

.MATH 1300B-MIDTERM # 1-2011.

NAME and I.D.# Solutions and Marking Scheme.

Instructions: This midterm exam consists of 4 multiple choice questions and 3 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 60 points.

Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages.

For long answer questions, YOU MUST SHOW YOUR WORK.

NO CALCULATORS. NO BOOKS. NO NOTES.

If you need additional scrap paper, it will be provided by the proctors.

Multiple Choice Answers:

B

#1

B

#2

C

#3

B

#4

Multiple Choice Questions (1-4)

Question 1 For what values of x is the following function continuous?

$$f(x) = \begin{cases} x-2 & \text{if } x < -1 \\ x^2 - 2x + 1 & \text{if } -1 \leq x \leq 2 \\ 3-x & \text{if } x > 2 \end{cases}$$

- A) all real numbers B) all real numbers, except -1 C) all real numbers, except 2
 D) all real numbers, except -1 and 2 E) all real numbers, except -1, 2 and 0.

For $x = -1$

$$\lim_{x \rightarrow -1^-} f(x) = -3 \neq \lim_{x \rightarrow -1^+} f(x) = 4.$$

So f is discontinuous

@ $x = -1$

f is continuous everywhere else.

For $x = 2$

$$\lim_{x \rightarrow 2^-} f(x) = 1 = \lim_{x \rightarrow 2^+} f(x)$$

$$f(2) = 1 = \lim_{x \rightarrow 2} f(x)$$

f is continuous

@ $x = 2$.

Question 2 Find the equation of the tangent line of the function

$$f(x) = x\sqrt{2x+5} \text{ at } x = 2.$$

- A) $y = \frac{13}{3}x - \frac{8}{3}$ B) $y = \frac{11}{3}x - \frac{4}{3}$ C) $y = \frac{17}{3}x - \frac{2}{3}$ D) $y = \frac{13}{3}x + \frac{7}{3}$
 E) $y = \frac{11}{3}x + \frac{2}{3}$

By product Rule $f'(x) = \sqrt{2x+5} + x \cdot 2 \left(\frac{1}{2\sqrt{2x+5}} \right)$

$$f'(2) = 3 + \frac{2}{3} = \frac{11}{3}$$

$$f(2) = 2\sqrt{9} = 6$$

$$y - 6 = \frac{11}{3}(x - 2)$$

$$y = \frac{11}{3}x - \frac{22}{3} + 6$$

$$y = \frac{13}{3}x - \frac{4}{3}$$

Question 3 Assuming that a bank is paying 5% annual interest which is compounded continuously, find the time needed for an initial deposit of 5,000 dollars to triple.

- A) $20 \ln(2)$ years B) $10 \ln(2)$ years C) $20 \ln(3)$ years D) $10 \ln(3)$ years
 E) $16 \ln(4)$ years

$$3P = P e^{rt}$$

$$3 = e^{0.05t}$$

$$\Rightarrow \ln 3 = 0.05t$$

$$t = \frac{\ln 3}{5 \times 10^{-2}} = 20 \ln 3$$

Question 4 Find the following limit.

$$\lim_{x \rightarrow 4^-} \frac{|x-4|}{x-4}$$

(Note: This is a one-sided limit)

- A) 1 B) -1 C) 0 D) ∞ E) The limit does NOT exist.

$$|x-4| = \begin{cases} x-4 & x \geq 4 \\ -(x-4) & x < 4 \end{cases}$$

$$\lim_{x \rightarrow 4^-} \frac{|x-4|}{x-4} = \lim_{x \rightarrow 4^-} \frac{-(x-4)}{x-4} = -1$$

Long Answer Questions (5-7)

Question 5 (14 points)

Using only the definition of derivative as a limit, calculate $f'(x)$ where

$$f(x) = \sqrt{x-2}.$$

Solution

4 points

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} =$$
$$\lim_{\Delta x \rightarrow 0} \frac{\sqrt{x+\Delta x-2} - \sqrt{x-2}}{\Delta x} =$$

4 points

Rationalize:

$$= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x+\Delta x-2} - \sqrt{x-2}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x-2} + \sqrt{x-2}}{\sqrt{x+\Delta x-2} + \sqrt{x-2}}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{x+\Delta x-2 - (x-2)}{\Delta x (\sqrt{x+\Delta x-2} + \sqrt{x-2})}$$

4 points

$$= \lim_{\Delta x \rightarrow 0} \frac{\cancel{\Delta x}}{\cancel{\Delta x} (\sqrt{x+\Delta x-2} + \sqrt{x-2})} = \frac{1}{2\sqrt{x-2}}$$

2 points

Question 6 (a)(6 points) The cost of producing x units of a product is given by

$$C(x) = \frac{4x}{2 + \sqrt{x}},$$

where $C(x)$ is the cost in dollars. Find the marginal cost for producing 4 units.

Solution

$$C'(4) = ?$$

Q. R \rightarrow $C'(x) = \frac{4(2 + \sqrt{x}) - 4x \left(\frac{1}{2\sqrt{x}}\right)}{(2 + \sqrt{x})^2} = \frac{4(4) - 16\left(\frac{1}{4}\right)}{16}$

\nearrow 4 points

$= \frac{16 - 4}{16} = \frac{12}{16} = \frac{3}{4}$

(b)(6 points) Solve the following logarithmic equation.

$$\ln(x) - \ln(x - 1) = 1$$

Solution

$$\ln(x) - \ln(x-1) = \ln \frac{x}{x-1} = \ln e$$

$\underbrace{\hspace{10em}}_{3 \text{ points}}$

\ln is
1-1
function \rightarrow

$$\frac{x}{x-1} = e$$

$\{x > 1\}$

~~$x = 1$~~

$$x = e(x-1)$$

$$x - ex = -e$$

$$x(1-e) = -e$$

or $\begin{cases} x = \frac{-e}{1-e} \\ x = \frac{e}{e-1} \end{cases}$

\updownarrow 3 points

Question 7 (14 points)

The function $y = f(x)$ is defined implicitly by

$$x^2 + 2y^2 = 3xy.$$

Find the equation of the tangent line to the graph determined by the above equation at $(1, 1)$.

Solution

8 points

$$\frac{d}{dx}(x^2 + 2y^2) = \frac{d}{dx}(3xy)$$
$$2x + 4y \frac{dy}{dx} = 3\left[y + x \frac{dy}{dx}\right]$$
$$\frac{dy}{dx}[4y - 3x] = 3y - 2x$$
$$\frac{dy}{dx} = \frac{3y - 2x}{4y - 3x} \Big|_{x=1} = \frac{1}{1} = 1.$$

2 points

4 points

$$y - 1 = 1(x - 1) \Rightarrow$$
$$\underline{y = x}$$

Space for additional work