STAT 2509 B - Assignment #1

SOLUTION

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Q.1: [6]

a) · parameter - is a descriptive (1) measure of a population (or numerical descriptive measure of a popul^n)
· it is a fixed constant (i.e. it does not vary) (1)

· statistic - is a (numerical) descriptive (1) measure of a sample
· it varies (1) from sample to sample

b) · population - is a collection of all items (1) of interest
· sample - is a subset of the units in a popul^n (1)

Q.2: [7]

a) quantitative (1/2) & continuous (1/2)
b) categorical (1) (or qualitative)
c) quantitative (1/2) & discrete (1/2)
d) categorical (1/2) (or qualitative) & ranked (1/2)
e) quantitative (1/2) & continuous (1/2)
f) categorical (1/2) & ranked (1/2)
g) quantitative (1/2) & discrete (1/2)

Q.3: [6]

(i) $\bar{x}$ - statistic (1), (ii) σ^2 - parameter (1)
(iii) μ - parameter (1), (iv) s^2 - statistic (1)
(v) β_1 - parameter (1), (vi) - $\hat{\beta}_1$ - statistic (1)

Q.4:

[4]

a) Ha (alternative hypothesis) (1) - is the one we want to show, ie. we are trying to find sufficient evidence for

Ho (null hypothesis) - is negating the statement in Ha. It is a "fall-back" hypothesis. It tests against Ha. (1)

b) Type I error = error we make when we reject Ho when it is true (1)

P[Type I error] = α

Type II error = error we make when we do not reject Ho when it is false (1)

P[Type II error] = β

Q.5: k- constant

X, Y - r.v.'s

[7]

a) (i) $E(k) = k$ (1), (ii) $E(kX) = kE(X)$ (1)

(iii) $E(X \pm Y) = E(X) \pm E(Y)$ (1)

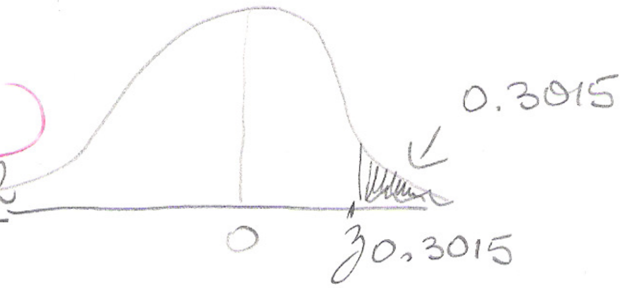
b) (i) $V(k) = 0$ (1), (ii) $V(kX) = k^2 V(X)$ (1)

(iii) $V(X \pm Y) = V(X) + V(Y) \pm 2\,Cov(X,Y)$ (1)

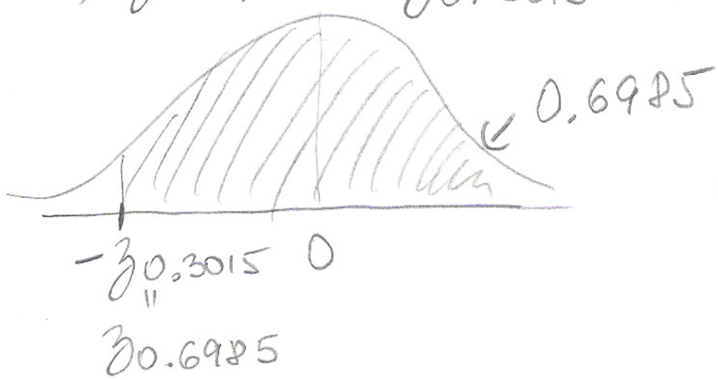
- when X & Y are indep. of each other, then $cov(X,Y) = 0$, i.e. $V(X \pm Y) = V(X) + V(Y)$ (1)

Q.6: [6]

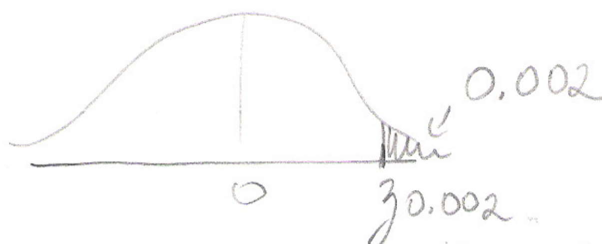
a) $z_{0.3015} = \underline{0.52}$ (1)



b) $z_{0.6985} = -z_{0.3015} = \underline{-0.52}$ (1)



