Université d'Ottawa
Faculté de génie

École de science informatique
et de génie électrique

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University of Ottawa
Faculty of Engineering

School of Electrical Engineering
and Computer Science

ELG3106 Applied Electromagnetism **Professor:** K. Hinzer **Date:** 19 October 2018
ELG3506 Électromagnétisme appliqué **Prof. :** K. Hinzer **Date:** 19 Octobre 2018

Instructions

- Allotted time: 80 minutes
- Closed book exam. Useful formulas are listed at the end of the exam.
- Programmable calculators are not allowed.
- Answer all questions. Explain clearly how you got to the final answer. Solutions without adequate explanation will be penalized.
- Your final answers must have units.
- The mark for each question is as indicated.
- You may detach and keep the Equation sheet.
- Do NOT write in the mark boxes.

Instructions

- Temps alloué : 80 minutes
- Examen à livres fermés. Des formules utiles sont fournies à la fin de l'examen.
- Calculatrices pré-programmables non autorisées.
- Répondre à toutes les questions. Expliquer clairement comment vous êtes arrivés à la solution finale. Les solutions sans justification adéquate ne seront pas prises en compte. Ne pas oublier de donner les unités.
- Les valeurs des questions sont telles qu'indiquées.
- Remettre TOUT le matériel à la professeur.

Q1/	**/ 14**
Q2/	**/ 4**
Q3/	**/ 4**
Q4/	**/ 12**
Mark/Note	**/ 34**

Name:

SOLUTIONS

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Student Number:

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<u>Question 1</u> The electric field of an electromagnetic wave traveling through a low loss non-magnetic medium with a conductivity of 0.0025 S/m is given by

Un champ électrique se propage dans un milieu à faible perte non-magnétique ayant une conductivité de 0.0025 S/m

$$\mathbf{E}(x,t) = (3\hat{\mathbf{x}} + 4\hat{\mathbf{y}})e^{-0.1\pi x}\sin\left(8\pi \times 10^9\, t + 20\pi x\right) \text{ V/m.}$$

(a) What is the (peak) amplitude of this field at $x = 0$?
Quelle est l'amplitude (maximale) pour ce champ à $x = 0$?

1|

Solution:

$$E_0 = \sqrt{(3\hat{\mathbf{x}} + 4\hat{\mathbf{y}})\cdot(3\hat{\mathbf{x}} + 4\hat{\mathbf{y}})} = 5 \text{ V}$$

(b) What type of polarization does this field have? What is the polarization direction?
Quelle est la polarisation de l'onde?
Quelle est la direction de polarisation?

1|

Solution:

Linear/linéaire. $\mathbf{a}_p = 3\hat{\mathbf{x}} + 4\hat{\mathbf{y}}$, or as a unit vector: $\hat{\mathbf{a}}_p = 0.6\hat{\mathbf{x}} + 0.8\hat{\mathbf{y}}$

(c) What is the direction of propagation?
Quelle est la direction de propagation?

1|

Solution:

$-\hat{\mathbf{x}}$

(d) What is the phasor representation of this electric field?
Quelle est la représentation phaseur de cette onde?

3|

Solution:

$$\tilde{\mathbf{E}}(x) = (3\hat{\mathbf{x}} + 4\hat{\mathbf{y}}) e^{-0.1\pi x} e^{+j20\pi x} e^{-j\pi/2} \text{ V/m}$$

(e) What is the wavenumber?
Quel est le nombre d'onde?

1|

Solution:

$$k = \beta = 20\pi \text{ rad/m}$$

(f) What is the attenuation constant?
Quelle est la constante d'atténuation?

1|

Solution:

$$\alpha = 0.1\pi \text{ Np/m}$$

(g) What is the frequency, in Hz?
Quelle est la fréquence, en Hz?

1|

Solution:

$$f = \frac{\omega}{2\pi} = \frac{8\pi \times 10^9}{2\pi} = 4 \times 10^9 \text{ Hz}$$

(h) What is the phase velocity?
Quelle est la vitesse de phase?

