A

1. (1 point) Suppose f is a differentiable function such that $f(3) = 2$ and $f'(3) = -1$. If $g = 2^{2f(x)}$, what is the value of $g'(3)$?

A. 2^5
B. $-2^5 \ln(2)$
C. -2^5
D. $-2^4 \ln(2)$
E. $\frac{-1}{2^5 \ln(2)}$
F. There's not enough information to say.

Your answer: B

$g'(x) = 2^{2f(x)} \cdot \ln 2 \cdot (2f(x))'$
$= 2^{2f(x)} \cdot \ln 2 \cdot 2f'(x)$

$x=3$

$= 2^4 \cdot \ln 2 \cdot 2 \cdot (-1)$
$= -2^5 \cdot \ln 2.$

2. (1 point) If $f(x) = \arccos(x^2)$, then $f'(x)$ is

A. $2x \arcsin(x)$
B. $\frac{x^2}{1+x^2}$
C. $\frac{2x}{\sqrt{1-x^4}}$
D. $2x \arcsin(x^2) + \frac{x^2}{\sqrt{1-x^2}}$
E. $\frac{x^2}{1+x^4}$
F. $\frac{-2x}{\sqrt{1-x^4}}$

Your answer: F

$f'(x) = \frac{(x^2)'}{-\sqrt{1-(x^2)^2}}$

$= \frac{2x}{-\sqrt{1-x^4}}.$

3. (1 point) Consider the curve implicitly defined by the equation

$$(y+2)^2 = x^2 + 15.$$

What is the equation of the tangent line to the curve at the point $(1, 2)$?

A. $y = 2$
B. $y = \frac{1}{2}(x^2+3)$
C. $y = 2x$
D. $y = \frac{1}{2}x + \frac{2}{3}$
E. $y = -5x + 7$
F. $y = \frac{1}{4}x + \frac{7}{4}$

Your answer: $\boxed{\text{F}}$

$2(y+2)\cdot y' = 2x.$

$y' = \left.\frac{2x}{2y+4}\right|_{(1,2)}$

$= \frac{2}{8} = \frac{1}{4}.$

4. (1 point) If $f(x) = \sqrt[3]{e^x + 9x^2}$ then

A. $f'(x) = \dfrac{e^x + 18x}{2\sqrt{e^x + 9x^2}}$
B. $f'(x) = \frac{1}{2}e^{x/3} + 3x^{\frac{2}{3}}$
C. $f'(x) = \dfrac{e^x + 18x}{3(e^x + 9x^2)^{\frac{2}{3}}}$
D. $f'(x) = \dfrac{1}{2\sqrt[3]{x}}(e^x + 18x)$
E. $f'(x) = \dfrac{1}{3\sqrt[3]{e^x + 18x}}$
F. $f'(x) = \sqrt[3]{e^x + 9x^2}\,(e^x + 18x)$

Your answer: $\boxed{\text{C}}$

$f'(x) = \frac{1}{3}(e^x + 9x^2)^{-\frac{2}{3}} \cdot (e^x + 9x^2)'$

$= \dfrac{e^x + 18x}{3(e^x + 9x^2)^{2/3}}$

5. (1 point) What is the degree 2 Taylor polynomial of the function $f(x) = \sqrt{x}$ centered at $x = 4$?

A. $2+\frac{1}{4}(x-4)+\frac{1}{64}(x-4)^2$ C. $2+\frac{1}{4}(x-4)-\frac{1}{32}(x-4)^2$ E. $4(x-4)-\frac{1}{64}(x-4)^2$

B. $2+\frac{1}{4}(x-4)-\frac{1}{64}(x-4)^2$ D. $2-\frac{1}{4}(x-4)-2(x-4)^2$ F. $2+2(x-4)$

Your answer: $\boxed{B}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$ $f''(x) = -\frac{1}{4}x^{-\frac{3}{2}}$

$f'(4) = \frac{1}{4}$ $f''(4) = -\frac{1}{4}\cdot\frac{1}{8} = -\frac{1}{32}$.

$f(4) + f'(4)(x-4) + \frac{f''(4)}{2!}(x-4)^2$

$= 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2$.

6. (2 points) Given function $f(x) = x^{\frac{1}{3}}(x-4)$, $x > 0$. Find all its local maximum values (if any) and minimum values (if any). Be sure to justify your conclusion with **First Derivative Test**. Are they also the global extrema?

$f'(x) = (x^{\frac{4}{3}} - 4x^{\frac{1}{3}})'$

$= \frac{4}{3}x^{\frac{1}{3}} - \frac{4}{3}x^{-\frac{2}{3}}$

$= \frac{4}{3}x^{-\frac{2}{3}}(x-1)$, $x>0$

$f'(x) = 0 \Rightarrow x = 1$

x:	$(0,1)$	$(1,+\infty)$
$f'(x)$:	<0	>0
$f(x)$:	$\searrow$	$\nearrow$

So $f(x)$ has a local minimum at $x=1$

the value is $f(1) = -3$.

It is also a global minimum.

