MFSK) = \frac{M}{2}Q\left(\sqrt{\frac{6309573}{10^6}\log_2 M}\right) = \frac{M}{2}Q\left(\sqrt{6.31\log_2 M}\right)$$

By increasing M, the BER will improve for this modulation, therefore, we apply $M = 2,4,8,...$ until we find a BER better than 10^{-5} which is $BER(16FSK) = \frac{16}{2}Q\left(\sqrt{6.31\times 4}\right) = 8Q(5.02) = 2\times 10^{-6} < 10^{-5}$.

We see that by increasing "M" from 2 to 32, BER improves. At M=16, we get to the right BER. If we increase "M" further, the BER will become better but to have low complex system we take M=16.

Now we consider MPSK or MQAM. For these systems, the bandwidth of the transmitted signal is

$$BW(MPSK \quad or \quad MQAM) = (1+\beta)R_s = (1+\beta)\frac{R_b}{\log_2 M} = (1+\beta)\frac{10^6}{\log_2 M} \le 4.5\times10^6$$

In the above inequality, $0 < \beta < 1$ and therefore the inequality is valid for any value of "M".

If we take M=2 or M=4, the BER is:

$$BER(BPSK\ or\ QPSK) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{2P_r}{R_b N_0}}\right) = Q\left(\sqrt{\frac{2\times6309573}{10^6}}\right) = Q(3.55) = 0.19\times10^{-3} > 10^{-5}$$

Obviously, this BER is not acceptable. Now if we increase "M", we know that BER will become even worse. Therefore, we cannot find any MPSK or MQAM system which has BER of less than or equal to 10^{-5}.

Conclusion: With the given specification, 16FSK is the solution.