Chapter 10: Dynamics of Rotational Motion

10.2. **IDENTIFY:** $\tau = Fl$ with $l = r\sin\phi$. Add the two torques to calculate the net torque.
SET UP: Let counterclockwise torques be positive.
EXECUTE: $\tau_1 = -F_1 l_1 = -(8.00\text{ N})(5.00\text{ m}) = -40.0\text{ N}\cdot\text{m}$.
$\tau_2 = +F_2 l_2 = (12.0\text{ N})(2.00\text{ m})\sin 30.0° = +12.0\text{ N}\cdot\text{m}$. $\Sigma\tau = \tau_1 + \tau_2 = -28.0\text{ N}\cdot\text{m}$. The net torque is $28.0\text{ N}\cdot\text{m}$, clockwise.
EVALUATE: Even though $F_1 < F_2$, the magnitude of τ_1 is greater than the magnitude of τ_2, because F_1

10.9. **IDENTIFY:** Apply $\Sigma\tau_z = I\alpha_z$.

SET UP: $\omega_{0z} = 0$. $\omega_z = (400\text{ rev/min})\left(\dfrac{2\pi\text{ rad/rev}}{60\text{ s/min}}\right) = 41.9\text{ rad/s}$

EXECUTE: $\tau_z = I\alpha_z = I\dfrac{\omega_z - \omega_{0z}}{t} = (1.60\text{ kg}\cdot\text{m}^2)\dfrac{41.9\text{ rad/s}}{8.00\text{ s}} = 8.38\text{ N}\cdot\text{m}$.

EVALUATE: In $\tau_z = I\alpha_z$, α_z must be in rad/s^2.

10.10. **IDENTIFY:** Apply $\Sigma\tau_z = I\alpha_z$ to the wheel. The acceleration a of a point on the cord and the angular acceleration α of the wheel are related by $a = R\alpha$.
SET UP: Let the direction of rotation of the wheel be positive. The wheel has the shape of a disk and $I = \frac{1}{2}MR^2$. The free-body diagram for the wheel is sketched in Figure 10.10a for a horizontal pull and in Figure 10.10b for a vertical pull. P is the pull on the cord and F is the force exerted on the wheel by the axle.

EXECUTE: **(a)** $\alpha_z = \dfrac{\tau_z}{I} = \dfrac{(40.0\text{ N})(0.250\text{ m})}{\frac{1}{2}(9.20\text{ kg})(0.250\text{ m})^2} = 34.8\text{ rad/s}^2$.

$a = R\alpha = (0.250\text{ m})(34.8\text{ rad/s}^2) = 8.70\text{ m/s}^2$.

(b) $F_x = -P$, $F_y = Mg$. $F = \sqrt{P^2 + (Mg)^2} = \sqrt{(40.0\text{ N})^2 + ([9.20\text{ kg}][9.80\text{ m/s}^2])^2} = 98.6\text{ N}$.

$\tan\phi = \dfrac{|F_y|}{|F_x|} = \dfrac{Mg}{P} = \dfrac{(9.20\text{ kg})(9.80\text{ m/s}^2)}{40.0\text{ N}}$ and $\phi = 66.1°$. The force exerted by the axle has magnitude 98.6 N and is directed at $66.1°$ above the horizontal, away from the direction of the pull on the cord.
(c) The pull exerts the same torque as in part (a), so the answers to part (a) don't change. In part (b), $F + P = Mg$ and $F = Mg - P = (9.20\text{ kg})(9.80\text{ m/s}^2) - 40.0\text{ N} = 50.2\text{ N}$. The force exerted by the axle has magnitude 50.2 N and is upward.
EVALUATE: The weight of the wheel and the force exerted by the axle produce no torque because they act at the axle.