7. **(2 points)** Evaluate the following limit using methods from calculus and algebra, if it exists. **Justify your steps clearly, identifying any indeterminate forms you encounter and any theorems that you use from class and why they apply.**

$$\lim_{x\to 0^+} (\sin x)^x \qquad 0^0$$

$$= \lim_{x\to 0^+} e^{x\ln(\sin x)}$$

$$= e^{\lim_{x\to 0^+} x\ln(\sin x)}$$

$$\lim_{x\to 0^+} x\ln(\sin x) = \lim_{x\to 0^+} \frac{\ln(\sin x)}{\frac{1}{x}} \quad \cdots \quad \frac{\infty}{\infty}$$

$$\overset{L'H}{=} \lim_{x\to 0^+} \frac{\frac{1}{\sin x}\cdot\cos x}{-\frac{1}{x^2}}$$

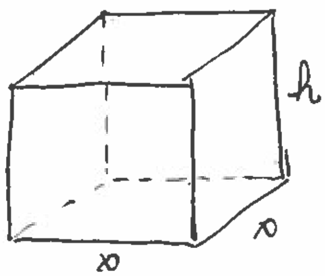
$$= \lim_{x\to 0^+} -\frac{x^2}{\tan x} \quad \cdots\cdots \quad \frac{0}{0}$$

$$\overset{L'H}{=} \lim_{x\to 0^+} -\frac{2x}{\sec^2 x}$$

$$= -\frac{0}{1} = 0$$

So, $\lim_{x\to 0^+} (\sin x)^x = e^{\lim_{x\to 0^+} x\ln(\sin x)} = e^0 = 1$

8. (4 points) A team of ecologists is conducting a research project on breeding robins in captivity. They first must construct suitable cages. They want a square base with a top made of the same mesh as the four walls. Find the minimum cost of building the cage if the cost of each m^2 of the four walls and the top is \$ 3 per m^2 and the total volume must be 200 m^3. **Make a diagram and label it; identify the function you need to optimize; justify, using Calculus, that your answer gives the minimum cost; and give the dimensions of the resulting cage.** Your answer must be legible, orderly, logical and well-justified to receive full marks.



Target

$$P(x,h) = 3\cdot(\underbrace{x^2}_{\text{Top}} + \underbrace{4xh}_{\text{4 sides}})$$

Restriction

$$x^2 h = 200 \Rightarrow h = \frac{200}{x^2}$$

$$P(x,h) = P(x) = 3\left(x^2 + \frac{800}{x}\right)$$

$$P'(x) = 3\left(2x - \frac{800}{x^2}\right) = 0$$

$$\Rightarrow 2x = \frac{800}{x^2} \Rightarrow x = \sqrt[3]{400}\ m$$

x:	$(0, \sqrt[3]{400})$	$(\sqrt[3]{400}, +\infty)$
$f'(x)$:	<0	>0
$f(x)$	$\searrow$	$\nearrow$

so $P(x)$ has a minimum value at $x = \sqrt[3]{400}\ m$

the minimum is $3\cdot\left(400^{\frac{2}{3}} + \frac{800}{(400)^{\frac{1}{3}}}\right)$ dollars.

Function whose minimal value was obtained (with notation as you have defined clearly above and in your diagram):

$$P(x) = 3\left(2x - \frac{800}{x^2}\right)$$

Width and height of each wall:

$$x = \sqrt[3]{400}\ m \qquad h = \frac{200}{(400)^{2/3}}\ m$$

9. (9 = 7 × 1+2 points)

The quantity of a drug being absorbed in the bloodstream, measured in mg/L, over time x, measured in hours, is modeled by the function $f(x) = 2xe^{-\frac{x}{3}}$.

Our goal is to produce its graph on the domain of interest $[0, \infty)$.

(a) Evaluate: $f(0) = \boxed{0}$, and $\lim_{x\to\infty} (2xe^{-\frac{x}{3}}) = \boxed{0}$

(b) Find the derivative of f: **Show your work.**

$$2(xe^{-\frac{x}{3}})' = 2(e^{-\frac{x}{3}} + xe^{-\frac{x}{3}}\cdot(-\tfrac{1}{3})) = 2e^{-\frac{x}{3}}(1-\tfrac{x}{3}).$$

$f'(x) = \boxed{2e^{-\frac{x}{3}}(1-\frac{x}{3})}$

(c) Give the critical point(s) of f (or write "none" if there aren't any). Give your answer(s) in EXACT form.

$x = \boxed{3}$

(d) In the space below, use the information above to make a table that indicates where f is increasing and where it is decreasing on the domain $[0, \infty)$. **Be sure to communicate your methods clearly.**

x :	$(0, 3)$	$(3, +\infty)$
$f'(x)$:	>0	<0
$f(x)$:	↗	↘

(e) Find the second derivative of f: **Show your work.**

$$2[e^{-\frac{x}{3}}(1-\tfrac{x}{3})]'$$
$$= 2[e^{-\frac{x}{3}}\cdot(-\tfrac{1}{3})(1-\tfrac{x}{3}) + e^{-\frac{x}{3}}\cdot(-\tfrac{1}{3})]$$
$$= -\tfrac{2}{3}e^{-\frac{x}{3}}(1-\tfrac{x}{3}+1).$$

$f''(x) = \boxed{-\frac{2}{3}e^{-\frac{x}{3}}(2-\frac{x}{3}).}$

(f) Give the critical point(s) of f' (candidates for inflection points) or write "none" if there aren't any. Give your answer(s) in EXACT form.

