Assignment 4 - Solutions

(Due: 19 November, 2018 by 17:00)

Q1. A bed rise is to be constructed on the bed of a 5.0 m-wide rectangular channel conveying a discharge of 25 m^3/s. The channel has a slope of 0.004 and the Manning's n = 0.014. Assume α = 1.2.

a). Determine the depth of uniform flow and the critical depth in the main channel. What is the flow regime upstream of the bed rise?

b). Determine the minimum height of the bed rise to just cause the flow over the rise to become critical (without choking). **Sketch the water surface profile.**

c). If the height of the bed rise found in b) is increased by 10 cm, what will be the water depth just upstream of the bed rise? **Sketch the new water surface profile.**

Solution :

a) Calculate the depth of uniform flow (i.e., the normal depth) in the main channel using Manning's eq:

$$Q = \frac{1}{n} \times \frac{A^{5/3}}{P^{2/3}} \times S_o^{1/2}$$

$$A = b y_n = 5 y_n$$

$$P = b + 2y_n = 5 + 2y_n$$

$n = 0.014$; $S_o = 0.004$; $Q = 25 \, m^3/s$

$$25.0 = \frac{1}{(0.014)} \times \frac{(5y_n)^{5/3}}{(5+2y_n)^{2/3}} \times (0.004)^{1/2}$$

Solve for y_n by trial & error:

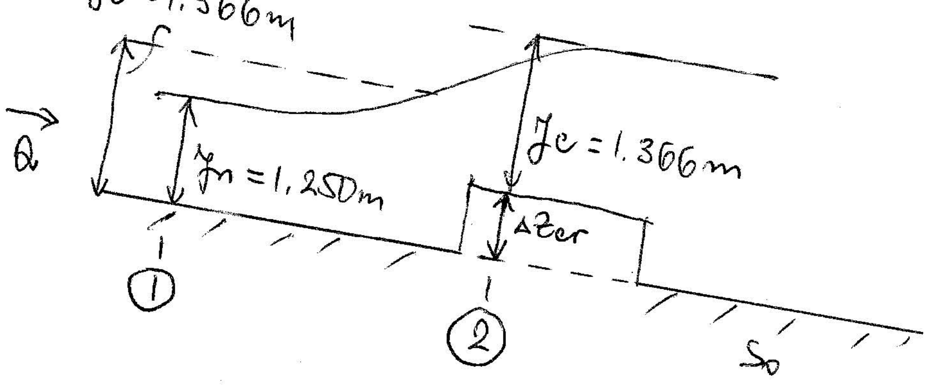
$\underline{y_n = 1.250 \text{ m}}$

$$q = \frac{Q}{b} = \frac{25.0}{5.0} = 5.0 \, m^3/s/m$$

$$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{(5.0)^2}{(9.81)}} = \underline{1.366 \text{ m}}$$

As $y_n = 1.250 m < y_c = 1.366 m$, the flow is <u>supercritical!</u>

b)



$E_{S1} = E_{SC_2} + \Delta z_{cr}$ (no choking!)

$E_{S1} = y_1 + \alpha \dfrac{q^2}{2g y_1^2}$; where $y_1 = y_n = 1.250\,m$, $\alpha = 1.2$