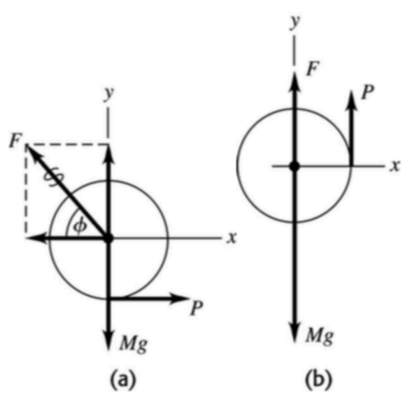


Figure 10.10

10.11. IDENTIFY: Use $\sum\tau_z = I\alpha_z$ to calculate α. Use a constant angular acceleration kinematic equation to relate α_z, ω_z, and t.

SET UP: For a solid uniform sphere and an axis through its center, $I = \frac{2}{5}MR^2$. Let the direction the sphere is spinning be the positive sense of rotation. The moment arm for the friction force is $l = 0.0150\text{ m}$ and the torque due to this force is negative.

EXECUTE: **(a)** $\alpha_z = \frac{\tau_z}{I} = \frac{-(0.0200\text{ N})(0.0150\text{ m})}{\frac{2}{5}(0.225\text{ kg})(0.0150\text{ m})^2} = -14.8\text{ rad/s}^2$

(b) $\omega_z - \omega_{0z} = -22.5\text{ rad/s}$. $\omega_z = \omega_{0z} + \alpha_z t$ gives $t = \frac{\omega_z - \omega_{0z}}{\alpha_z} = \frac{-22.5\text{ rad/s}}{-14.8\text{ rad/s}^2} = 1.52\text{ s}$.

EVALUATE: The fact that α_z is negative means its direction is opposite to the direction of spin. The negative α_z causes ω_z to decrease.

10.13. IDENTIFY: Apply $\sum\vec{F} = m\vec{a}$ to each book and apply $\sum\tau_z = I\alpha_z$ to the pulley. Use a constant acceleration equation to find the common acceleration of the books.

SET UP: $m_1 = 2.00\text{ kg}$, $m_2 = 3.00\text{ kg}$. Let T_1 be the tension in the part of the cord attached to m_1 and T_2 be the tension in the part of the cord attached to m_2. Let the $+x$-direction be in the direction of the acceleration of each book. $a = R\alpha$.

EXECUTE: **(a)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $a_x = \frac{2(x - x_0)}{t^2} = \frac{2(1.20\text{ m})}{(0.800\text{ s})^2} = 3.75\text{ m/s}^2$. $a_1 = 3.75\text{ m/s}^2$ so

$T_1 = m_1 a_1 = 7.50\text{ N}$ and $T_2 = m_2(g - a_1) = 18.2\text{ N}$.

(b) The torque on the pulley is $(T_2 - T_1)R = 0.803\text{ N}\cdot\text{m}$, and the angular acceleration is $\alpha = a_1/R = 50\text{ rad/s}^2$, so $I = \tau/\alpha = 0.016\text{ kg}\cdot\text{m}^2$.

EVALUATE: The tensions in the two parts of the cord must be different, so there will be a net torque on the pulley.

10.15. IDENTIFY: The constant force produces a torque which gives a constant angular acceleration to the wheel.

SET UP: $\omega_z = \omega_{0z} + \alpha_z t$ because the angular acceleration is constant, and $\sum\tau_z = I\alpha_z$ applies to the wheel.

EXECUTE: $\omega_{0z} = 0$ and $\omega_z = 12.0\text{ rev/s} = 75.40\text{ rad/s}$. $\omega_z = \omega_{0z} + \alpha_z t$, so

$\alpha_z = \frac{\omega_z - \omega_{0z}}{t} = \frac{75.40\text{ rad/s}}{2.00\text{ s}} = 37.70\text{ rad/s}^2$. $\sum\tau_z = I\alpha_z$ gives

$I = \frac{Fr}{\alpha_z} = \frac{(80.0\text{ N})(0.120\text{ m})}{37.70\text{ rad/s}^2} = 0.255\text{ kg}\cdot\text{m}^2$.

EVALUATE: The units of the answer are the proper ones for moment of inertia.

