Assignment 3 - Solutions

(Due: 29 October, 2018 by 17:00)

Q1. Water at 20°C is pumped from reservoir 1 to reservoir 2 through a 0.95 m-diameter copper pipe 1300 m in length (including the suction line), fitted with three regular 90° flanged elbows in order to change the direction of the flow, as illustrated in Figure 1. The pump is located 10 m above the water surface elevation of reservoir 1 and the suction line is 100 m long. Assume a friction factor, λ = 0.15; a minor head loss coefficient due to an elbow, k_L = 0.3; a minor head loss coefficient due to a reentrant pipe entrance, k_L = 1.0; and a minor head loss coefficient due to a reentrant pipe exit, k_L = 1.0. The hydraulic power delivered to the flow by the pump is 500 kW.

a). Determine the **maximum allowable velocity and discharge** in the pipe without causing cavitation at the suction side, *s* of the pump.

b). For the maximum allowable discharge, determine the corresponding **maximum allowable head** delivered to the flow by the pump.

c). If the maximum allowable head is delivered by the pump, determine the magnitude of the **pressure head registered at the suction and discharge sides** of the pump.

d). Determine the **maximum allowable water surface elevation** of reservoir 2 to which the flow may be pumped.

Assume: P_{atm} = 101.33 kN/m^2, P_{vap} = 2.34 kN/m^2

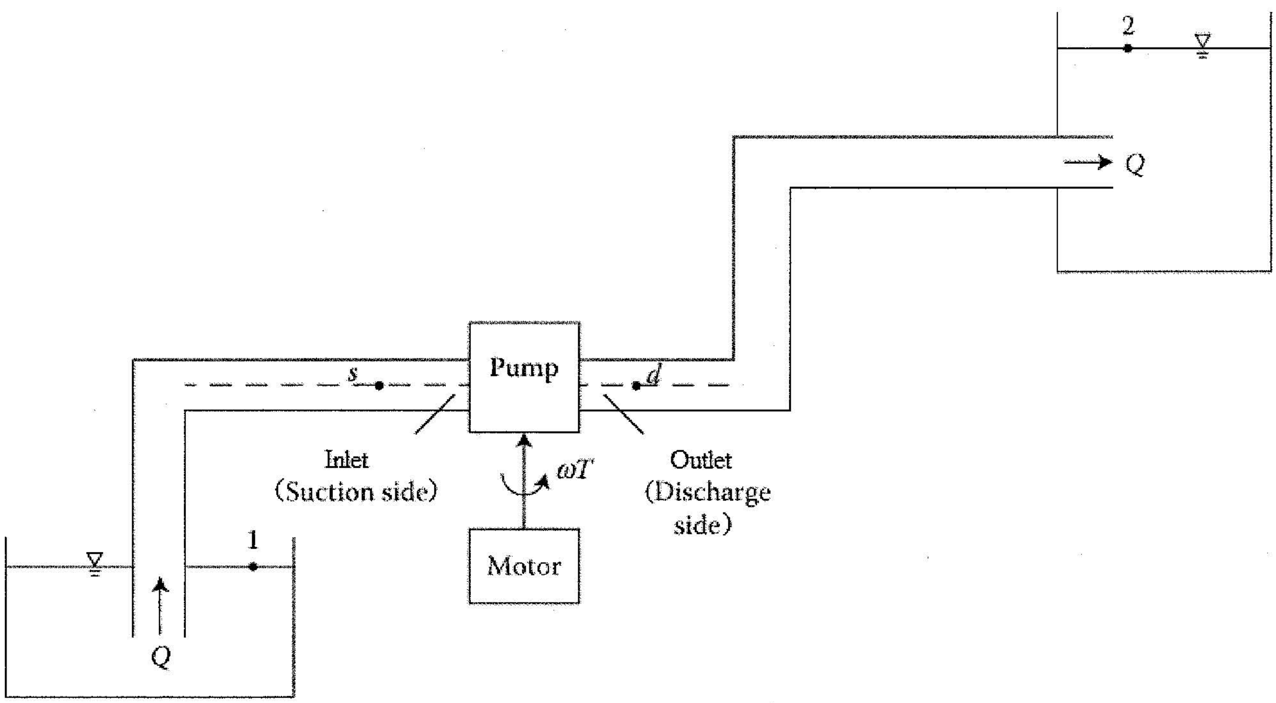


Figure 1

Solution:

a) Set datum at the level of point 1 (lower reservoir) and apply Bernoulli's eq. between points 1 and s:

$$\frac{p_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{p_s}{\rho g} + z_s + \frac{V_s^2}{2g} + h_{f,s} + h_{L,s}$$, where