$E_{S1} = 1.250 + (1.2)\dfrac{(5.0)^2}{(2)(9.81)(1.250)^2} = 2.23\,m$

$E_{SC_2} = \dfrac{3}{2} y_c = \left(\dfrac{3}{2}\right)(1.366) = 2.05\,m$

$2.23 = 2.05 + \Delta z_{cr}$

$\Delta z_{cr} = \underline{0.18\,m}$

c) $\Delta z_{new} = \Delta z_{cr} + 0.10 = 0.18 + 0.10 = 0.28\,m$

$E_{S1,new} = E_{SC_2} + \Delta z_{new} = 2.05 + 0.28 = 2.33\,m$

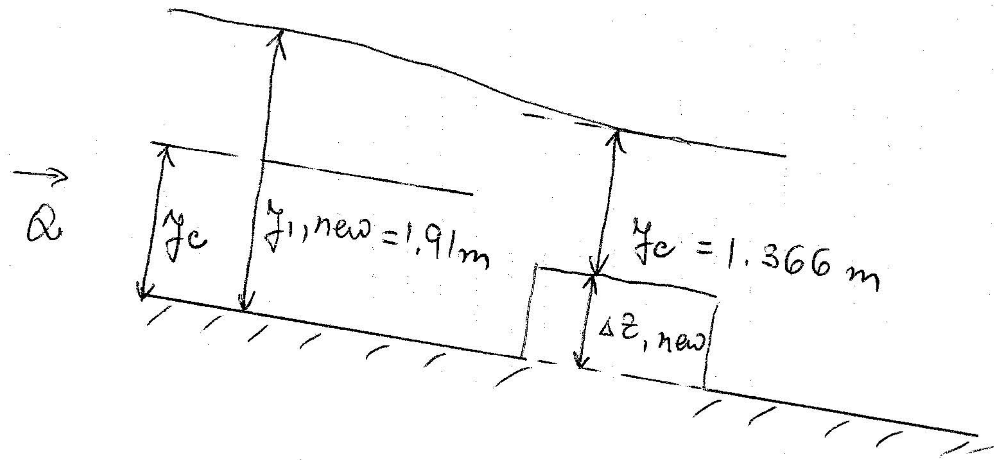
$E_{S1,new} = y_{1,new} + \alpha \dfrac{q^2}{2g\, y_{1,new}^2}$

$2.33 = y_{1,new} + (1.2)\dfrac{(5.0)^2}{(2)(9.81)\, y_{1,new}^2}$

Solve for $y_{1,new} > y_c = 1.366\,m$ by trial & error:

$y_{1,new} = 1.91\,m$

Important: as a result of choking the flow regime upstream of the bed rise changes from supercritical to subcritical !



Q2. Water flows in a rectangular channel of width 3 m and a mild slope. The depth of uniform flow in the channel is 2 m. A sluice gate is inserted in the channel as illustrated in Figure 1. The opening of the gate is set at 1 m, the coefficient of contraction is 0.6 and the depth of flow just upstream of the sluice gate is measured to be 4.78 m.

a). Assuming no energy losses through the gate and $\alpha = 1.2$, determine the flowrate in the channel.

b). Verify that a hydraulic jump will occur downstream of the gate with a sequent depth equal to y_n and determine the initial depth of the jump;

c). If the gate is raised to give an opening of 1.6 m, determine whether or not a hydraulic jump will form downstream;

d). For the scenario described in c), calculate the depths immediately upstream of the gate and at the position of the vena contracta.

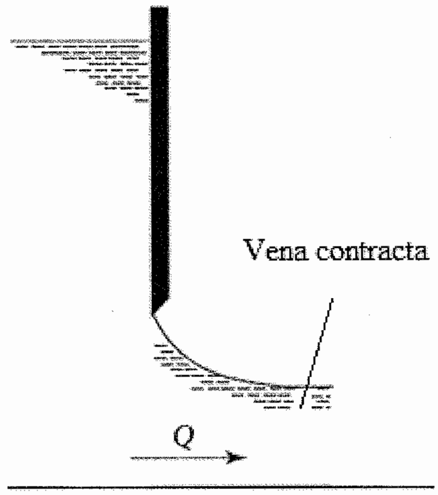
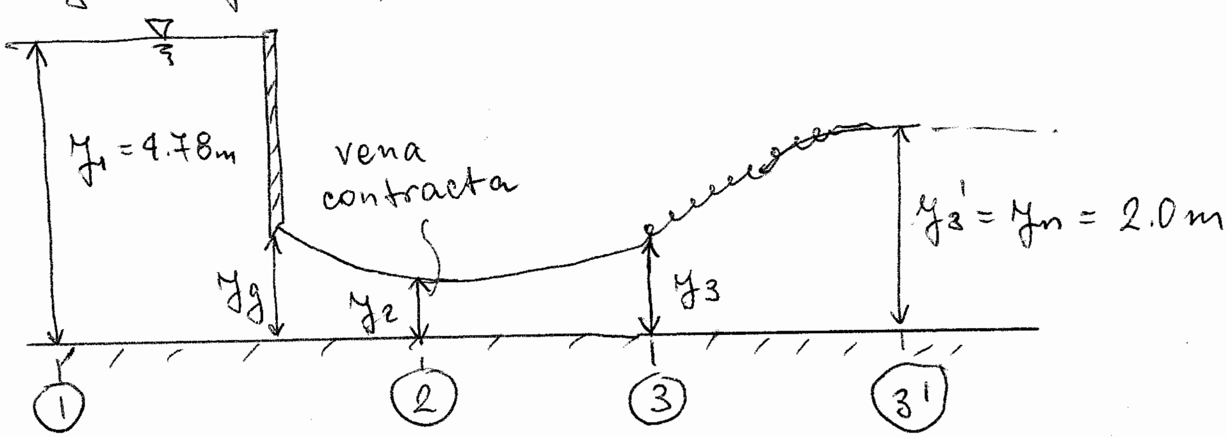