10.16. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each box and $\sum \tau_z = I\alpha_z$ to the pulley. The magnitude a of the acceleration of each box is related to the magnitude of the angular acceleration α of the pulley by $a = R\alpha$.

SET UP: The free-body diagrams for each object are shown in Figure 10.16. For the pulley, $R = 0.250$ m and $I = \frac{1}{2}MR^2$. T_1 and T_2 are the tensions in the wire on either side of the pulley. $m_1 = 12.0$ kg, $m_2 = 5.00$ kg and $M = 2.00$ kg. $\vec{F}$ is the force that the axle exerts on the pulley. For the pulley, let clockwise rotation be positive.

EXECUTE: **(a)** $\sum F_x = ma_x$ for the 12.0 kg box gives $T_1 = m_1 a$. $\sum F_y = ma_y$ for the 5.00 kg weight gives $m_2 g - T_2 = m_2 a$. $\sum \tau_z = I\alpha_z$ for the pulley gives $(T_2 - T_1)R = (\frac{1}{2}MR^2)\alpha$. $a = R\alpha$ and $T_2 - T_1 = \frac{1}{2}Ma$.

Adding these three equations gives $m_2 g = (m_1 + m_2 + \frac{1}{2}M)a$ and

$$a = \left(\frac{m_2}{m_1 + m_2 + \frac{1}{2}M}\right)g = \left(\frac{5.00 \text{ kg}}{12.0 \text{ kg} + 5.00 \text{ kg} + 1.00 \text{ kg}}\right)(9.80 \text{ m/s}^2) = 2.72 \text{ m/s}^2. \text{ Then}$$

$T_1 = m_1 a = (12.0 \text{ kg})(2.72 \text{ m/s}^2) = 32.6$ N. $m_2 g - T_2 = m_2 a$ gives

$T_2 = m_2(g - a) = (5.00 \text{ kg})(9.80 \text{ m/s}^2 - 2.72 \text{ m/s}^2) = 35.4$ N. The tension to the left of the pulley is 32.6 N and below the pulley it is 35.4 N.

(b) $a = 2.72 \text{ m/s}^2$

(c) For the pulley, $\sum F_x = ma_x$ gives $F_x = T_1 = 32.6$ N and $\sum F_y = ma_y$ gives

$F_y = Mg + T_2 = (2.00 \text{ kg})(9.80 \text{ m/s}^2) + 35.4 \text{ N} = 55.0$ N.

EVALUATE: The equation $m_2 g = (m_1 + m_2 + \frac{1}{2}M)a$ says that the external force $m_2 g$ must accelerate all three objects.

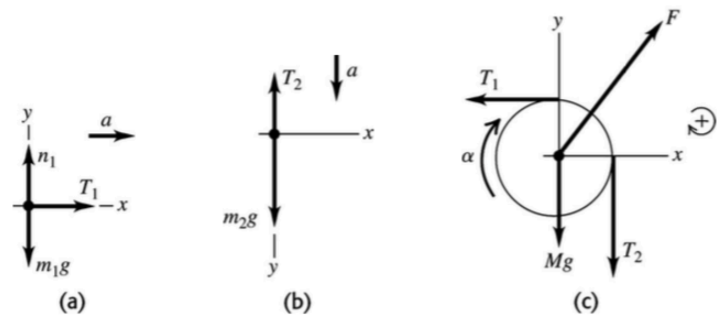
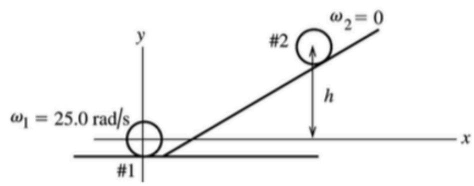


Figure 10.16

10.23. IDENTIFY: Apply conservation of energy to the motion of the wheel.

SET UP: The wheel at points 1 and 2 of its motion is shown in Figure 10.23.



Take $y = 0$ at the center of the wheel when it is at the bottom of the hill.