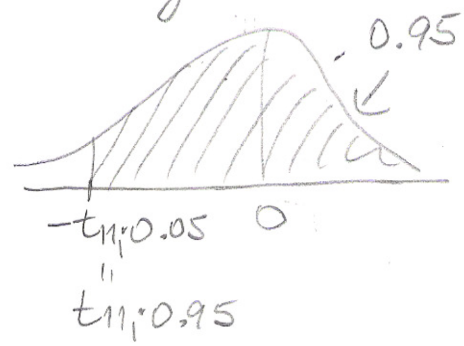
c) $z_{0.002} = \underline{2.88}$ (1)



d) $t_{11;0.05} = \underline{1.796}$ (1)

e) $-t_{11;0.05} = \underline{-1.796}$ (1)

f) $t_{11;0.95} = -t_{11;0.05} = -\underline{1.796}$ (1)



Q.7: $N(\mu, \sigma^2)$, σ^2-known [6]

a) (i) $\bar{X} \pm 1.96\, \sigma/\sqrt{n} \Rightarrow z_{\alpha/2} = 1.96 \Rightarrow \alpha/2 = 0.025$
$\alpha = 0.05$
$1-\alpha = 0.95$

$\therefore$ $\underline{95\%\ \text{C.I.}}$ for μ (1)

(ii) $\bar{X} \pm 2.24\, \sigma/\sqrt{n} \Rightarrow z_{\alpha/2} = 2.24 \Rightarrow \alpha/2 = 0.0125$
$\alpha = 0.025$
$1-\alpha = 0.975$

$\therefore$ $\underline{97.5\%\ \text{C.I.}}$ for μ (1)

(iii) $\bar{X} \pm 3.09\, \sigma/\sqrt{n} \Rightarrow z_{\alpha/2} = 3.09 \Rightarrow \alpha/2 = 0.0010$
$\alpha = 0.0020$
$1-\alpha = 0.998$

$\therefore$ $\underline{99.8\%\ \text{C.I.}}$ for μ (1)

b) (i) $\bar{x} \pm z_{\alpha/2}\sigma/\sqrt{n} \Rightarrow 1-\alpha = 0.8968$

$\alpha = 0.1032$ (1)

$\alpha/2 = 0.0576 \Rightarrow \underline{\underline{z_{\alpha/2} = 1.63}}$

(ii) $\bar{x} \pm z_{\alpha/2}\sigma/\sqrt{n} \Rightarrow 1-\alpha = 0.9920$

$\alpha = 0.0080$

$\alpha/2 = 0.0040 \Rightarrow \underline{\underline{z_{\alpha/2} = 2.65}}$ (1)

(iii) $\bar{x} \pm z_{\alpha/2}\sigma/\sqrt{n} \Rightarrow 1-\alpha = 0.7540$

$\alpha = 0.2460$ (1)

$\alpha/2 = 0.1230 \Rightarrow \underline{\underline{z_{\alpha/2} = 1.16}}$

<u>Q8:</u>
[4]

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

show $s^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}\right]$

ie. $\sum_{i=1}^{n}(x_i-\bar{x})^2 = \sum_{i=1}^{n}(x_i^2 + \bar{x}^2 - 2\bar{x}x_i) =$ (1)

$= \sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n}\bar{x}^2 - 2\bar{x}\sum_{i=1}^{n} x_i =$

$= \sum_{i=1}^{n} x_i^2 + n\bar{x}^2 - 2\bar{x}\sum_{i=1}^{n} x_i =$

$= \sum_{i=1}^{n} x_i^2 + \cancel{n}\frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n^{\cancel{2}}} - 2\frac{\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} x_i\right)}{n} =$

$= \sum_{i=1}^{n} x_i^2 + \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n} - 2\frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n} =$ (2)

$= \sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}$

$\therefore \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}\right]$ (1) $\square$.