1. The driver of a car starts to brake when she sees a tree blocking the road. The car slows down uniformly with an acceleration of –5.60 m/s² for 4.20 s, making straight skid marks 62.4 m long ending at the tree. With what speed does the car then strike the tree?

SOLUTION:
In the simultaneous equations:

$$\left\{\begin{array}{c} v_{xf} = v_{xi} + a_x t \\ x_f - x_i = \frac{1}{2}\left(v_{xi} + v_{xf}\right)t \end{array}\right\} \text{ we have } \left\{\begin{array}{c} v_{xf} = v_{xi} - \left(5.60 \text{ m/s}^2\right)\left(4.20 \text{ s}\right) \\ 62.4 \text{ m} = \frac{1}{2}\left(v_{xi} + v_{xf}\right)\left(4.20 \text{ s}\right) \end{array}\right\}.$$

So substituting for v_{xi} gives $62.4 \text{ m} = \frac{1}{2}\left[v_{xf} + \left(56.0 \text{ m/s}^2\right)\left(4.20 \text{ s}\right) + v_{xf}\right]\left(4.20 \text{ s}\right)$

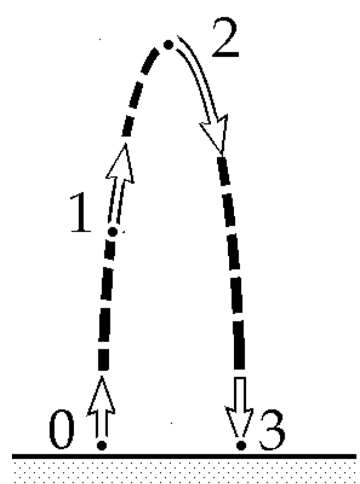
$$14.9 \text{ m/s} = v_{xf} + \frac{1}{2}\left(5.60 \text{ m/s}^2\right)\left(4.20 \text{ s}\right).$$

Thus

$$v_{xf} = \boxed{3.10 \text{ m/s}}.$$

2 A test rocket is fired vertically upward from a well. A catapult gives it initial velocity 80.0 m/s at ground level. Its engines then fire and it accelerates upward at 4.00 m/s² until it reaches an altitude of 1 000 m. At that point its engines fail and the rocket goes into free fall, with an acceleration of –9.80 m/s². (a) How long is the rocket in motion above the ground? (b) What is its maximum altitude? (c) What is its velocity just before it collides with the Earth? (You will need to consider the motion while the engine is operating separate from the free-fall motion.

SOLUTION:
Let point 0 be at ground level and point 1 be at the end of the engine burn. Let point 2 be the highest point the rocket reaches and point 3 be just before impact.
Below are the data found for each phase of the rocket's motion.



(0 to 1) $v_f^2 - (80.0)^2 = 2(4.00)(1\,000)$ so $v_f = 120 \text{ m/s}$

$120 = 80.0 + (4.00)t$ giving $t = 10.0 \text{ s}$

(1 to 2) $0 - (120)^2 = 2(-9.80)(x_f - x_i)$ giving $x_f - x_i = 735 \text{ m}$

$0 - 120 = -9.80t$ giving $t = 12.2 \text{ s}$

This is the time of maximum height of the rocket.

(2 to 3) $v_f^2 - 0 = 2(-9.80)(-1\,735)$

$v_f = -184 = (-9.80)t$ giving $t = 18.8 \text{ s}$

(a) $t_{\text{total}} = 10 + 12.2 + 18.8 = \boxed{41.0 \text{ s}}$

(b) $\left(x_f - x_i\right)_{\text{total}} = \boxed{1.73 \text{ km}}$

(c) $v_{\text{final}} = \boxed{-184 \text{ m/s}}$

		t	*x*	*v*	*a*
0	Launch	0.0	0	80	+4.00
#1	End Thrust	10.0	1 000	120	+4.00
#2	Rise Upwards	22.2	1 735	0	–9.80
#3	Fall to Earth	41.0	0	–184	–9.80

4 The height of a helicopter above the ground is given by $h = 3.00t^3$, where h is in meters and t is in seconds. After 2.00 s, the helicopter releases a small mailbag. How long after its release does the mailbag reach the ground?

SOLUTION: $y = 3.00t^3$: At $t = 2.00$ s, $y = 3.00(2.00)^3 = 24.0$ m and

$$v_y = \frac{dy}{dt} = 9.00t^2 = 36.0 \text{ m/s} \uparrow .$$

If the helicopter releases a small mailbag at this time, the equation of motion of the mailbag is

$$y_b = y_{bi} + v_i t - \frac{1}{2}gt^2 = 24.0 + 36.0t - \frac{1}{2}(9.80)t^2 .$$

Setting $y_b = 0$,

$$0 = 24.0 + 36.0t - 4.90t^2 .$$

Solving for t, (only positive values of t count), $\boxed{t = 7.96 \text{ s}}$.

5 A rock is dropped from rest into a well. (a) The sound of the splash is heard 2.40 s after the rock is released from rest. How far below the top of the well is the surface of the water? The speed of sound in air (at the ambient temperature) is 336 m/s. (b) **What If?** If the travel time for the sound is neglected, what percentage error is introduced when the depth of the well is calculated?