$p_1 = p_{atm.} = 101.33\ kN/m^2$ (absolute)

$p_s = p_{vap.} = 2.34\ kN/m^2$ (absolute)

$z_1 = 0$; $V_1 = 0$; $z_s = 10m$; $L_s = 100\ m$

The frictional head losses on the suction line are:

$$h_{f,s} = \frac{\lambda L_s V_s^2}{2gD} = \frac{(0.15)(100)(V_s^2)}{(2)(9.81)(0.95)} = 0.80 V_s^2$$

The minor (local) head losses on the suction line:

$$h_{L,s} = \Sigma k_L \frac{V_s^2}{2g} = (1.0 + 0.3)\frac{V_s^2}{(2)(9.81)} = 0.066\ V_s^2$$

$$\frac{P_{atm} - P_{vap}}{\rho g} = z_s + \frac{V_s^2}{2g} + h_{f,s} + h_{L,s}$$

$$\frac{(101.33 - 2.34)\times 10^3}{(1000)(9.81)} = 10.0 + \frac{V_s^2}{(2)(9.81)} + 0.80 V_s^2 + 0.066\ V_s^2$$

$V_s = 0.315\ m/s = V_{pipe}$ (for continuity)

$$Q = V_s A_{pipe} = V_{pipe} A_{pipe} = (0.315)\left(\frac{\pi \times 0.95^2}{4}\right) = 0.223\ m^3/s$$

b) $P_{out} = 500\ kW$ (given), $Q = 0.223\ m^3/s$ (from a)

$P_{out} = \rho g H_p Q$

$$H_p = \frac{P_{out}}{\rho g Q} = \frac{500\times 10^3}{(1000)(9.81)(0.223)} = 228.56\ m$$

c) $H_p = 228.56\,m$ (from b) ; $P_s = ?$ $P_d = ?$

$P_s = P_{rap} = 2.34\ kN/m^2$ (from a)

$P_s(gage) = P_{rap} - P_{atm} = 2.34 - 101.33 = -98.99\ kN/m^2$

The pressure head at the suction side is

$$\frac{P_s}{\rho g} = \frac{-98.99 \times 10^3}{(1000)(9.81)} = \underline{-10.09\ m}$$

To determine the pressure head at the discharge side of the pump, apply Bernoulli's eq between points s and d:

$$\frac{P_s}{\rho g} + z_s + \frac{V_s^2}{2g} + H_p = \frac{P_d}{\rho g} + z_d + \frac{V_d^2}{2g}$$

$z_s = z_d = 10\ m$ and $V_s = V_d = 0.315\ m/s$ (for cont.)

$$\frac{-98.99 \times 10^3}{(1000)(9.81)} + 228.56 = \frac{P_d}{(1000)(9.81)}$$

$P_d = 2.14 \times 10^6\ N/m^2$

The pressure head at the discharge side is:

$$\frac{P_d}{\rho g} = \frac{2.14 \times 10^6}{(1000)(9.81)} = \underline{218.47\ m}$$

d) In order to determine the maximum allowable elevation of 2 to which water may be pumped, apply Bernoulli's eq. between 1 and 2:

$$\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} + H_p = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g} + h_{L(s+d)} + h_{f(s+d)}$$

$P_1 = P_2 = 0$ (atm.) ; $z_1 = 0$ (datum)

$V_{pipe} = 0.315\ m/s$ and $V_1 = V_2 = 0$ (surface)