Figure 10.23

The wheel has both translational and rotational motion so its kinetic energy is $K = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}Mv_{cm}^2$.

EXECUTE: $K_1 + U_1 + W_{other} = K_2 + U_2$

$W_{other} = W_{fric} = -2600$ J (the friction work is negative)

$K_1 = \frac{1}{2}I\omega_1^2 + \frac{1}{2}Mv_1^2$; $v = R\omega$ and $I = 0.800MR^2$ so

$K_1 = \frac{1}{2}(0.800)MR^2\omega_1^2 + \frac{1}{2}MR^2\omega_1^2 = 0.900MR^2\omega_1^2$

$K_2 = 0,\ U_1 = 0,\ U_2 = Mgh$

Thus $0.900MR^2\omega_1^2 + W_{fric} = Mgh$

$M = w/g = 392\ \text{N}/(9.80\ \text{m/s}^2) = 40.0$ kg

$$h = \frac{0.900MR^2\omega_1^2 + W_{fric}}{Mg}$$

$$h = \frac{(0.900)(40.0\ \text{kg})(0.600\ \text{m})^2(25.0\ \text{rad/s})^2 - 2600\ \text{J}}{(40.0\ \text{kg})(9.80\ \text{m/s}^2)} = 14.0\ \text{m}.$$

EVALUATE: Friction does negative work and reduces h.

10.27. IDENTIFY: As the ball rolls up the hill, its kinetic energy (translational and rotational) is transformed into gravitational potential energy. Since there is no slipping, its mechanical energy is conserved.

SET UP: The ball has moment of inertia $I_{cm} = \frac{2}{3}mR^2$. Rolling without slipping means $v_{cm} = R\omega$. Use coordinates where $+y$ is upward and $y = 0$ at the bottom of the hill, so $y_1 = 0$ and $y_2 = h = 5.00$ m. The ball's kinetic energy is $K = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$ and its potential energy is $U = mgh$.

EXECUTE: **(a)** Conservation of energy gives $K_1 + U_1 = K_2 + U_2$. $U_1 = 0$, $K_2 = 0$ (the ball stops).

Therefore $K_1 = U_2$ and $\frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2 = mgh$. $\frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}(\frac{2}{3}mR^2)\left(\frac{v_{cm}}{R}\right)^2 = \frac{1}{3}mv_{cm}^2$, so

$\frac{5}{6}mv_{cm}^2 = mgh$. Therefore $v_{cm} = \sqrt{\frac{6gh}{5}} = \sqrt{\frac{6(9.80\ \text{m/s}^2)(5.00\ \text{m})}{5}} = 7.67$ m/s and

$$\omega = \frac{v_{cm}}{R} = \frac{7.67\ \text{m/s}}{0.113\ \text{m}} = 67.9\ \text{rad/s}.$$

(b) $K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{3}mv_{cm}^2 = \frac{1}{3}(0.426\ \text{kg})(7.67\ \text{m/s})^2 = 8.35$ J.

EVALUATE: Its translational kinetic energy at the base of the hill is $\frac{1}{2}mv_{cm}^2 = \frac{3}{2}K_{rot} = 12.52$ J. Its total kinetic energy is 20.9 J, which equals its final potential energy:

$mgh = (0.426\ \text{kg})(9.80\ \text{m/s}^2)(5.00\ \text{m}) = 20.9$ J.

10.29. **(a) IDENTIFY:** Use $\sum \tau_z = I\alpha_z$ to find α_z and then use a constant angular acceleration equation to find ω_z.

SET UP: The free-body diagram is given in Figure 10.29.

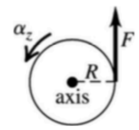


Figure 10.29

EXECUTE: Apply $\sum \tau_z = I\alpha_z$ to find the angular acceleration:

$FR = I\alpha_z$

$$\alpha_z = \frac{FR}{I} = \frac{(18.0\ \text{N})(2.40\ \text{m})}{2100\ \text{kg}\cdot\text{m}^2} = 0.02057\ \text{rad/s}^2$$

SET UP: Use the constant α_z kinematic equations to find ω_z.