$x = \boxed{6.}$

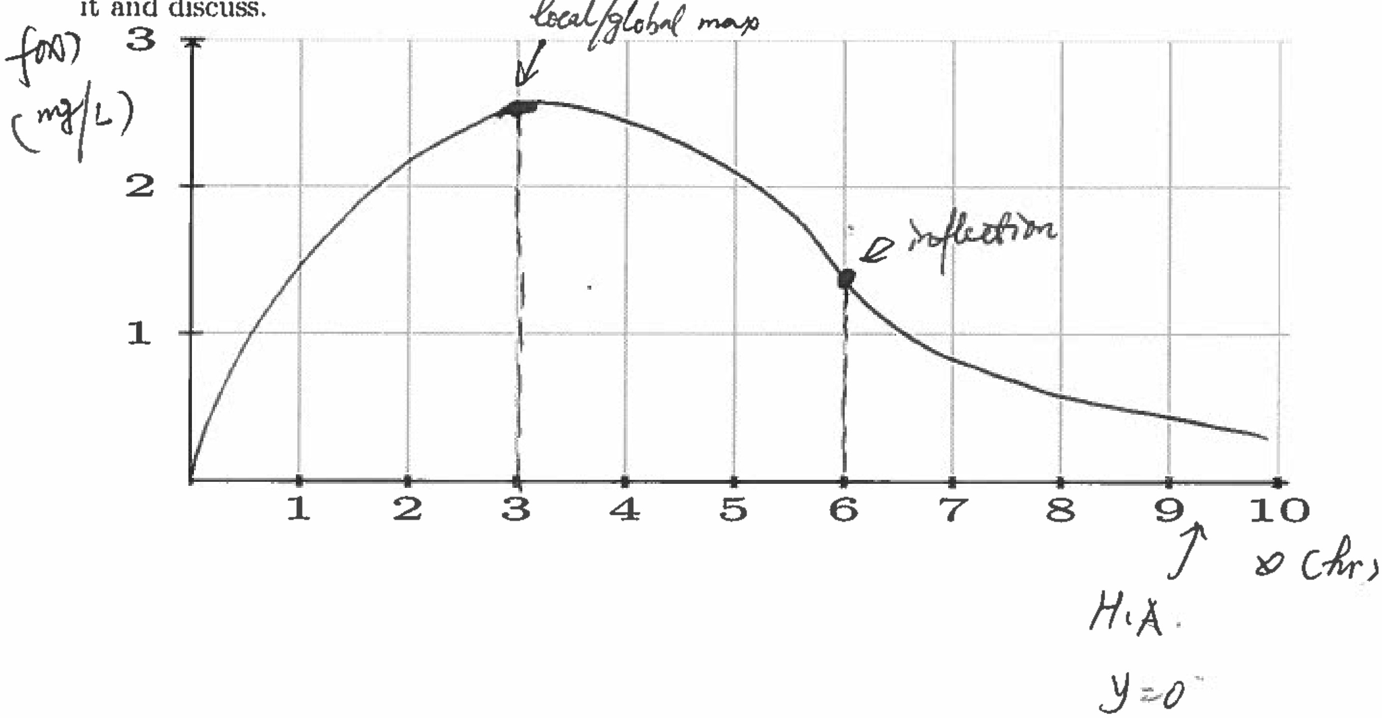
(g) In the space below, use the information above to make a table that indicates where f is concave up and where it is concave down on the domain $[0, \infty)$. **Be sure to communicate your methods clearly.**

x	$(0, 6)$	$(6, +\infty)$
$f''(x)$	<0	>0
$f(x)$	concave down	concave up

(h) On the axes below sketch the graph of $y = f(x)$ on the domain $[0, \infty)$ and:

- Plot all points on the graph of $y = f(x)$ corresponding to critical points of f and of f';
- Indicate **all** local and global extrema, and inflection points, on this domain, if any;
- Indicate all horizontal and vertical asymptotes, if any; and
- Label the axes, by name and with units of measurement.

If there is disagreement between your graph and any of your results in (a)-(g) you must identify it and discuss.



B

1. (1 point) Suppose f is a differentiable function such that $f(2) = 3$ and $f'(2) = -5$. If $g = 2^{3f(x)}$, what is the value of $g'(2)$?