Figure 1

Solution :

Diagram for a) and b).



a) $y_2 = y_g C_c$ where $y_g = 1.0\,m$, $C_c = 0.6$

$y_2 = (1.0)(0.6) = \underline{0.6\,m}$

Specific energy eq. between ① and ②:

$$y_1 + \alpha \frac{V_1^2}{2g} = y_2 + \alpha \frac{V_2^2}{2g}$$ (no energy losses at gate)

$\alpha = 1.2$; $y_1 = 4.78\,m$; $b = 3.0\,m$

$$V_1 = \frac{Q}{A_1} = \frac{Q}{by_1} = \frac{Q}{3y_1} = \frac{Q}{(3)(4.78)} = \frac{Q}{14.34}$$

$$V_2 = \frac{Q}{A_2} = \frac{Q}{by_2} = \frac{Q}{(3.0)(0.6)} = \frac{Q}{1.8}$$

$$4.78 + \frac{(1.2)(Q)^2}{(2)(9.81)(14.34)^2} = 0.6 + \frac{(1.2)(Q)^2}{(2)(9.81)(1.8)^2}$$

$\underline{Q = 15.00\,m^3/s}$

b) Hydraulic jump with $y_3' = y_n = 2.0\,m$

$$V_2 = \frac{Q}{A_2} = \frac{(15.00)}{(3.0)(0.6)} = 8.333\,m/s$$

$$Fr_2 = \frac{V_2}{\sqrt{g y_2}} = \frac{8.333}{\sqrt{(9.81)(0.6)}} = 3.43 > 1.0 \Rightarrow$$ supercritical flow under the gate!

$$V_3' = \frac{Q}{A_3'} = \frac{15.00}{(3.0)(2.0)} = 2.500\,m/s$$

$$Fr_3' = \frac{V_3'}{\sqrt{g y_3'}} = \frac{2.500}{\sqrt{(9.81)(2.0)}} = 0.56 < 1.0 \Rightarrow$$ subcritical flow at $y_3' = y_n = 2.0\,m$!

The transition from supercritical flow under the gate to subcritical flow at normal depth downstream of the gate can only happen through a hydraulic jump!
Sequent depth:

$$y_3 = \left(\frac{y_{3'}}{2}\right)\left(\sqrt{1+8Fr_{3'}^2}-1\right) = \left(\frac{2.0}{2}\right)\left(\sqrt{1+(8)(0.56)^2}-1\right) = 0.87m$$

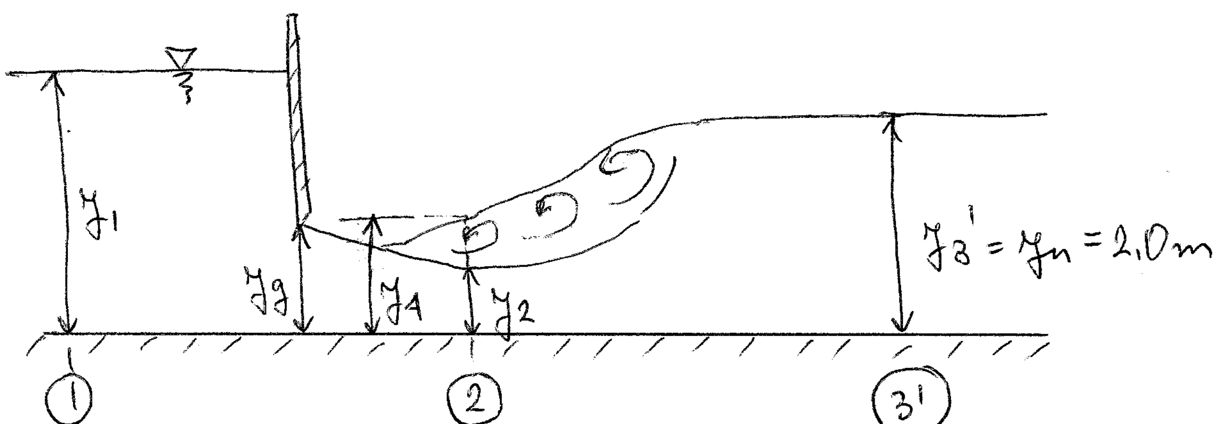
As $y_3 = 0.87m > y_2 = 0.6m$, hydraulic jump will form downstream of vena contracta!