$\omega_z = ?$; ω_{0z} (initially at rest); $\alpha_z = 0.02057\ \text{rad/s}^2$; $t = 15.0\ \text{s}$

EXECUTE: $\omega_z = \omega_{0z} + \alpha_z t = 0 + (0.02057\ \text{rad/s}^2)(15.0\ \text{s}) = 0.309\ \text{rad/s}$

(b) IDENTIFY and SET UP: Calculate the work from $W = \tau_z \Delta\theta$, using a constant angular acceleration equation to calculate $\theta - \theta_0$, or use the work-energy theorem. We will do it both ways.

EXECUTE: (1) $W = \tau_z \Delta\theta$

$$\Delta\theta = \theta - \theta_0 = \omega_{0z}t + \tfrac{1}{2}\alpha_z t^2 = 0 + \tfrac{1}{2}(0.02057\ \text{rad/s}^2)(15.0\ \text{s})^2 = 2.314\ \text{rad}$$

$\tau_z = FR = (18.0\ \text{N})(2.40\ \text{m}) = 43.2\ \text{N}\cdot\text{m}$

Then $W = \tau_z \Delta\theta = (43.2\ \text{N}\cdot\text{m})(2.314\ \text{rad}) = 100\ \text{J}$.

or

(2) $W_{\text{tot}} = K_2 - K_1$

$W_{\text{tot}} = W$, the work done by the child

$K_1 = 0$; $K_2 = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(2100\ \text{kg}\cdot\text{m}^2)(0.309\ \text{rad/s})^2 = 100\ \text{J}$

Thus $W = 100\ \text{J}$, the same as before.

EVALUATE: Either method yields the same result for W.

(c) IDENTIFY and SET UP: Use $P_{\text{av}} = \dfrac{\Delta W}{\Delta t}$ to calculate P_{av}.

EXECUTE: $P_{\text{av}} = \dfrac{\Delta W}{\Delta t} = \dfrac{100\ \text{J}}{15.0\ \text{s}} = 6.67\ \text{W}$.

EVALUATE: Work is in joules, power is in watts.

10.31. **IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$ and constant angular acceleration equations to the motion of the wheel.

SET UP: $1\ \text{rev} = 2\pi\ \text{rad}$. $\pi\ \text{rad/s} = 30\ \text{rev/min}$.

EXECUTE: **(a)** $\tau_z = I\alpha_z = I\dfrac{\omega_z - \omega_{0z}}{t}$.

$$\tau_z = \frac{\left((1/2)(2.80\ \text{kg})(0.100\ \text{m})^2\right)(1200\ \text{rev/min})\left(\dfrac{\pi\ \ \text{rad/s}}{30\ \text{rev/min}}\right)}{2.5\ \text{s}} = 0.704\ \text{N}\cdot\text{m}$$

(b) $\omega_{\text{av}}\Delta t = \dfrac{(600\ \text{rev/min})(2.5\ \text{s})}{60\ \text{s/min}} = 25.0\ \text{rev} = 157\ \text{rad}$.

(c) $W = \tau\Delta\theta = (0.704\ \text{N}\cdot\text{m})(157\ \text{rad}) = 111\ \text{J}$.

(d) $K = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left((1/2)(2.80\ \text{kg})(0.100\ \text{m})^2\right)\left((1200\ \text{rev/min})\left(\dfrac{\pi\ \ \text{rad/s}}{30\ \text{rev/min}}\right)\right)^2 = 111\ \text{J}$.

the same as in part (c).

EVALUATE: The agreement between the results of parts (c) and (d) illustrates the work-energy theorem.

10.35. (a) IDENTIFY: Use $L = mvr\sin\phi$.

SET UP: Consider Figure 10.35 (next page).