1|

Solution:

$$u_p = \frac{\omega}{\beta} = \frac{8\pi \times 10^9}{20\pi} = 4 \times 10^8 \text{ m/s}$$

(i) What is the intrinsic impedance of the medium?
Quelle est l'impédance intrinsèque du milieu?

2|

Solution:

For a low loss medium, $\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{\sigma}{2}\eta_c$ therefore $\eta_c = \frac{2\alpha}{\sigma} = 2(0.1\pi)(400) = 80\pi\ \Omega$

Or: $\eta_c = \sqrt{\frac{\mu}{\varepsilon}} = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{\eta_0}{n} = \frac{120\pi}{1.5} = 80\pi$

(j) What is the refractive index of the medium?
Quel est l'indice de réfraction du milieu?

Solution:

2|

For a non-magnetic medium, $\eta_c = \sqrt{\frac{\mu}{\varepsilon}} = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{\eta_0}{n}$, therefore $n = \frac{\eta_0}{\eta_c} = \frac{120\pi}{80\pi} = 1.5$

Or: $u_p = \frac{c}{n}$, so $n = \frac{c}{u_p} = \frac{3\times10^8}{2\times10^8} = 1.5$

Question 2

In free space, your source emits a wave having

Une onde plane uniforme dont le champ électrique est donné par

$$\mathbf{E}(z,t) = \hat{\mathbf{x}} 10 \sin(\omega t - \beta z) \text{ V/m.}$$

Determine $\mathbf{H}(z,t)$.

Déterminez $\mathbf{H}(z,t)$.

Solution:

4|

$$\mathbf{H}(z,t) = \frac{\hat{\mathbf{k}} \times \mathbf{E}}{\eta} = \frac{\hat{\mathbf{z}} \times \hat{\mathbf{x}} 10 \sin(\omega t - \beta z)}{120\pi}$$

$$= \hat{\mathbf{y}}\, 0.0265 \sin(\omega t - \beta z) \text{ A/m}$$

$$= \hat{\mathbf{y}}\, 0.0265 \cos\left(\omega t - \beta z + \pi/2\right) \quad \text{A/m}$$

Sine and cosine answers are acceptable

Question 3

A wave in medium 1, travelling in the +z-direction, is normally incident on medium 2 at z = 0. The media are nonmagnetic. The **total** electric fields in the two media are

Une onde dans le milieu 1, se propage dans la direction +z. Elle rencontre à incidence normale le milieu 2 à z=0. Les milieux sont non-magnétiques. Les champs électriques totaux pour chaque milieu sont

$$\mathbf{E}_1(z,t) = \hat{\mathbf{a}}_x 12\cos\left(2\pi\times 10^9 t - 60z\right) - \hat{\mathbf{a}}_x 3\cos\left(2\pi\times 10^9 t + 60z\right) \text{ V/m},$$
$$\mathbf{E}_2(z,t) = \hat{\mathbf{a}}_x 9\cos(\omega t - kz) \text{ V/m}$$

(a) What is the reflection coefficient?
Quel est le coefficient de réflexion?

2|

Solution:

$$\Gamma = \frac{E_{r0}}{E_{i0}} = -\frac{3}{12} = -\frac{1}{4}$$

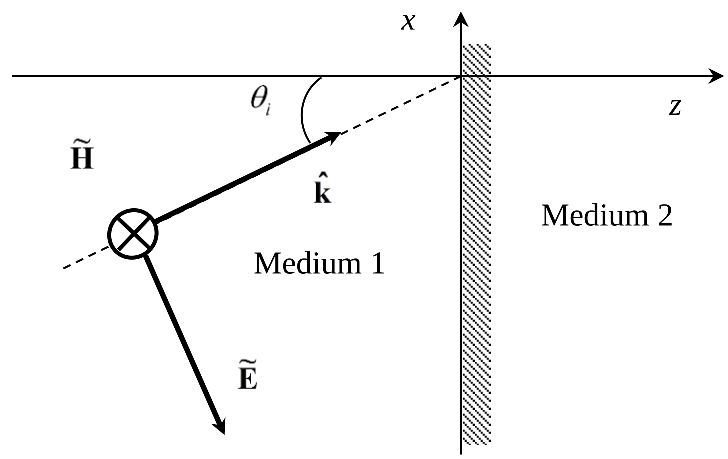
(b) What is the transmission coefficient?
Quel est le coefficient de transmission?