The frictional head losses on both the suction and discharge lines are:

$$h_f(s+d) = \frac{\lambda L V_{pipe}^2}{2gD} \quad \text{where } L = 1300\,m,\ D = 0.95\,m$$

$$h_f(s+d) = \frac{(0.15)(1300)(0.315)^2}{(2)(9.81)(0.95)} = 1.038\,m$$

The minor head losses on both the suction and discharge lines are

$$h_L(s+d) = \Sigma k_L \frac{V_{pipe}^2}{2g} = (1 + 0.3 \times 3 + 1)\frac{(0.315)^2}{(2)(9.81)} = 0.015\,m$$

$$228.56 = z_2 + 1.038 + 0.015$$

$$z_2 = \underline{227.51\ m}$$

Q2. The pump–motor system in Question 1 is illustrated in Figure 2, where the hydraulic power delivered to the flow by the pump is 500 kW. The total electric input power supplied to the motor is 700 kW, and the rotating shaft of the motor has an angular velocity of 300 rad/sec and a shaft torque of 2000 N-m.

a). Determine the hydraulic input power delivered by the motor to the pump.

b). Determine the motor efficiency.

c). Determine the pump efficiency.

d). Determine the overall pump–motor system efficiency.

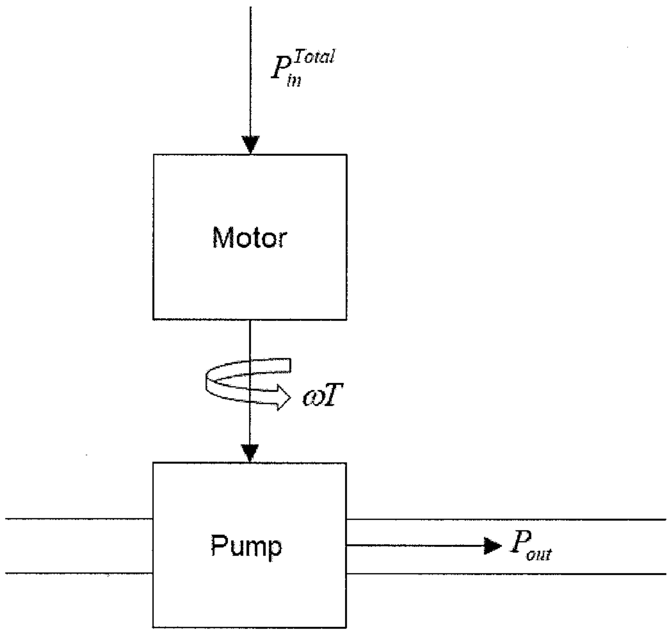


Figure 2

Solution:

a) $P_{in} = ?$ $\omega = 300\,rad/s$, $T = 2000\ N\cdot m$

$P_{in} = \omega T = (300)(2000) = 600\ 000\ W = 600\ kW$

b) $\xi = ?$ $P_{in}^{Total} = 700\ kW$, $P_{in} = 600\ kW$ (from a)

$$\xi = \frac{P_{in}}{P_{in}^{Total}} = \frac{600}{700} = 0.857 = 85.7\%$$

c) $\eta = ?$ $P_{out} = 500\ kW$, $P_{in} = 600\ kW$ (from a)

$$\eta = \frac{P_{out}}{P_{in}} = \frac{500}{600} = 0.833 = 83.3\%$$

d) Overall system efficiency = ?

$\eta_{s} = (0.857)(0.833) = 0.714 = 71.4\%$

Q3. A pump having the characteristics tabulated below delivers water from a river at elevation of 52 m to a reservoir with a water level elevation of 85 m through a 350 mm-diameter coated cast iron pipeline, 2000 m long (k_s = 0.15 mm). The minor losses in the system amount to 10 $V^2/2g$.

a). Determine the pump's operating condition (discharge, head, efficiency and power consumption);

b). Determine the operating condition, if a second identical pump is installed and the two pumps are connected in **parallel**;

c). Determine the operating condition, if a second identical pump is installed and the two pumps are connected in **series**;

d). Determine the operating condition when the **discharge is regulated** (by valve control) to **175 L/s** in the case of (i) **parallel** operation and (ii) **series** operation.

Assume $\nu = 1.13 \times 10^{-6}$ m²/s.