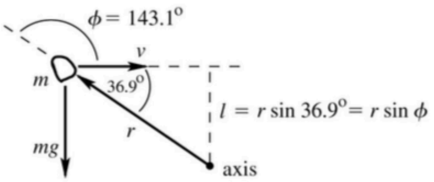


Figure 10.35

EXECUTE: $L = mvr\sin\phi =$
$(2.00\text{ kg})(12.0\text{ m/s})(8.00\text{ m})\sin 143.1°$
$L = 115\text{ kg}\cdot\text{m}^2/\text{s}$

To find the direction of $\vec{L}$ apply the right-hand rule by turning $\vec{r}$ into the direction of $\vec{v}$ by pushing on it with the fingers of your right hand. Your thumb points into the page, in the direction of $\vec{L}$.

(b) IDENTIFY and SET UP: By $\vec{\tau} = \dfrac{d\vec{L}}{dt}$ the rate of change of the angular momentum of the rock equals the torque of the net force acting on it.

EXECUTE: $\tau = mg(8.00\text{ m})\cos 36.9° = 125\text{ kg}\cdot\text{m}^2/\text{s}^2$

To find the direction of $\vec{\tau}$ and hence of $d\vec{L}/dt$, apply the right-hand rule by turning $\vec{r}$ into the direction of the gravity force by pushing on it with the fingers of your right hand. Your thumb points out of the page, in the direction of $d\vec{L}/dt$.

EVALUATE: $\vec{L}$ and $d\vec{L}/dt$ are in opposite directions, so L is decreasing. The gravity force is accelerating the rock downward, toward the axis. Its horizontal velocity is constant but the distance l is decreasing and hence L is decreasing.

10.37. IDENTIFY and SET UP: Use $L = I\omega$.

EXECUTE: The second hand makes 1 revolution in 1 minute, so
$\omega = (1.00\text{ rev/min})(2\pi\text{ rad/1 rev})(1\text{ min/60 s}) = 0.1047\text{ rad/s}$.

For a slender rod, with the axis about one end,
$I = \frac{1}{3}ML^2 = \frac{1}{3}(6.00\times10^{-3}\text{ kg})(0.150\text{ m})^2 = 4.50\times10^{-5}\text{ kg}\cdot\text{m}^2$.

Then $L = I\omega = (4.50\times10^{-5}\text{ kg}\cdot\text{m}^2)(0.1047\text{ rad/s}) = 4.71\times10^{-6}\text{ kg}\cdot\text{m}^2/\text{s}$.

EVALUATE: $\vec{L}$ is clockwise.

10.43. IDENTIFY: Apply conservation of angular momentum to the motion of the skater.

SET UP: For a thin-walled hollow cylinder $I = mR^2$. For a slender rod rotating about an axis through its center, $I = \frac{1}{12}ml^2$.

EXECUTE: $L_i = L_f$ so $I_i\omega_i = I_f\omega_f$.

$I_i = 0.40\text{ kg}\cdot\text{m}^2 + \frac{1}{12}(8.0\text{ kg})(1.8\text{ m})^2 = 2.56\text{ kg}\cdot\text{m}^2$. $I_f = 0.40\text{ kg}\cdot\text{m}^2 + (8.0\text{ kg})(0.25\text{ m})^2 = 0.90\text{ kg}\cdot\text{m}^2$.

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{2.56\text{ kg}\cdot\text{m}^2}{0.90\text{ kg}\cdot\text{m}^2}\right)(0.40\text{ rev/s}) = 1.14\text{ rev/s}.$$

EVALUATE: $K = \frac{1}{2}I\omega^2 = \frac{1}{2}L\omega$. ω increases and L is constant, so K increases. The increase in kinetic energy comes from the work done by the skater when he pulls in his hands.

10.49. IDENTIFY: Apply conservation of angular momentum to the collision.

SET UP: The system before and after the collision is sketched in Figure 10.49. Let counterclockwise rotation be positive. The bar has $I = \frac{1}{3}m_2L^2$.