A. 2^9

B. $2^9 \ln(2)$

C. $-15(2^9)\ln(2)$

D. $2^6 \ln(2)$

E. $\frac{-15}{2^9 \ln(2)}$

F. There's not enough information to say.

Your answer: C

$$g'(x) = 2^{3f(x)} \cdot \ln 2 \cdot (3f(x))'$$
$$= 2^{3f(x)} \cdot \ln 2 \cdot 3 \cdot f'(x)$$
$$g'(2) = 2^9 \cdot \ln 2 \cdot 3 \cdot (-5)$$
$$= -15(2^9)\ln(2)$$

2. (1 point) If $f(x) = \arcsin(x^2)$, then $f'(x)$ is

A. $2x \arcsin(x)$

B. $\dfrac{x^2}{1+x^2}$

C. $\dfrac{2x}{\sqrt{1-x^2}}$

D. $2x\arcsin(x^2) + \dfrac{x^2}{\sqrt{1-x^2}}$

E. $\dfrac{x^2}{1+x^4}$

F. $\dfrac{2x}{\sqrt{1-x^4}}$

Your answer: F

$$f'(x) = \frac{(x^2)'}{\sqrt{1-(x^2)^2}}$$
$$= \frac{2x}{\sqrt{1-x^4}}$$

3. (1 point) Consider the curve implicitly defined by the equation

$$y^3 + 2y = x^2 + 11.$$

What is the equation of the tangent line to the curve at the point $(1, 2)$?

A. $y = 2$

B. $y = 2(x^2 + 11)$

C. $y = \frac{1}{7}x + \frac{13}{7}$

D. $y = 14x - 12$

E. $y = -5x + 7$

F. $y = \frac{3}{7}x + \frac{11}{7}$

Your answer: C

$3y^2 \cdot y' + 2y' = 2x.$

$y' = \left.\dfrac{2x}{3y^2 + 2}\right|_{(1,2)}$

$= \dfrac{2}{12 + 2} = \dfrac{1}{7}.$

4. (1 point) If $f(x) = \sqrt{e^x + 9x^2}$ then

A. $f'(x) = \dfrac{e^x + 18x}{2\sqrt{e^x + 9x^2}}$

B. $f'(x) = \dfrac{1}{2}e^{x/2} + 3$

C. $f'(x) = \dfrac{1}{\sqrt{e^x + 9x^2}}$

D. $f'(x) = \dfrac{1}{2\sqrt{x}}(e^x + 18x)$

E. $f'(x) = \dfrac{1}{2\sqrt{e^x + 18x}}$

F. $f'(x) = \sqrt{e^x + 9x^2}\,(e^x + 18x)$

Your answer: A

$f'(x) = \dfrac{(e^x + 9x^2)'}{2\sqrt{e^x + 9x^2}}$

$= \dfrac{e^x + 18x.}{2\sqrt{e^x + 9x^2}}$

5. (1 point) What is the degree 2 Taylor polynomial of the function $f(x) = \ln(x)$ centered at $x = e^2$?

A. $2+e^{-2}(x-e^2)+\frac{1}{2}e^{-4}(x-e^2)^2$

B. $e^2 + e^{-2}x - \frac{1}{2}e^{-2}x^2$

C. $2+e^{-2}(x-e^2)-\frac{1}{2}e^{-4}(x-e^2)^2$

D. $e^2+e-2x+e^{-4}x^2+e^{-8}x^3$

E. $e^{-2}(x+e^2)-\frac{1}{4}e^{-4}(x+e^2)^2$

F. $2+(e^{-2}-\frac{1}{2}e^{-4})(x-e^2)$

Your answer: C

$f'(x) = \frac{1}{x}$, $f''(x) = -\frac{1}{x^2}$

$f(e^2)+f'(e^2)(x-e^2)+\frac{f''(e^2)}{2!}(x-e^2)^2$

$= 2 + e^{-2}(x-e^2) - \frac{e^{-4}}{2}(x-e^2)^2$

$f(e^2)=2$

$f'(e^2)=\frac{1}{e^2}=e^{-2}$

$f''(e^2)=-\frac{1}{e^4}=-e^{-4}.$

6. (2 points) Given function $f(x) = x^{\frac{1}{2}}(x-4), x > 0$. Find all its local maximum values (if any) and minimum values (if any). Be sure to justify your conclusion with **First Derivative Test**. Are they also the global extrema?

$f'(x) = (x^{\frac{1}{2}})'(x-4) + x^{\frac{1}{2}}(x-4)'$

$= \frac{1}{2}x^{-\frac{1}{2}}(x-4) + x^{\frac{1}{2}}(1)$

$= \frac{1}{2}x^{-\frac{1}{2}}[x-4+2\cdot x^{\frac{1}{2}}\cdot x^{\frac{1}{2}}]$

$= \frac{1}{2}x^{-\frac{1}{2}}(3x-4)$, $x>0$.

$f'(x)=0 \Rightarrow x=\frac{4}{3}$

x :	$(0, \frac{4}{3})$	$(\frac{4}{3}, +\infty)$
$f'(x)$:	<0	>0
$f(x)$:	$\searrow$	$\nearrow$

So. $f_{min} = f(\frac{4}{3}) = (\frac{4}{3})^{\frac{1}{2}}(\frac{4}{3}-4)$

It is also the global ~~extrema~~. minimum

There is no global max.