Q (L/s)	H_p (m)	η (%)
0	60.0	-
50	58.0	44
100	52.0	65
150	41.0	64
200	25.0	48

Solution;

First develop the system curve:

$$H_{SH} = H_s + \left(\frac{\lambda L}{D} + \sum k_L\right)\frac{V^2}{2g} = 33.0 + \left(\frac{\lambda \times 2000}{0.35} + 10\right)\frac{V^2}{2g}$$

$$k_s/D = \frac{0.15 \times 10^{-3}}{0.35} = 4.286 \times 10^{-4} \; ; \; H_s = 85.0 - 52.0 = 33.0 \text{ m}$$

Assume values of Q and calculate system head, H_{SH}:

<u>Q = 0</u>

$V = 0$; $\lambda = 0$; $Re = 0$, $h_f = 0$, $h_L = 0$

<u>Q = 50 L/s = 0.05 m³/s</u>

$$V = \frac{Q}{A} = \frac{0.05}{\left(\frac{\pi \times 0.35^2}{4}\right)} = 0.520 \text{ m/s}$$

$$Re = \frac{DV}{\nu} = \frac{(0.35)(0.520)}{1.13 \times 10^{-6}} = 1.61 \times 10^5$$

λ is calculated from the Moody formula:

$$\lambda = 0.0055 \left[1 + \left(\frac{20000\, ks}{D} + \frac{10^6}{Re} \right)^{1/3} \right] =$$

$$= 0.0055 \left[1 + \left(\frac{20000 \times 0.15 \times 10^{-3}}{0.35} + \frac{10^6}{1.61 \times 10^5} \right)^{1/3} \right] = 0.0190$$

$$h_L = \Sigma k_L \frac{V^2}{2g} = (10)\left(\frac{0.520^2}{2 \times 9.81}\right) = 0.138\ m \approx 0.14\ m$$

$$h_f = \frac{\lambda L V^2}{2gD} = \frac{(0.019)(2000)(0.520)^2}{(2)(9.81)(0.35)} = 1.49\ m$$

$H_{SH} = 33.0 + 0.14 + 1.49 = 34.63\ m$

Repeat the above steps for $Q = 100, 150, 200\ L/s$

a) Single pump operation

Plot the given Q vs H_p and Q vs η. Obtain the coordinates of the duty point (Q, H_p, η) as the point of intersection of the pump curve and the system curve. See plot:

$Q = 136\ L/s$; $H_p = 44\ m$; $\eta = 66\%$

$$P_{in}\ (\text{pump}) = \frac{P_{out}}{\eta} = \frac{\rho g H_p Q}{\eta} = \frac{(1000)(9.81)(44.0)(0.136)}{0.66} =$$

$$= \underline{88.94\ kW}$$

b) Two identical pumps in parallel operation

Plot double the given Q vs the pump head ($2Q$ vs H_p), and $2Q$ vs η. Obtain coordinates of duty point:

$Q = 182\ L/s$; $H_p = 53\ m$; $\eta = 62\%$

$$P_{in} = \frac{(1000)(9.81)(53.0)(0.182)}{0.62} = 152.62\ kW$$

c) Two identical pumps in series operation
Plot given Q vs double the pump head (Q vs $2H_p$) and Q vs η. Obtain duty point coordinates:

$Q = 192\ L/s$; $H_p = 55\ m$; $\eta = 50\%$

$$P_{in} = \frac{(1000)(9.81)(55.0)(0.192)}{0.5} = 207.19\ kW$$

d) As the discharge is regulated to $175\ L/s$, obtain the point of intersection of the $Q = 175\ L/s$ line and the respective pump curves:

i) Pumps in parallel operation.