c) $y_g = 1.6m$ $\quad y_2 = y_g C_c = (1.6)(0.6) = 0.96m$

$$V_2 = \frac{Q}{A_2} = \frac{(15.00)}{(3)(0.96)} = 5.208 \ m/s$$

$$Fr_2 = \frac{V_2}{\sqrt{g y_2}} = \frac{5.208}{\sqrt{(9.81)(0.96)}} = 1.70 > 1.0 \rightarrow \text{supercritical flow under the gate!}$$

Hydraulic jump will still form downstream of vena contracta.
As $y_2 = 0.96m > y_3 = 0.87m$, the jump will be <u>submerged!</u>



d) Apply the momentum equation between (2) and (3'):

$$\frac{y_4^2}{2} + \frac{Q^2}{g b^2 y_2} = \frac{(y_3')^2}{2} + \frac{Q^2}{g b^2 y_3'}$$

$$\frac{y_4^2}{2} + \frac{(15.00)^2}{(9.81)(3.0)^2(0.96)} = \frac{(2.0)^2}{2} + \frac{(15.00)^2}{(9.81)(3.0)^2(2.0)}$$

$$y_4 = \underline{1.11\ m}$$

Apply the energy equation between (1) and (2):

$$y_1 + \alpha \frac{Q^2}{2 g b^2 y_1^2} = y_4 + \alpha \frac{Q^2}{2 g b^2 y_2^2}$$

$$y_1 + \frac{(1.2)(15.00)^2}{(2)(9.81)(3.0)^2(y_1)^2} = 1.11 + \frac{(1.2)(15.00)^2}{(2)(9.81)(3.0)^2(0.96)^2}$$

Solve for y_1 by trial & error:

$$y_1 = \underline{2.53\ m}$$

Q3. Consider the channel and the sluice gate in Question 2 with a gate opening of 1 m, a Manning's roughness coefficient $n = 0.014$ and a bed slope of 0.0015. Determine the **distance downstream from vena contracta** at which a hydraulic jump will form.

a). Use the direct step method with $\Delta y = 0.03$ m to determine the water surface profile downstream of vena contracta. Show **sample calculations**.

b). **Plot and label** the water surface profile outlined in a) and **show channel bed, and the critical and normal depth lines on the same plot**.

Solution:

a) $y_n = 2.0$ m from Q2.

Find critical depth in the channel:

$$q = \frac{Q}{b} = \frac{15.00}{3.0} = 5.00 \ m^3/s/m$$

$$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{(5.00)^2}{(9.81)}} = 1.366 \ m$$

As $y_n = 2.00m > y_c = 1.366m$, the channel is Mild!

The depth at vena contracta, $y_2 = 0.6m$ is less than both y_n and y_c. Therefore, we have Region 3 and M-3 type curve. The water depth increases in the direction of flow downstream until it reaches y_3, at which point a hydraulic jump will form. The profile is found by starting at vena contracta ($y_2 = 0.6m$) and proceeding downstream at small intervals ($\Delta y = 0.03m$) until we reach $y_3 = 0.87m$. Fr, S_o, S_f are evaluated at each intermediate depth:

Step 1: Sample calculations:

$y_1 = 0.6m \qquad \Delta y = 0.03m$ (given)

$y_2 = 0.6 + 0.03 = 0.63 \ m$

$A_1 = by = (3)(0.6) = 1.8 \, m^2$

$P_1 = b + 2y_1 = 3 + (2)(0.6) = 4.2 \, m$

$$Fr_1^2 = \frac{q^2}{g y_1^3} = \frac{(5.00)^2}{(9.81)(0.6)^3} = 11.7982$$

$1 - Fr_1^2 = 1 - 11.7982 = -10.7982$

$$Sf_1 = \left(\frac{n Q P_1^{2/3}}{A_1^{5/3}}\right)^2 = \left[\frac{(0.014)(15.00)(4.2)^{2/3}}{(1.8)^{5/3}}\right]^2 = 0.0421$$