7. **(2 points)** Evaluate the following limit using methods from calculus and algebra, if it exists. **Justify your steps clearly, identifying any indeterminate forms you encounter and any theorems that you use from class and why they apply.**

$$\lim_{x\to 0^+} x^{\sin x} \qquad 0^0$$

$$= \lim_{x\to 0^+} e^{\sin x \cdot \ln x} = e^{\lim_{x\to 0^+} \sin x \cdot \ln x}$$

Now consider

$$\lim_{x\to 0^+} \sin x \ln x \quad \cdots \quad 0\cdot\infty$$

$$= \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{\sin x}} \quad \cdots \quad \frac{\infty}{\infty}$$

$$\stackrel{L'H}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{\cos x}{\sin^2 x}}$$

$$= \lim_{x\to 0^+} -\frac{\sin^2 x}{x\cos x} \quad \cdots \quad \frac{0}{0}$$

$$\stackrel{L'H}{=} \lim_{x\to 0^+} -\frac{2\sin x\cos x}{\cos x - x\sin x}$$

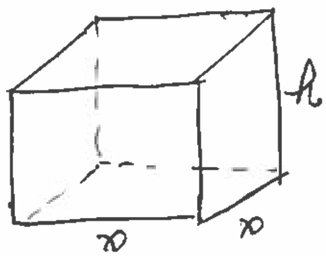
$$= -\frac{2\sin 0\cos 0}{\cos 0 - 0\cdot\sin 0} = 0.$$

So. $\lim_{x\to 0^+} x^{\sin x} = e^{\lim_{x\to 0^+} \sin x\cdot\ln x}$

$= e^0$

$= 1$

8. **(4 points)** A team of ecologists is conducting a research project on breeding robins in captivity. They first must construct suitable cages. They want a square base with a top made of the same mesh as the four walls. Find the minimum cost of building the cage if the cost of each m^2 of the four walls and the top is \$ 5 per m^2 and the total volume must be 100 m^3. **Make a diagram and label it; identify the function you need to optimize; justify, using Calculus, that your answer gives the minimum cost; and give the dimensions of the resulting cage.** Your answer must be legible, orderly, logical and well-justified to receive full marks.



Target: Top 4 walls.

$$P(x,h) = 5\cdot(\underbrace{x^2}_{\text{Top}} + \underbrace{4xh}_{\text{4 walls}})$$

Restriction

$$x^2\cdot h = 100 \Rightarrow h = \frac{100}{x^2}$$

$$P(x,h) = P(x) = 5\cdot\left(x^2 + \frac{400}{x}\right) \quad x \geq 0.$$

$$P'(x) = 5\left(2x - \frac{400}{x^2}\right) = 0$$

$$\Rightarrow 2x^3 = 400 \Rightarrow x = \sqrt[3]{200} \text{ m}$$

or $(200)^{\frac{1}{3}}$ m

x:	$(0, \sqrt[3]{200})$	$(\sqrt[3]{200}, +\infty)$
$f'(x)$:	<0	>0
$f(x)$:	$\searrow$	$\nearrow$

So $P(x)$ has a minimum value of $5\cdot\left[(200)^{\frac{2}{3}} + \frac{400}{(200)^{\frac{1}{3}}}\right]$ dollars

when $x = \sqrt[3]{200}$ m, $y = \frac{100}{(200)^{\frac{2}{3}}}$ m.

Function whose minimal value was obtained (with notation as you have defined clearly above and in your diagram):

$$\boxed{P(x) = 5\left(x^2 + \frac{400}{x}\right)}$$

Width and height of each wall:

$$\boxed{x = \sqrt[3]{200} \text{ m} \qquad h = \frac{100}{(200)^{2/3}} \text{ m.}}$$

9. (9 = 7 × 1+2 points)

The quantity of a drug being absorbed in the bloodstream, measured in mg/L, over time x, measured in hours, is modeled by the function $f(x) = 4xe^{-\frac{x}{2}}$.

Our goal is to produce its graph on the domain of interest $[0, \infty)$.

(a) Evaluate: $f(0) = \boxed{0}$, and $\lim_{x\to\infty} (4xe^{-\frac{x}{2}}) = \boxed{0}$

$\lim_{x\to\infty} \frac{x}{e^{\frac{x}{2}}} \overset{L'H}{=} \lim_{x\to\infty} \frac{1}{\frac{1}{2}e^{\frac{x}{2}}} = 0$

(b) Find the derivative of f **(Show your work.)**:

$f'(x) = \boxed{e^{-\frac{x}{2}}(4-2x)}$

$(4xe^{-\frac{x}{2}})'$
$= 4[e^{-\frac{x}{2}} - xe^{-\frac{x}{2}} \cdot \frac{1}{2}]$
$= e^{-\frac{x}{2}}(4-2x)$

(c) Give the critical point(s) of f (or write "none" if there aren't any). Give your answer(s) in EXACT form.

$x = \boxed{2}$

(d) In the space below, use the information above to make a table that indicates where f is increasing and where it is decreasing on the domain $[0, \infty)$. **Be sure to communicate your methods clearly.**

x	$[0, 2)$	$(2, +\infty)$
$f'(x)$	>0	<0
$f(x)$	↗	↘

(e) Find the second derivative of f: **(Show your work.)**

$f''(x) = \boxed{e^{-\frac{x}{2}}(x-4)}$

$[e^{-\frac{x}{2}}(4-2x)]'$
$= (e^{-\frac{x}{2}})'(4-2x) + e^{-\frac{x}{2}} \cdot (4-2x)'$
$= e^{-\frac{x}{2}} \cdot (-\frac{1}{2})(4-2x) + e^{-\frac{x}{2}} \cdot (-2)$
$= e^{-\frac{x}{2}}(-2+x-2)$

(f) Give the critical point(s) of f' (candidates for inflection points) or write "none" if there aren't any. Give your answer(s) in EXACT form.