$Q = 175 L/s$; $H_p = 54\ m$; $\eta = 62\%$

$$P_{in} = \frac{(1000)(9.81)(54.0)(0.175)}{0.62} = 149.52\ kW$$

ii) Pumps in series operation

$Q = 175\ L/s$; $H_p = 67\ m$; $\eta = 57\%$

$$P_{in} = \frac{(1000)(9.81)(67)(0.175)}{0.57} = 201.79\ kW$$

Two pumps in series

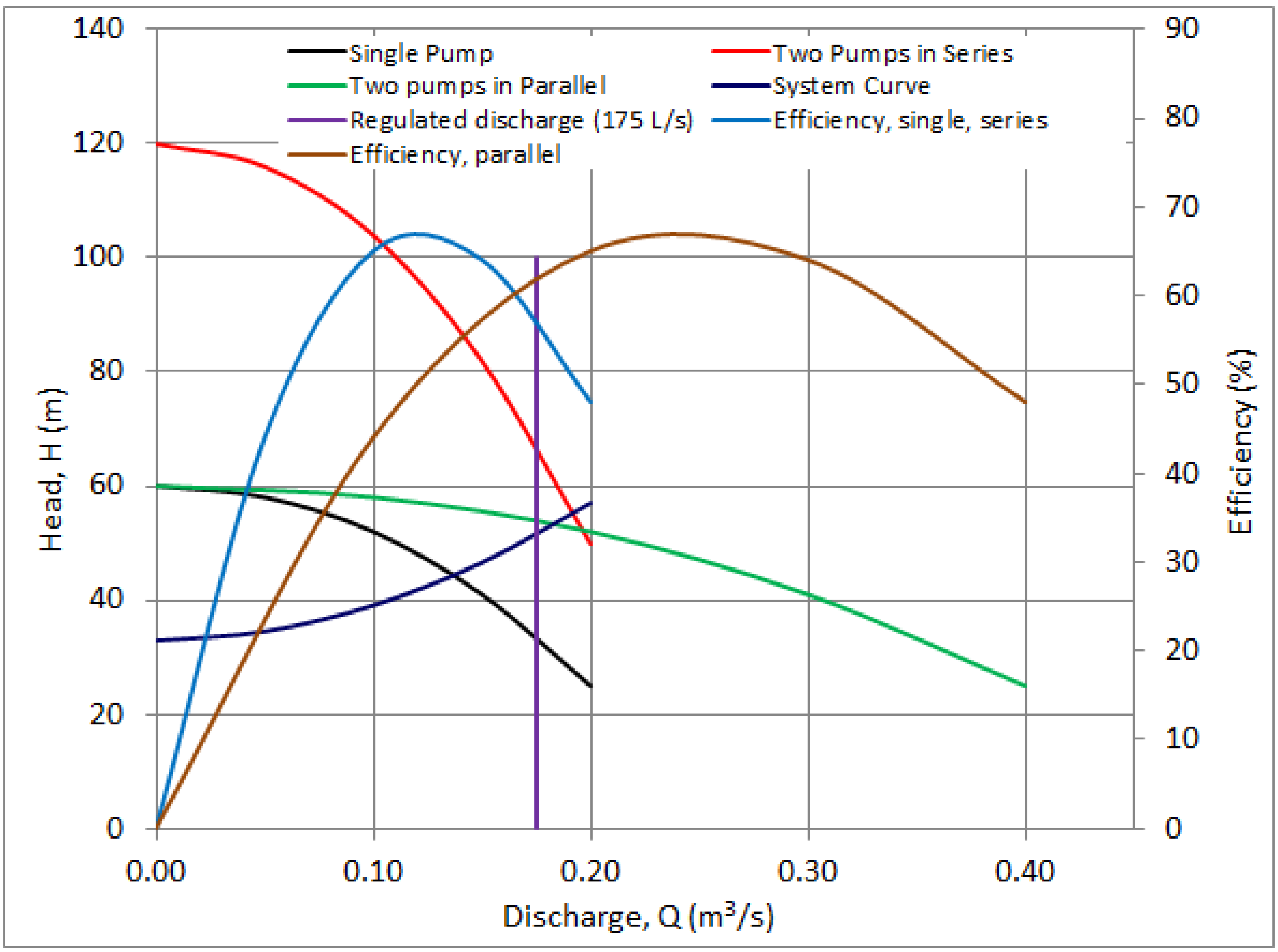
Q (m^3/s)	H_p (m)	η (%)	$2H_p$ (m)	v (m/s)	Re (-)	λ (-)	h_f (m)	h_L (m)	H_s (m)	H_{SH} (m)
0.00	60.00	-	120.00	0.000	0.00E+00	0.0000	0.00	0.00	33.00	33.00
0.05	58.00	44.00	116.00	0.520	1.61E+05	0.0190	1.49	0.14	33.00	34.63
0.10	52.00	65.00	104.00	1.039	3.22E+05	0.0180	5.66	0.55	33.00	39.21
0.15	41.00	64.00	82.00	1.559	4.83E+05	0.0176	12.46	1.24	33.00	46.70
0.20	25.00	48.00	50.00	2.079	6.44E+05	0.0174	21.90	2.20	33.00	57.10