2|

Solution:

$$\tau = \frac{E_{t0}}{E_{i0}} = \frac{9}{12} = \frac{3}{4}$$

Question 4



Consider a uniform plane wave obliquely incident from medium 1 onto medium 2, as shown; the incident angle is θ_i = 30°, and the reflection coefficient is -0.017. The constitutive parameters of medium 1 are ε_r = 4 and μ_r = 16, while those of medium 2 are ε_r = 9 and μ_r = 27; the wavenumber in medium 1 is 4 rad/m. The magnitude of the incident magnetic field is H_{i0} = 0.01 A/m, and its direction is $\hat{\mathbf{a}}_p = -\hat{\mathbf{y}}$ as shown in the figure.

Considérons une onde plane uniforme à incidence oblique se propageant du milieu 1 vers le milieu 2 comme montré; l'angle d'incidence est θ_i = 30°, et le coefficient de réflexion est -0.017. Les paramètres constitutifs du milieu 1 sont ε_r = 4 and μ_r = 16, tandis que ceux du milieu 2 sont ε_r = 9 and μ_r = 27; le nombre d'onde du milieu 1 est 4 rad/m. La magnitude du champ magnétique incident est H_{i0} = 0.01 A/m, et sa direction de propagation est $\hat{\mathbf{a}}_p = -\hat{\mathbf{y}}$ comme dans la figure.

(a) What is the plane of incidence? Is this wave transverse electric or transverse magnetic? Justify your answer.
Quel est le plan d'incidence? Est-ce que l'onde est transverse électrique ou transverse magnétique?

3|

Solution:

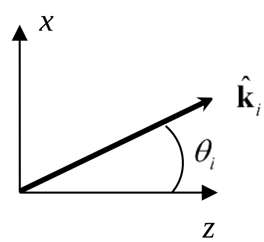
The plane of incidence is the *x*-*z* plane. This is a TM wave, since **H** is normal to the plane of incidence (**E** is in the plane of incidence).

(b) Write down the incident magnetic field in the phasor representation.

4|

Écrivez le champ magnétique incident en représentation phaseur.

Solution:



$$\tilde{\mathbf{H}}_i = H_{i0}\hat{\mathbf{a}}_p e^{-j\mathbf{k}_i\cdot\mathbf{R}}$$

$$H_{i0} = 0.01 \text{ A/m}$$

$$\hat{\mathbf{a}}_p = -\hat{\mathbf{y}}$$

$$\mathbf{k}_1 = k_1\left(\hat{\mathbf{z}}\cos\theta_i + \hat{\mathbf{x}}\sin\theta_i\right) = 4\left(\hat{\mathbf{z}}\cos 30^\circ + \hat{\mathbf{x}}\sin 30^\circ\right)$$

$$= 4\left(\hat{\mathbf{z}}0.866 + \hat{\mathbf{x}}0.5\right) = 3.46\hat{\mathbf{z}} + 2\hat{\mathbf{x}}$$