$x = \boxed{4}$

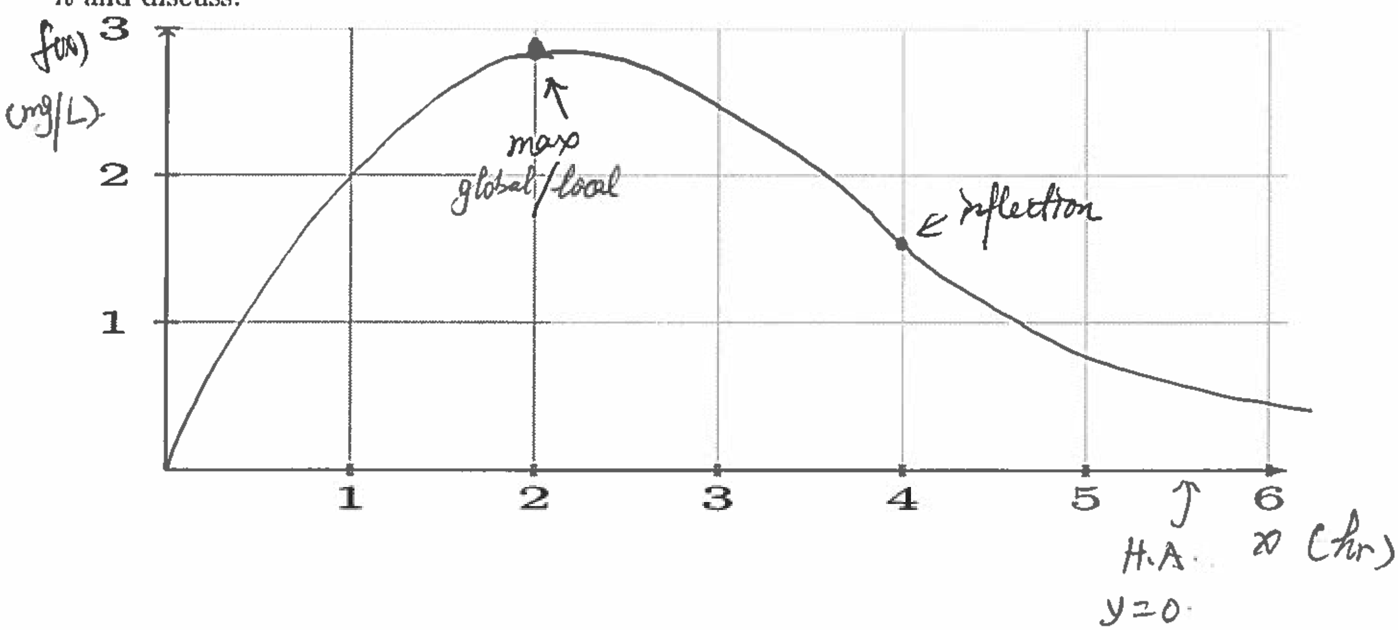
(g) In the space below, use the information above to make a table that indicates where f is concave up and where it is concave down on the domain $[0, \infty)$. **Be sure to communicate your methods clearly.**

x :	$[0,4)$	$(4,+\infty)$
$f''(x)$:	<0	>0
$f(x)$:	concave down	concave up

(h) On the axes below sketch the graph of $y = f(x)$ on the domain $[0, \infty)$ and:

- Plot all points on the graph of $y = f(x)$ corresponding to critical points of f and of f';
- Indicate **all** local and global extrema, and inflection points, on this domain, if any;
- Indicate all horizontal and vertical asymptotes, if any; and
- Label the axes, by name and with units of measurement.

If there is disagreement between your graph and any of your results in (a)-(g) you must identify it and discuss.



C

1. (1 point) Suppose f is a differentiable function such that $f(3) = -2$ and $f'(3) = \frac{-1}{\ln 2}$. If $g = 2^{2f(x)}$, what is the value of $g'(3)$?

A. 2^{-6}

B. $2^{-4}\ln(2)$

C. 2^{-4}

D. 2^{-5}

E. $\frac{-1}{2^3}$

F. There's not enough information to say.

Your answer: E

$(2^{2f(x)})' = 2^{2f(x)} \cdot \ln 2 \cdot (2f(x)')$

$x = 3$

$= 2^{-4} \cdot \ln 2 \cdot 2 \cdot (-\frac{1}{\ln 2})$

$2 \cdot 2^{-3} = -1/2^3$

2. (1 point) If $f(x) = \arctan(x^2)$, then $f'(x)$ is

A. $2x\arctan(x)$

B. $\dfrac{x^2}{1+x^2}$

C. $\dfrac{2x}{1+x^2}$

D. $2x\arctan(x^2) + \dfrac{x^2}{\sqrt{1+x^2}}$

E. $\dfrac{2x}{1+x^4}$

F. $\dfrac{-2x}{1-x^2}$

Your answer: E

$f'(x) = \dfrac{(x^2)'}{1+(x^2)^2} = \dfrac{2x}{1+x^4}$

3. (1 point) Consider the curve implicitly defined by the equation

$$y^3 - y^2 = x^2 + 3.$$

What is the equation of the tangent line to the curve at the point $(1, 2)$?