ν = 1.13E-06 m^2/s
L = 2000 m
D = 0.35 m
Σk_L = 10
ks = 1.50E-04 m

Two pumps in parallel

Q (m^3/s)	H_p (m)	η (%)	2Q (m^3/s)	v (m/s)	Re (-)	λ (-)	h_f (m)	h_L (m)	H_s (m)	H_{SH} (m)
0.00	60.00	-	0.00	0.00	0.00E+00	0.0000	0.00	0.00	33.00	33.00
0.05	58.00	44.00	0.10	0.52	1.61E+05	0.0190	1.49	0.14	33.00	34.63
0.10	52.00	65.00	0.20	1.04	3.22E+05	0.0180	5.66	0.55	33.00	39.21
0.15	41.00	64.00	0.30	1.56	4.83E+05	0.0176	12.46	1.24	33.00	46.70
0.20	25.00	48.00	0.40	2.08	6.44E+05	0.0174	21.90	2.20	33.00	57.10

ν = 1.13E-06 m^2/s
L = 2000 m
D = 0.35 m
Σk_L = 10
ks = 1.50E-04 m



Q4. A pump is required to deliver 125 L/s of water from reservoir *A* with water surface elevation of 385.7 m to reservoir *B* with water surface elevation of 402.5 m. The pipeline (concrete, k_s = 0.36 mm) is 300 m long with a diameter of 0.20 m. The local losses on the pipeline amount to 2.9 $V^2/2g$. Use figures 3 and 4 to determine the appropriate pump model and operating conditions (speed, discharge, head, efficiency and power consumption).

Assume $\nu = 1.13 \times 10^{-6}$ m²/s.

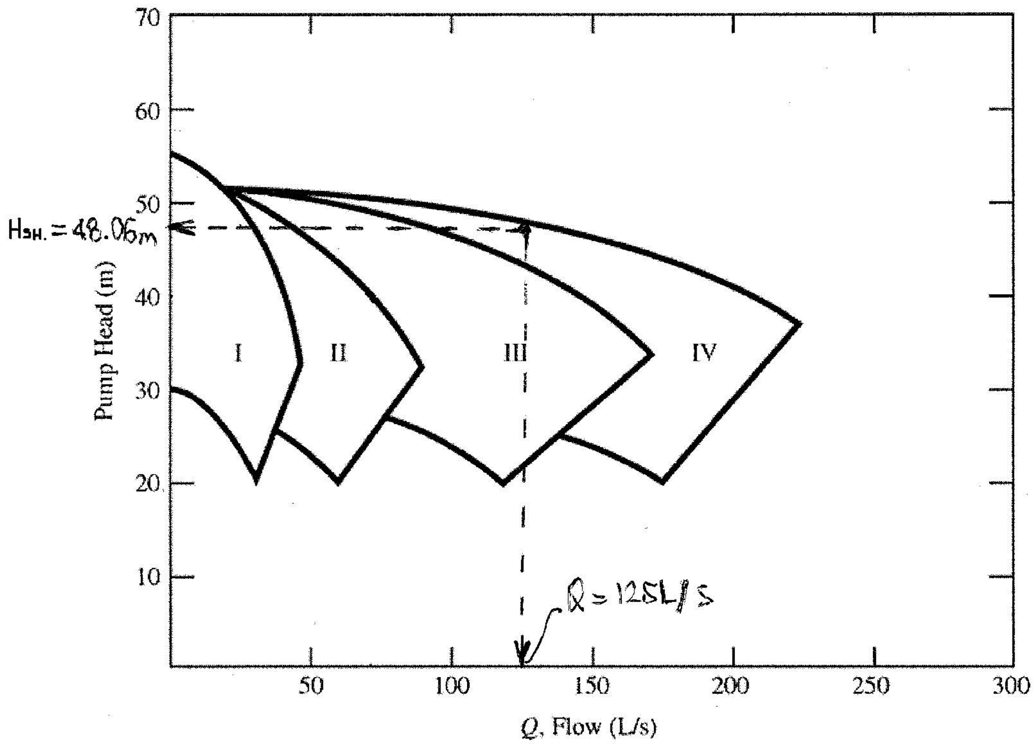


Figure 3 Pump model selection chart (Houghtalen et al., 2010)