(a) $d = \frac{1}{2}(9.80)t_1^2$ $\qquad d = 336t_2$

$t_1 + t_2 = 2.40$ $\qquad 336t_2 = 4.90(2.40 - t_2)^2$

$4.90t_2^2 - 359.5t_2 + 28.22 = 0$

$$t_2 = \frac{359.5 \pm \sqrt{359.5^2 - 4(4.90)(28.22)}}{9.80}$$

$t_2 = \frac{359.5 \pm 358.75}{9.80} = 0.0765$ s $\qquad$ so $\qquad d = 336t_2 = \boxed{26.4 \text{ m}}$

(b) Ignoring the sound travel time, $d = \frac{1}{2}(9.80)(2.40)^2 = 28.2$ m, an error of.

$\boxed{6.82\%}$

6 Two railroad tracks intersect at right angles at station O. At 11AM the train A, moving west with constant speed of 50 km/h, leaves the station O. One hour later train B, moving south with the constant speed of 60 km/h, passes through the station O. Find minimum distance between these trains.SOLUTION: BELOW FOR vA=50km/h and vB=70km/h

Train A moves along z axis and at time t it will have position: $x_A = V_A t = 50t$

Train B moves along the y axis and at time t it will have position: $y_B = 60 - V_B t = 60 - 60t$

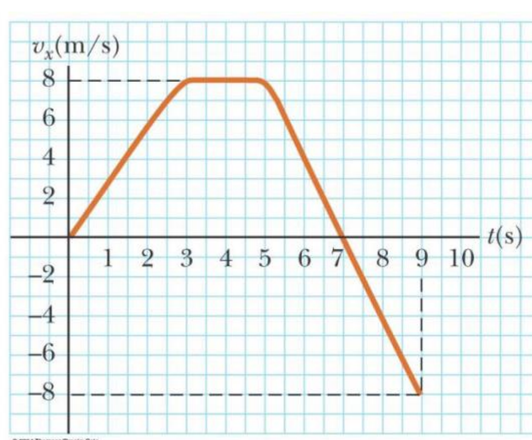
The distance between the two trains is given by: $D = \sqrt{x_A^2 + y_B^2} = \sqrt{(50t)^2 + (60 - 60t)^2}$

The minimum distance is given by the condition:

$$\frac{dD}{dt} = 0 \Rightarrow \frac{1}{2}\frac{1}{\sqrt{2500t^2 + (60 - 60t)^2}} \times [2(2500t) + 2(60 - 60t)(-60)] = 0$$

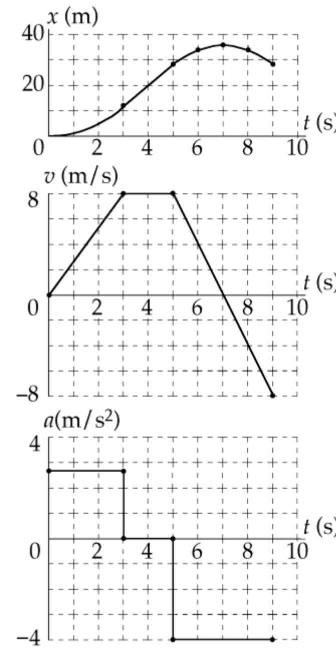
$$(2500t) + (60 - 60t)(-60) = 0 \Rightarrow (2500 + 3600)t = 3600 \Rightarrow t = \frac{3600}{6100} = \frac{36}{61}(hr) = 35.41\text{min} = 35\text{min}\,24.6\text{sec}$$

3 A student drives a moped along a straight road as described by the velocity-versus-time graph. Sketch this graph in the middle of a sheet of graph paper. (a) Directly above your graph, sketch a graph of the position versus time, aligning the time coordinates of the two graphs. (b) Sketch a graph of the acceleration versus time directly below the v_x-t graph, again aligning the time coordinates. On each graph, show the numerical values of x and a_x for all points of inflection. (c) What is the acceleration at $t = 6$ s? (d) Find the position (relative to the starting point) at $t = 6$ s. (e) What is the moped's final position at $t = 9$ s?



Q3 SOLUTIONS:

(a) See the graphs at the right.



Choose $x = 0$ at $t = 0$.

At $t = 3\text{ s}$, $x = \frac{1}{2}(8\text{ m/s})(3\text{ s}) = 12\text{ m}$.

At $t = 5\text{ s}$, $x = 12\text{ m} + (8\text{ m/s})(2\text{ s}) = 28\text{ m}$.

At $t = 7\text{ s}$, $x = 28\text{ m} + \frac{1}{2}(8\text{ m/s})(2\text{ s}) = 36\text{ m}$.

(b) For $0 < t < 3\text{ s}$, $a = \frac{8\text{ m/s}}{3\text{ s}} = 2.67\text{ m/s}^2$.

For $3 < t < 5\text{ s}$, $a = 0$.

(c) For $5\text{ s} < t < 9\text{ s}$, $a = -\frac{16\text{ m/s}}{4\text{ s}} = \boxed{-4\text{ m/s}^2}$.

(d) At $t = 6\text{ s}$, $x = 28\text{ m} + (6\text{ m/s})(1\text{ s}) = \boxed{34\text{ m}}$.

(e) At $t = 9\text{ s}$, $x = 36\text{ m} + \frac{1}{2}(-8\text{ m/s})(2\text{ s}) = \boxed{28\text{ m}}$.