A. $y = 2$

B. $y = \frac{1}{2}(x^2 + 3)$

C. $y = 2x$

D. $y = \frac{1}{2}x + \frac{2}{3}$

E. $y = -5x + 7$

F. $y = \frac{1}{4}x + \frac{7}{4}$

Your answer: $\boxed{F}$

$$3y^2 \cdot y' - 2y \cdot y' = 2x$$

$$\Rightarrow y' = \frac{2x}{3y^2 - 2y}\Big|_{(1,2)}$$

$$= \frac{2}{12-4} = \frac{1}{4}.$$

4. (1 point) If $f(x) = \sqrt{e^{2x} + x^2}$ then

A. $f'(x) = \dfrac{e^x + x}{2\sqrt{e^{2x} + x^2}}$

B. $f'(x) = e^x + 1$

C. $f'(x) = \dfrac{e^{2x} + x}{2\sqrt{e^{2x} + x^2}}$

D. $f'(x) = \dfrac{1}{2\sqrt{x}}\left(e^{2x} + x^2\right)$

E. $f'(x) = \dfrac{e^{2x} + x}{\sqrt{e^{2x} + x^2}}$

F. $f'(x) = \sqrt{e^{2x} + x^2}\left(e^{2x} + x^2\right)$

Your answer: $\boxed{E}$

$$(\sqrt{e^{2x} + x^2})'$$

$$= \frac{1}{2}(e^{2x} + x^2)^{-\frac{1}{2}} \cdot (e^{2x} + x^2)'$$

$$= \frac{\cancel{2}e^{2x} + \cancel{2}x}{\cancel{2}\sqrt{e^{2x} + x^2}}$$

5. (1 point) What is the degree 2 Taylor polynomial of the function $f(x) = \sqrt[3]{x}$ centered at $x = 1$?

A. $1 + \frac{1}{3}(x-1) + \frac{1}{9}(x-1)^2$ C. $1 + \frac{1}{3}(x-1) - \frac{2}{9}(x-1)^2$ E. $\frac{1}{3}(x-1) - \frac{2}{9}(x-1)^2$

B. $1 + \frac{1}{3}(x-1) - \frac{1}{9}(x-1)^2$ D. $1 - \frac{1}{4}(x-1) - 2(x-1)^2$ F. $1 + \frac{1}{3}(x-1)$

Your answer: $\boxed{B}$

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$ $f''(x) = -\frac{2}{9}x^{-\frac{5}{3}}$

$f(1) = 1$ $f'(1) = \frac{1}{3}$ $f''(1) = -\frac{2}{9}$.

$1 + \frac{1}{3}(x-1) - \frac{2}{9}/2!\,(x-1)^2$

6. (2 points) Given function $f(x) = x^{\frac{1}{4}}(x-4)$, $x > 0$. Find all its local maximum values (if any) and minimum values (if any). Be sure to justify your conclusion with **First Derivative Test.** Are they also the global extrema?

$$f'(x) = (x^{\frac{5}{4}} - 4x^{\frac{1}{4}})'$$
$$= \frac{5}{4}x^{\frac{1}{4}} - 4\cdot\frac{1}{4}\cdot x^{-\frac{3}{4}}$$
$$= \frac{x^{-\frac{3}{4}}}{4}(5x-4).$$

$f'(x) = 0 \Rightarrow x = \frac{4}{5}$

x:	$(0, \frac{4}{5})$	$(\frac{4}{5}, +\infty)$
$f'(x)$:	< 0	> 0
$f(x)$	$\searrow$	$\nearrow$

so $f(x)$ has a local min when $x = \frac{4}{5}$.

the value is $(\frac{4}{5})^{\frac{1}{4}}(\frac{4}{5} - 4)$.

It is also a global min.

7. (2 points) Evaluate the following limit using methods from calculus and algebra, if it exists. **Justify your steps clearly, identifying any indeterminate forms you encounter and any theorems that you use from class and why they apply.**

$$\lim_{x\to\infty} (x)^{\tan(x)}$$

$\infty^{\tan\infty}$ .... NOT the indeterminate form in our lectures!

The limit does not exist!

$$\boxed{\begin{array}{l}\end{array}}$$

- It actually depends on HOW does $x \to \infty$.
- If $x \to \infty$ by this sequence

  $\{ \frac{\pi}{2}^-, \frac{3\pi}{2}^-, \ldots, \ldots \}$.

  $\lim_{x\to\infty} x^{\tan x} = \infty$.