Solution:

The pump must generate a total head equal to the static head (elevation difference) plus the pipeline friction and local head losses. Develop the system curve:

$$H_{SH} = H_s + \left(\frac{\lambda L}{D} + \Sigma k_L\right)\frac{V^2}{2g}, \text{ where}$$

$H_s = 402.5 - 385.7 = 16.8$ m ; $L = 300$ m ; $D = 0.20$ m

$\Sigma k_L = 2.9$

$$H_{SH} = 16.8 + \left(\frac{\lambda \times 300}{0.2} + 2.9 \right) \frac{V^2}{(2)(9.81)}$$

Assume values of Q and calculate H_{SH} :

$Q = 0\ L/s = 0\ m^3/s$

$V = 0$; $h_L = 0$ $H_{SH} = H_s = 16.8\ m$

$Q = 25\ L/s = 0.025\ m^3/s$

$$V = \frac{Q}{A} = \frac{0.025}{\left(\frac{\pi \times 0.2^2}{4}\right)} = 0.796\ m/s$$

$$Re = \frac{DV}{\nu} = \frac{(0.2)(0.796)}{1.13 \times 10^{-6}} = 1.41 \times 10^5$$

$$\frac{k_s}{D} = \frac{0.36 \times 10^{-3}}{0.2} = 1.8 \times 10^{-3}$$

$$\lambda = 0.0055 \left[1 + \left(\frac{20000\ k_s}{D} + \frac{10^6}{Re} \right)^{1/3} \right] =$$

$$= 0.0055 \left[1 + \left(20000 \times 1.8 \times 10^{-3} + \frac{10^6}{1.41 \times 10^5} \right)^{1/3} \right] = 0.0248$$

$$h_f = \frac{\lambda L V^2}{2gD} = \frac{(0.0248)(300)(0.796)^2}{(2)(9.81)(0.2)} = 1.20\ m$$

$$h_L = \Sigma k_L \frac{V^2}{2g} = (2.9) \frac{(0.796)^2}{(2)(9.81)} = 0.09\ m$$

$H_{SH} = 16.8 + 1.20 + 0.09 = 18.09\ m$

Repeat previous steps until enough points on the system curve are obtained.

Note that for $Q = 125\ L/s$, $H_{SH} = 48.06\ m$.

Plot the point with coordinates ($Q = 125\ L/s$, $H_p = 48.06\ m$) on Figure 3. Pump model IV is the logical choice.

Plot the system curve on the chart for Pump IV on Figure 4. The pump can deliver the required $Q = 125\ L/s$ against a head of $H_p = 48.10\ m > H_{SH} = 48.06\ m$ if it operates at a speed of 4350 rpm.

The final pump operating conditions are:

Speed = 4350 rpm

$Q = 125\ L/s$ $\eta = 61\%$

$H = 48.1\ m$ HP input = 135

System Curve

Q (L/s)	Q (m^3/s)	v (m/s)	R_e (-)	λ (-)	h_f (m)	h_L (m)	H_s (m)	H_{SH} (m)
0.00	0.000	0.000	0.00E+00	0.0000	0.00	0.00	16.80	16.80
25.00	0.025	0.796	1.41E+05	0.0248	1.20	0.09	16.80	18.09
50.00	0.050	1.592	2.82E+05	0.0242	4.69	0.37	16.80	21.87
75.00	0.075	2.387	4.23E+05	0.0241	10.48	0.84	16.80	28.12
100.00	0.100	3.183	5.63E+05	0.0240	18.56	1.50	16.80	36.85
125.00	0.125	3.979	7.04E+05	0.0239	28.92	2.34	16.80	48.06
150.00	0.150	4.775	8.45E+05	0.0239	41.58	3.37	16.80	61.75
175.00	0.175	5.570	9.86E+05	0.0238	56.53	4.59	16.80	77.92
200.00	0.200	6.366	1.13E+06	0.0238	73.77	5.99	16.80	96.56
225.00	0.225	7.162	1.27E+06	0.0238	93.30	7.58	16.80	117.68

ν = 1.13E-06 m^2/s
L = 300 m
D = 0.2 m
Σk_L = 2.9
k_s = 0.00036 m
k_s/D = 1.80E-03

Pump, IV