$S_0 - Sf_1 = 0.0015 - 0.0421 = -0.0406$

$A_2 = b y_2 = (3)(0.63) = 1.89 \, m^2$

$P_2 = b + 2y_2 = 3 + (2)(0.63) = 4.26 \, m$

$$Fr_2^2 = \frac{q^2}{g y_2^3} = \frac{(5.00)^2}{(9.81)(0.63)^3} = 10.1918$$

$1 - Fr_2^2 = 1 - 10.1918 = -9.1918$

$$Sf_2 = \left(\frac{n Q P_2^{2/3}}{A_2^{5/3}}\right)^2 = \left[\frac{(0.014)(15.00)(4.26)^{2/3}}{(1.89)^{5/3}}\right]^2 = 0.0365$$

$S_0 - Sf_2 = 0.0015 - 0.0365 = -0.0350$

$$(S_0 - Sf)_{mean} = \frac{(S_0 - Sf_1) + (S_0 - Sf_2)}{2} = \frac{(-0.0406) + (-0.0350)}{2} =$$

$$= -0.0378$$

$$(1 - Fr^2)_{mean} = \frac{(1 - Fr_1^2) + (1 - Fr_2^2)}{2} = \frac{(-10.7982) + (-9.1918)}{2} =$$

$$= -9.9950$$

$$\Delta x_1 = \Delta y \left(\frac{1 - Fr^2}{S_0 - Sf}\right)_{mean} = (0.03)\left(\frac{-9.9950}{-0.0378}\right) = \underline{7.93 \, m}$$

Continue downstream until $y = y_3 = 0.87 m$

The distance from vena contracta (control point) at which a hydraulic jump will form is

$$\underline{x = 66.84 \text{ m}}$$

See table below:

Delineating water surface profile - M-3 curve

y (m)	A (m^2)	P (m)	Fr^2 (-)	$1-Fr^2$ (-)	$1-Fr^2$ (mean)	S_f (m/m)	S_0-S_f (m/m)	S_0-S_f (mean)	x (m)	Bed (m)	y_c (m)	y_n (m)	y (vs bed) (m)
0.600	1.80	4.20	11.7982	-10.798		4.21E-02	-4.06E-02		0.00	0.00	1.37	2.00	0.60
0.630	1.89	4.26	10.1918	-9.192	-9.995	3.65E-02	-3.50E-02	-3.78E-02	7.93	-0.01	1.35	1.99	0.62
0.660	1.98	4.32	8.8642	-7.864	-8.528	3.18E-02	-3.03E-02	-3.27E-02	15.77	-0.02	1.34	1.98	0.64
0.690	2.07	4.38	7.7575	-6.758	-7.311	2.80E-02	-2.65E-02	-2.84E-02	23.49	-0.04	1.33	1.96	0.65
0.720	2.16	4.44	6.8277	-5.828	-6.293	2.47E-02	-2.32E-02	-2.48E-02	31.09	-0.05	1.32	1.95	0.67
0.750	2.25	4.50	6.0407	-5.041	-5.434	2.20E-02	-2.05E-02	-2.18E-02	38.56	-0.06	1.31	1.94	0.69
0.780	2.34	4.56	5.3702	-4.370	-4.705	1.96E-02	-1.81E-02	-1.93E-02	45.88	-0.07	1.30	1.93	0.71
0.810	2.43	4.62	4.7953	-3.795	-4.083	1.76E-02	-1.61E-02	-1.71E-02	53.05	-0.08	1.29	1.92	0.73
0.840	2.52	4.68	4.2997	-3.300	-3.547	1.59E-02	-1.44E-02	-1.52E-02	60.04	-0.09	1.28	1.91	0.75
0.870	2.61	4.74	3.8700	-2.870	-3.085	1.43E-02	-1.28E-02	-1.36E-02	66.84	-0.10	1.27	1.90	0.77

Q = 15.00 m^3/s
S_0 = 0.0015 m/m
n = 0.014
b = 3.00 m
y_c = 1.366 m
y_n = 2.000 m

b) See plot below: