CONCORDIA UNIVERSITY **Department of Mathematics and Statistics**

Course	Number		Section
Mathematics	**STAT 249**		**BB**
Examination	Date	Time	Pages
Final	**April 2014**	**3 hours**	**4**
Instructor			Course Examiner
Ewa Duma			**Ewa Duma**

Instructions: Only approved calculators permitted. The value for each problem is indicated in square brackets in the margin (out of a possible total of 100). Show all your steps. Write the complete solution on the right hand pages of your examination booklet only.
A Table of Normal (0,1) distribution is attached at the end of the booklet.

MARKS: marks for each problem are shown in front of the problems.

↓MARKS

[**10**] **Problem 1 :** (a) In the definition of the independence of two events, there are three equalities to check:
$P(A|B) = P(A)$ or $P(B|A) = P(B)$ or $P(B \cap A) = P(A)P(B)$
If any of one of these equalities holds, A and B are independent.
Show that if any of these equalities holds, the other two also hold.

(b) If two events, A and B, are such that: $P(A) = 0.4$, $P(B) = 0.5$, $P(A \cap B) = 0.2$, find $P(A|A \cup B)$ and $P(A \cap B|A \cup B)$.

[**7**] **Problem 2 :** A student answers a multiple-choice examination question that offers four possible answers. The probability that the student knows the answer to the question is 0.7 and the probability that the student will guess is 0.3. Assume that if the student guesses, the probability of selecting the correct answer is 0.25.
If the student correctly answers a question, what is the probability that the student really knew the answer?

[**10**] **Problem 3 :** Five identical bowls are labeled 1, 2, 3, 4 and 5. Bowl i contains i white and $5 - i$ black balls, with $i = 1, 2, ..., 5$. A bowl is randomly chosen and two balls are randomly selected from this bowl without replacement.
(a) What is the probability that both selected balls are white?
(b) Given that both selected balls are white, what is the probability that the third bowl was chosen, i.e. that the balls were selected from the third bowl?

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[**7**] **Problem 4 :** In a gambling game a person draws a single card from an ordinary 52-card playing deck. A person is paid \$15 for drawing a jack or a queen and \$5 for drawing a king or an ace. A person who draws any other card has to pay \$4.

If Mary plays this game, what is her expected gain?

[**13**] **Problem 5 :** The store sold 4 bicycles randomly selected from a large lot that is known to contain 10% defectives.

Let X denote the random number of defectives among the 4 sold.

(a) Compute the probabilities for each possible value of X.

(b) The purchaser of the bicycles will return the defectives for repair and the repair cost is given by: $C = 3X^2 + X + 2$. Find the expected repair cost.

[**10**] **Problem 6 :** Let the probability density function of X be given by:

$$f(x) = \begin{cases} cx^2 + x, & 0 \le x \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Find the constant c.

(b) Find the mean and variance of X.

(c) Use the result from (b) to find the mean and variance of the random cost W where $W = 5 - 0.5X$.

[**10**] **Problem 7 :** (a) Given the moment generating function $M_X(t) = e^{2t+6t^2}$, find the moment generating function of $Z = 2X + 3$.

Use it to determine the mean and variance of Z.

(b) Customers arrive at a bank according to a Poisson distribution at an average of 12 customers per hour.

Find the probability that at least 3 customers arrive in the next 10 minutes.

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[10] **Problem 8 :** The width of bolts of fabric is normally distributed with mean 950 mm and standard deviation 10 mm.

(a) What is the probability that a randomly chosen bolt has a width of between 947 and 958 mm?

(b) What is the appropriate value of C such that a randomly chosen bolt has a width less than C with probability 0.8531?

[13] **Problem 9 :** Suppose that the random variables X and Y have the joint probability density function:

$$f(x,y) = \begin{cases} 6x^2y & 0 \le x \le y,\ x+y \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Find the marginal densities of X and Y.

(b) Find the conditional density of Y given $X = x$.

(c) Find $P(\ Y < 1.1 \mid X = 0.6)$.

[10] **Problem 10 :** The joint distribution of X and Y, $p(x,y)$, is given by:

$p(0,0) = \frac{1}{9}$, $p(1,0) = \frac{2}{9}$, $p(2,0) = \frac{1}{9}$

$p(0,1) = \frac{2}{9}$, $p(1,1) = \frac{2}{9}$, $p(2,1) = 0$

$p(0,2) = \frac{1}{9}$, $p(1,2) = 0$, $p(2,2) = 0$

(a) Find the marginal distribution of X and Y. Are they independent?

(b) Find the conditional distribution of the random variable X knowing that $Y = 1$.

(c) Find $P(X + Y \ge 2 \mid X \le 1)$.

GOOD LUCK !!!

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Table 2.3 Area $\Phi(x)$ under the Standard Normal Curve to the Left of x

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5597	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8557	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

FIGURE 1. standard normal distribution

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