EXECUTE: **(a)** Conservation of angular momentum: $m_1v_0d = -m_1vd + \frac{1}{3}m_2L^2\omega$.

$$(3.00\text{ kg})(10.0\text{ m/s})(1.50\text{ m}) = -(3.00\text{ kg})(6.00\text{ m/s})(1.50\text{ m}) + \frac{1}{3}\left(\frac{90.0\text{ N}}{9.80\text{ m/s}^2}\right)(2.00\text{ m})^2\omega$$

$\omega = 5.88$ rad/s.

(b) There are no unbalanced torques about the pivot, so angular momentum is conserved. But the pivot exerts an unbalanced horizontal external force on the system, so the linear momentum is not conserved.

EVALUATE: Kinetic energy is not conserved in the collision.

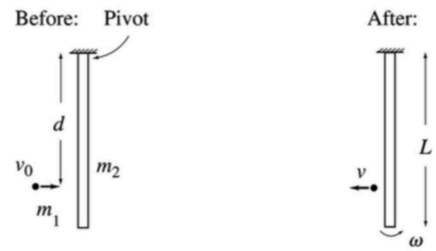


Figure 10.49

10.55. IDENTIFY: Use the kinematic information to solve for the angular acceleration of the grindstone. Assume that the grindstone is rotating counterclockwise and let that be the positive sense of rotation. Then apply $\sum\tau_z = I\alpha_z$ to calculate the friction force and use $f_k = \mu_k n$ to calculate μ_k.

SET UP: $\omega_{0z} = 850\text{ rev/min}(2\pi\text{ rad/1 rev})(1\text{ min/60 s}) = 89.0\text{ rad/s}$

$t = 7.50$ s; $\omega_z = 0$ (comes to rest); $\alpha_z = ?$

EXECUTE: $\omega_z = \omega_{0z} + \alpha_z t$

$$\alpha_z = \frac{0 - 89.0\text{ rad/s}}{7.50\text{ s}} = -11.9\text{ rad/s}^2$$

SET UP: Apply $\sum\tau_z = I\alpha_z$ to the grindstone. The free-body diagram is given in Figure 10.55.

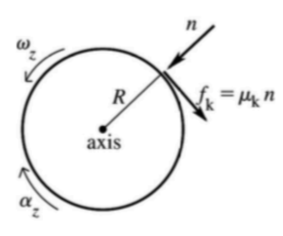


Figure 10.55

The normal force has zero moment arm for rotation about an axis at the center of the grindstone, and therefore zero torque. The only torque on the grindstone is that due to the friction force f_k exerted by the ax; for this force the moment arm is $l = R$ and the torque is negative.

EXECUTE: $\sum\tau_z = -f_kR = -\mu_k nR$

$I = \frac{1}{2}MR^2$ (solid disk, axis through center)

Thus $\sum\tau_z = I\alpha_z$ gives $-\mu_k nR = (\frac{1}{2}MR^2)\alpha_z$

$$\mu_k = -\frac{MR\alpha_z}{2n} = -\frac{(50.0\text{ kg})(0.260\text{ m})(-11.9\text{ rad/s}^2)}{2(160\text{ N})} = 0.483$$

EVALUATE: The friction torque is clockwise and slows down the counterclockwise rotation of the grindstone.

10.65. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to each object and apply $\sum \tau_z = I\alpha_z$ to the pulley.

SET UP: Call the 75.0 N weight A and the 125 N weight B. Let T_A and T_B be the tensions in the cord to the left and to the right of the pulley. For the pulley, $I = \frac{1}{2}MR^2$, where $Mg = 80.0\text{ N}$ and $R = 0.300\text{ m}$. The 125 N weight accelerates downward with acceleration a, the 75.0 N weight accelerates upward with acceleration a and the pulley rotates clockwise with angular acceleration α, where $a = R\alpha$.