$$\mathbf{R} = z\hat{\mathbf{z}} + x\hat{\mathbf{x}}$$

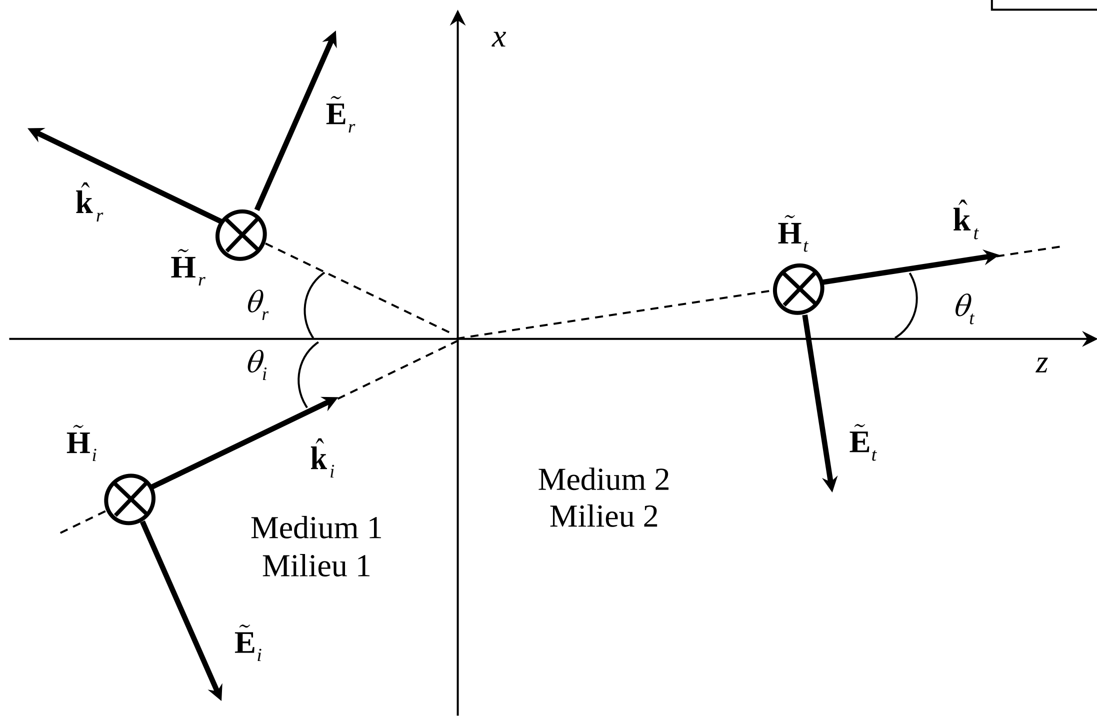
$$\mathbf{k}_1\cdot\mathbf{R} = 3.46z + 2x$$

$$\tilde{\mathbf{H}}_i = H_{i0}\hat{\mathbf{a}}_p e^{-j\mathbf{k}_i\cdot\mathbf{R}} = -0.01\hat{\mathbf{y}}e^{-j(3.46z+2x)} \text{ A/m}$$

(c) Sketch the vector triplets (**E**, **H** et **k**) for the reflected and transmitted fields seen at the interface. Make sure you indicate the actual field and propagation directions, the coordinate system, and the relevant angles.
*Faîtes un croquis des trois vecteurs (***E***,* **H** *et* **k***) pour les champs réfléchis et transmis à l'interface des deux milieux. Assurez-vous d'indiquer chaque champ et sa direction de propagation, le système des coordonées et les angles qui s'appliquent.*

5|

Solution:



Extra page/ *page supplémentaire*

Equation sheet – feuilles d'équation

$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$ $\qquad \nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$ $\qquad \nabla \cdot \mathbf{D} = \rho_v$ $\qquad \nabla \cdot \mathbf{B} = 0$

$\mathbf{D} = \varepsilon \mathbf{E}$ $\qquad \mathbf{B} = \mu \mathbf{H}$ $\qquad \mathbf{J} = \sigma \mathbf{E}$ $\qquad \mathbf{H} = \dfrac{1}{\eta_c} \hat{\mathbf{k}} \times \mathbf{E}$

$\eta_0 = 120\,\pi = 377\ \Omega$ $\qquad \lambda = \dfrac{2\pi}{k}$ $\qquad \omega = 2\pi f$ $\qquad \eta_c = \sqrt{\dfrac{\mu}{\varepsilon_c}}$ $\qquad k = \omega\sqrt{\mu\varepsilon}$

$\varepsilon = \varepsilon_r \varepsilon_0$ $\qquad \varepsilon_c = \varepsilon - \dfrac{j\sigma}{\omega}$ $\qquad \varepsilon_0 = 8.85 \times 10^{-12} \cong \dfrac{1}{36\pi} \times 10^{-9}\ \text{F/m}$ $\qquad \mu_0 = 4\pi \times 10^{-7}\ \text{H/m}$