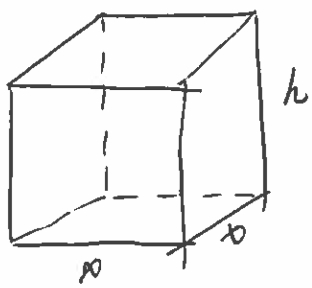
- If $x \to \infty$ by another sequence

  $\{ \pi, 2\pi, 3\pi, \ldots \}$.

  $\lim_{x\to\infty} x^{\tan x} = \lim_{x\to\infty} x^0 = \lim_{x\to\infty} 1 = 1$.

- You can have other values ....

8. (4 points) A team of ecologists is conducting a research project on breeding robins in captivity. They first must construct suitable cages. They want a square base with a top made of the same mesh as the four walls. Find the minimum cost of building the cage if the cost of each m^2 of the four walls and the top is \$ 4 per m^2 and the total volume must be 150 m^3. **Make a diagram and label it; identify the function you need to optimize; justify, using Calculus, that your answer gives the minimum cost; and give the dimensions of the resulting cage.** Your answer must be legible, orderly, logical and well-justified to receive full marks.



Target: top, 4 walls

$$P(x,h) = 4(x^2 + 4xy), \quad x>0$$

Constraints:

$$x^2 h = 150 \Rightarrow h = \frac{150}{x^2}$$

$$P(x,h) = 4\left(x^2 + \frac{600}{x}\right).$$

$$P'(x) = 4\left(2x - \frac{600}{x^2}\right) = 0$$

$$\Rightarrow 2x^3 = 600 \Rightarrow x = \sqrt[3]{300} \text{ m.}$$

x	$(0, \sqrt[3]{300})$	$(\sqrt[3]{300}, +\infty)$
$f'(x)$	<0	>0
$f(x)$	$\searrow$	$\nearrow$

So $f(x)$ has a minimum at $x = \sqrt[3]{300}$ m

the value is $4\left(300^{\frac{2}{3}} + \frac{600}{\sqrt[3]{300}}\right)$ dollars

Function whose minimal value was obtained (with notation as you have defined clearly above and in your diagram):

$$4\left(x^2 + \frac{600}{x}\right).$$

Width and height of each wall:

$x = \sqrt[3]{300}$ m, $\quad h = \frac{150}{(300)^{2/3}}$ m.

9. (9 = 7 × 1+2 points)

The quantity of a drug being absorbed in the bloodstream, measured in mg/L, over time x, measured in hours, is modeled by the function $f(x) = \frac{6 \ln x}{x}$.

$\lim_{x\to\infty} \frac{6 \frac{1}{x}}{1} = \lim_{x\to\infty} \frac{6}{x} = 0$

Our goal is to produce its graph on the domain of interest $(0, \infty)$.

(a) Evaluate: $\lim_{x\to 0^+} f(x) = \boxed{-\infty}$, and $\lim_{x\to\infty} f(x) = \boxed{0}$ ← L'H rule.

(b) Find the derivative of f: **Show your work.**

$6\left(\frac{\ln x}{x}\right)' = 6\,\frac{\frac{1}{x}\cdot x - \ln x}{x^2}$

$= \frac{6(1-\ln x)}{x^2}$

$f'(x) = \boxed{\frac{6(1-\ln x)}{x^2}}$

(c) Give the critical point(s) of f (or write "none" if there aren't any). Give your answer(s) in EXACT form.

$x = \boxed{e,\ 0.}$

(d) In the space below, use the information above to make a table that indicates where f is increasing and where it is decreasing on the domain $(0, \infty)$. **Be sure to communicate your methods clearly.**

x	$(0, e)$	$(e, +\infty)$
$f'(x)$	>0	<0
$f(x)$	$\nearrow$	$\searrow$

(e) Find the second derivative of f: **Show your work.**

$6\left[\frac{1-\ln x}{x^2}\right]'$

$= 6\,\frac{-\frac{1}{x}\cdot x^2 - 2x(1-\ln x)}{x^4}$

$= 6\,\frac{-1-2+2\ln x}{x^3}.$

$f''(x) = \boxed{\frac{6(2\ln x-3)}{x^3}}$

(f) Give the critical point(s) of f' (candidates for inflection points) or write "none" if there aren't any. Give your answer(s) in EXACT form.

$x = \boxed{e^{3/2},\ 0}$

$\ln x = \frac{3}{2}$

$x = e^{3/2}$

(g) In the space below, use the information above to make a table that indicates where f is concave up and where it is concave down on the domain $(0, \infty)$. **Be sure to communicate your methods clearly.**

x :	$(0, e^{3/2})$	$(e^{3/2}, +\infty)$
$f''(x)$:	< 0	> 0
$f(x)$	concave down	concave up.

(h) On the axes below sketch the graph of $y = f(x)$ on the domain $[0, \infty)$ and:

- Plot all points on the graph of $y = f(x)$ corresponding to critical points of f and of f';
- Indicate **all** local and global extrema, and inflection points, on this domain, if any;
- Indicate all horizontal and vertical asymptotes, if any; and
- Label the axes, by name and with units of measurement.

If there is disagreement between your graph and any of your results in (a)-(g) you must identify it and discuss.