EXECUTE: $\sum \vec{F} = m\vec{a}$ applied to the 75.0 N weight gives $T_A - w_A = m_A a$. $\sum \vec{F} = m\vec{a}$ applied to the 125.0 N weight gives $w_B - T_B = m_B a$. $\sum \tau_z = I\alpha_z$ applied to the pulley gives $(T_B - T_A)R = (\frac{1}{2}MR^2)\alpha_z$ and $T_B - T_A = \frac{1}{2}Ma$. Combining these three equations gives $w_B - w_A = (m_A + m_B + M/2)a$ and

$$a = \left(\frac{w_B - w_A}{w_A + w_B + w_{\text{pulley}}/2}\right)g = \left(\frac{125\text{ N} - 75.0\text{ N}}{75.0\text{ N} + 125\text{ N} + 40.0\text{ N}}\right)g = 0.2083g.$$

$T_A = w_A(1 + a/g) = 1.2083 w_A = 90.62\text{ N}$. $T_B = w_B(1 - a/g) = 0.792 w_B = 98.96\text{ N}$. $\sum \vec{F} = m\vec{a}$ applied to the pulley gives that the force F applied by the hook to the pulley is $F = T_A + T_B + w_{\text{pulley}} = 270\text{ N}$. The force the ceiling applies to the hook is 270 N.

EVALUATE: The force the hook exerts on the pulley is less than the total weight of the system, since the net effect of the motion of the system is a downward acceleration of mass.

10.68. IDENTIFY: Apply conservation of energy to the motion of the shell, to find its linear speed v at points A and B. Apply $\sum \vec{F} = m\vec{a}$ to the circular motion of the shell in the circular part of the track to find the normal force exerted by the track at each point. Since $r << R$ the shell can be treated as a point mass moving in a circle of radius R when applying $\sum \vec{F} = m\vec{a}$. But as the shell rolls along the track, it has both translational and rotational kinetic energy.

SET UP: $K_1 + U_1 = K_2 + U_2$. Let 1 be at the starting point and take $y = 0$ to be at the bottom of the track, so $y_1 = h_0$. $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. $I = \frac{2}{3}mr^2$ and $\omega = v/r$, so $K = \frac{5}{6}mv^2$. During the circular motion, $a_{\text{rad}} = v^2/R$.

EXECUTE: (a) $\sum \vec{F} = m\vec{a}$ at point A gives $n + mg = m\dfrac{v^2}{R}$. The minimum speed for the shell not to fall off the track is when $n \to 0$ and $v^2 = gR$. Let point 2 be A, so $y_2 = 2R$ and $v_2^2 = gR$. Then $K_1 + U_1 = K_2 + U_2$ gives $mgh_0 = 2mgR + \frac{5}{6}m(gR)$. $h_0 = (2 + \frac{5}{6})R = \frac{17}{6}R$.

(b) Let point 2 be B, so $y_2 = R$. Then $K_1 + U_1 = K_2 + U_2$ gives $mgh_0 = mgR + \frac{5}{6}mv_2^2$. With $h = \frac{17}{6}R$ this gives $v^2 = \frac{11}{5}gR$. Then $\sum \vec{F} = m\vec{a}$ at B gives $n = m\dfrac{v^2}{R} = \frac{11}{5}mg$.

(c) Now $K = \frac{1}{2}mv^2$ instead of $\frac{5}{6}mv^2$. The shell would be moving faster at A than with friction and would still make the complete loop.

(d) In part (c): $mgh_0 = mg(2R) + \frac{1}{2}mv^2$. $h_0 = \frac{17}{6}R$ gives $v^2 = \frac{5}{3}gR$. $\sum \vec{F} = m\vec{a}$ at point A gives $mg + n = m\dfrac{v^2}{R}$ and $n = m\left(\dfrac{v^2}{R} - g\right) = \frac{2}{3}mg$. In part (a), $n = 0$, since at this point gravity alone supplies the net downward force that is required for the circular motion.

EVALUATE: The normal force at A is greater when friction is absent because the speed of the shell at A is greater when friction is absent than when there is rolling without slipping.