$\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ $\qquad \Gamma = |\Gamma| e^{j\theta_\Gamma}$ $\qquad \tau = 1 + \Gamma$ $\qquad \dfrac{\sin\theta_t}{\sin\theta_i} = \dfrac{k_1}{k_2}$ $\qquad u_p = \dfrac{c}{n}$ $\qquad c = 3 \times 10^8\ \text{m/s}$

$\Gamma_\perp = \Gamma_{TE} = \dfrac{E_o^r}{E_o^i} = \dfrac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$ $\qquad \tau_\perp = \tau_{TE} = \dfrac{E_o^{tr}}{E_o^i} = \dfrac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$

$\Gamma_{||} = \Gamma_{TM} = \dfrac{E_o^r}{E_o^i} = \dfrac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$ $\qquad \tau_{||} = \tau_{TM} = \dfrac{E_o^{tr}}{E_o^i} = \dfrac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$

$\nabla^2 \mathbf{E} + k_c^2 \mathbf{E} = 0$ $\qquad \nabla^2 \mathbf{H} + k_c^2 \mathbf{H} = 0$

$\mathbf{S} = \mathbf{E} \times \mathbf{H}$ $\qquad \tilde{\mathbf{S}}_{av} = \tfrac{1}{2} \text{Re}[\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^*]\ (\text{W/m}^2)$ $\qquad \tilde{\mathbf{S}}_{av}^i(z) = \hat{\mathbf{z}} \dfrac{|E_{i0}|^2}{2\eta_1^*}$ $\qquad l_{\max} = \dfrac{\theta_r \lambda_1}{4\pi} + n\dfrac{\lambda_1}{2}\ (n = 0, 1, 2 \ldots)$

$\tilde{\mathbf{S}}_{1av}(z) = \hat{\mathbf{z}} \dfrac{|E_{i0}|^2}{2\eta_1^*}\left(1 - |\Gamma|^2\right)$ $\qquad \tilde{\mathbf{S}}_{2av}(z) = \hat{\mathbf{z}} \dfrac{|\tau|^2 |E_{i0}|^2}{2\eta_2^*}$ $\qquad S = \dfrac{|\tilde{\mathbf{E}}_i|_{\max}}{|\tilde{\mathbf{E}}_i|_{\min}} = \dfrac{1 + |\Gamma|}{1 - |\Gamma|}$

For lossless and low loss media $k = \beta$ - *pour milieu sans perte ou peu de perte* $k = \beta$

	General Case *Cas général*	Lossless *Sans perte*	Low loss *Peu de perte*	Good conductor *Bon conducteur*
α (Np/m)	$\omega\left\{\tfrac{1}{2}\mu\varepsilon'\left[\sqrt{1+\left(\dfrac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right]\right\}^{1/2}$	0	$\dfrac{\sigma}{2}\sqrt{\dfrac{\mu}{\varepsilon}}$	$\sqrt{\pi f \mu\sigma}$
β (rad/m)	$\omega\left\{\tfrac{1}{2}\mu\varepsilon'\left[\sqrt{1+\left(\dfrac{\varepsilon''}{\varepsilon'}\right)^2} + 1\right]\right\}^{1/2}$	$\omega\sqrt{\mu\varepsilon}$	$\omega\sqrt{\mu\varepsilon}$	$\sqrt{\pi f \mu\sigma}$
η_c (Ω)	$\sqrt{\dfrac{\mu}{\varepsilon'}}\left(1 - j\dfrac{\varepsilon''}{\varepsilon'}\right)^{-1/2}$	$\sqrt{\dfrac{\mu}{\varepsilon}}$	$\sqrt{\dfrac{\mu}{\varepsilon}}$	$(1+j)\dfrac{\alpha}{\sigma}$
u_p (m/s)	$\dfrac{\omega}{\beta}$	$\dfrac{1}{\sqrt{\mu\varepsilon}}$	$\dfrac{1}{\sqrt{\mu\varepsilon}}$	$\sqrt{\dfrac{4\pi f}{\mu\sigma}}$